

Name:

IN THIS LESSON, YOU WILL:

- Understand the limitations of two-way tables
- Understand sampling error and how sample size impacts accuracy
- Investigate selection bias and its effect on statistical error
- Explore four sampling techniques to reduce selection bias
- Understand the p-value in the context of hypothesis testing
- Interpret conclusions correctly for hypothesis testing
- Explore different factors in college admissions and scholarship awards



INTRO

A New Treatment

Imagine you're a medical researcher testing a new treatment. You have two groups: 500 patients in Treatment 1 and 500 in Treatment 2. Here is the recovery data:

Treatment	Recovered
Treatment 1	300
Treatment 2	200

1. What percent of patients recovered using each treatment?
2. What recommendation would you give based on this data? Explain your reasoning.



LEARN IT

Sampling Error

We use samples instead of entire populations because it's easier and faster. Sample measurements are called **statistics**. These statistics give our best estimate of the true population values, called **parameters**. Parameters are usually hard to measure directly, but they're important because they help us make predictions.

Common Variables for Sample Statistics and Population Parameters

Measurement	Sample	Population
Number of items	n	N
Mean	$\bar{x}$	μ
Standard deviation	s	σ
Correlation	r	ρ

You can take many different samples from a population. Each sample gives a slightly different estimate. This difference between the sample and the actual population value is called **sampling error**. You can't fully avoid it, but taking a larger sample typically reduces sampling error and makes your estimate more accurate.

Example:

To find the average starting salary for a Bachelor's graduate, you could:

- Ask **1,000 recent graduates** for their starting salaries and calculate the average.
- Ask **only your brother** who recently graduated

1. **Which of the two samples would you put more trust in, the 1000 students or your brother? Explain.**
2. **Let's say that you asked a different set of 1000 students from that same university their salaries. Would you expect that the average for this second group was close to the average for the first? Why or why not?**
3. **Instead of your brother, you ask one other random graduate their starting salary. Would you expect their salary to be close to your brother's salary? Why or why not?**

In this example, you're not interested in just your brother's salary or even the specific group you survey. You want to predict the average salary for **any graduate**.

In statistics, always note if you're working with a **sample** (a smaller group) or the **population** (the whole group). This helps you understand how reliable your prediction might be.



PRACTICE IT - Think - Pair - Share

You need to proceed carefully when selecting a sample to represent a whole population. Try to come up with a GOOD example of BAD sampling.

1. **In the space below, write down an example of a population and specific sample of that population.**
 2. **How might using your sample lead to sampling error?**
 3. **Turn to your partner and share your example. Write down their sample then work together to create a new LARGER sample and explain why that newer sample might lead to smaller sampling error.**
-

Selection Bias

Selection bias happens when the sample doesn't accurately represent the whole population. This can affect the results by making one outcome look more likely than it is.

Example

Let's go back to our treatment scenario from the Intro. Imagine that most severe cases were assigned to Treatment 2, while Treatment 1 had mostly minor cases. This could make Treatment 1 look better simply because it had easier cases.

- Of the 500 patients assigned Treatment 1, 400 had a minor case and 100 had a major case.
- Of the 500 patients assigned Treatment 2, 50 had a minor case and 450 had a major case.

1. **What percentage of the patients with treatment 1 had a major case of the illness?**

2. What percentage of the patients with treatment 2 had a major case of the illness?

We can see then that an unequal percentage of patients with a serious case were assigned treatment 2. Below are the recovery numbers for each sub-group/treatment pair.

Treatment	Minor illness recovery	Major illness recovery
1	280	20
2	38	162

1. Calculate the recovery % and complete the table for the patients divided by subgroups.

Treatment	Minor illness recovery	Major illness recovery
1		
2		

2. Would your conclusions about treatment 1 and treatment 2 change? Why?

ARTICLE: [Sampling Methods | Types, Techniques & Examples](#)

When taking samples to make accurate claims about a population, it's important to avoid selection bias. Here are four sampling methods to help ensure your sample is meaningful. Each method has its own strengths, so choose the one that best fits your needs. Read the article up to "Non-probability sampling methods" and answer the questions.

1. Fill in this Summary chart for the four types of random sampling techniques from the article

Sampling Method	Description	Example	Limitations
Simple Random 	Assigns random numbers to each item, then selects the first n items up to the sample size		Does not guarantee representation of all groups
Systematic 	Leaves items ordered, then samples via a structured method from the whole group		Does not guarantee representation of all groups; could exaggerate an unknown pattern
Stratified 	Divides items into groups based on similarities, then samples from each group		May under-represent large subgroups or overrepresent small subgroups
Cluster 	Divides items into groups that represent the population, then chooses one whole group as a sample		May not have large enough population to match exact representation in each group

2. Choose ONE sampling method and describe in your own words:

a. One advantage of using that particular technique

b. Give an example of a scenario that fits this sampling method

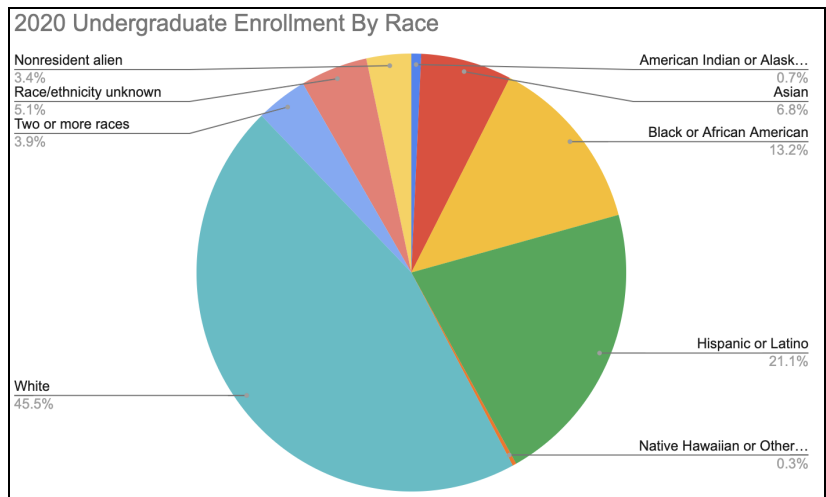


PRACTICE IT

Dealing with Selection Bias

Lyra wants to find the average scholarships awarded to American college students and ensure her sample reflects U.S. college demographics. She surveys her 48-person senior class using cluster sampling, dividing them into four groups.

Race/Ethnicity	Percentage
White	45.5%
Black or African American	13.2%
Hispanic or Latino	21.1%
Asian	6.8%
American Indian or Alaskan Native	0.7%
Hawaiian or Other Pacific Islander	0.3%
Two or More Races	3.9%
Nonresident Alien	3.4%
Race/Ethnicity Unknown	5.1%



1. How does cluster sampling help create a good estimate of the population?
2. What challenge might Lyra face using cluster sampling with her 48-person class?
3. Which sampling method would you recommend to Lyra and why?
4. Lyra says, "I want a completely unbiased sample so no one can challenge my study!" How would you respond?



Hypothesis Testing

Your friend believes 50% of students at your college are business majors. You suspect it's less. To test this, you survey 100 students and use **hypothesis testing** to see if the data supports your suspicion.

How Hypothesis Testing Works:

Set up your Null Hypothesis (H_0): This is the claim or assumption we start with, which we'll try to disprove if the evidence suggests otherwise. Here, H_0 is that 50% of students are business majors (your friend's claim).

Set up your Alternative Hypothesis (H_1): This is the opposite of the null and what you're trying to support. For our example, H_1 is that fewer than 50% of students are business majors (your belief).

Why Both Hypotheses?

The null hypothesis acts as a starting point. If your data strongly suggests that fewer than 50% of students are business majors, you'll have reason to reject H_0 in favor of H_1 . However, if the data doesn't show a strong difference, we'll keep H_0 as a likely explanation. Hypothesis testing is a way to measure if the observed difference (in this case, the percentage of business majors) is meaningful.

Example

For your sample of 100 community college students, you can express the null and alternative hypotheses as follows:

- $H_0: \mu = 50$ students
- $H_1: \mu < 50$ students (Note: $H_1: \mu \neq 50$ and $H_1: \mu > 50$ are different alternative hypotheses)

Identify the Hypotheses

1. **You read in a Forbes article that the median student loan debt for college borrowers was \$28,950. From what you've heard from family, that seems too low to you.**
 - a. $H_0 =$
 - b. $H_1 =$
2. **You saw a statistic from the Education Data Initiative that college tuition increases by an average of 8% each year. You don't know if that's too high or too low, but you think that it's definitely not 8%.**
 - a. $H_0 =$
 - b. $H_1 =$

VIDEO: [Understanding the p-Value and What It Tells Us](#)

Whenever you take a sample, you introduce sampling error. But how can you tell if your results happened by chance or because the null hypothesis is wrong? A **p-value** helps determine if your results are statistically significant. A low p-value means your sample is different enough to suggest the null hypothesis might be incorrect. Watch the video to learn more about p-values in action, then answer the questions.

1. Sample statistics may possibly differ from the population parameters by mere chance. The p-value tells us the probability that ____

EXAMPLE

Let's consider the example from the video on Choconutty bars. Use the video to answer these questions.

2. Write the null and alternative hypotheses for this Choconutty test.
 - a. H_0 :
 - b. H_1 :
3. Assuming the null hypothesis is true, would it become increasingly more likely or increasingly less likely to find a sample mean weight further and further away from 70g?
4. What would happen to the p-value of your test as the mean of the samples gets further and further away from 70g?
5. The significance level is the threshold we choose for how unlikely is unlikely enough to reject the null hypothesis. The most common level of significance is 0.05, which means that if the p-value tells us there is a less than a 5% chance, then we can be reasonably sure (95% confidence) it was NOT just chance, but in fact the null hypothesis is wrong. Give an example where you might either want more or accept less confidence in your test.

Making Appropriate Claims With Statistics

You have to be careful about how you report results from hypothesis testing. Here are some of the claims you can and cannot make with statistical testing.

If your test fails to reject the null hypothesis:

It's possible that another sample WOULD reject the null hypothesis, even if your particular sample did not. After all, we don't know for sure if the null hypothesis is true or if we just haven't found the right sample yet. The following are ways to phrase your conclusion:

- Fail to reject null hypothesis
- The alternative hypothesis is not likely given this sample data
- We have insufficient evidence to indicate the null hypothesis is wrong
- The sample data are not statistically significantly different than expected by the null hypothesis

If your test rejects the null hypothesis:

Similarly, you must be careful about the conclusions you draw being limited to statistical likelihood when your sample disagrees with the null hypothesis. Here are some ways to phrase your conclusion:

- Data indicates the alternative is likely true
- We have 95% confidence that the null hypothesis is false
- The truth of the null hypothesis is statistically unlikely

- The sample's difference from the null hypothesis is statistically significant at the 5% level

Phrases that you should NOT use include:

- The alternative hypothesis must be true
- The null hypothesis is false
- The null hypothesis is true



PRACTICE IT

Hypothesis Testing and p-values

Answer each question in the space provided to show your understanding of hypothesis testing and p-values.

- 1. Imagine you're curious about the average number of hours students sleep per night. You believe it's lower than the national average of 7 hours. Write the null and alternative hypotheses for this scenario.**
- 2. Your friend claims that at least 70% of college students have a part-time job. Based on your own experience, you think it might be less. How would you set up the null and alternative hypotheses?**
- 3. A study claims that the average monthly rent for apartments in your city is \$1,500. You're suspicious that it might actually be higher. Write the null and alternative hypotheses.**
- 4. Imagine you conducted a hypothesis test about the average commute time of college students and obtained a p-value of 0.03. Interpret this p-value in the context of a significance level of 0.05.**
- 5. If a sample shows that only 48% of surveyed students prefer online learning (with a p-value of 0.08), should you reject the null hypothesis that 50% of students prefer online learning? Assume a significance level of 0.05. Explain your answer.**

6. Suppose you tested whether the average score on a final exam for a sample of students is higher than the passing mark of 50. Your p-value was 0.02. What is the appropriate way to report this finding? Choose one of the following:
- a. The null hypothesis is false, so students scored higher than 50.
 - b. The sample's difference from the passing mark of 50 is statistically significant at the 5% level.
 - c. The alternative hypothesis must be true.
7. Imagine you performed a test and failed to reject the null hypothesis. Does this mean the null hypothesis is true? Explain why or why not.



APPLY IT

Follow your teacher's directions to complete the Application Problems.

Teachers, you can find the Application problems linked in the Lesson Guide.