Fixing Misbehaving Models

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Fixing Misbehaving Models

Portrait of a Bad Moment

fixmodel.pdf ch 9
newbook.pdf ch 2,17
GAMSchk.pdf

I finish a model

I eagerly send it to the solver

I fix my GAMS typing mistakes

The model has 10,000 variables and 4,000 constraints (Actually I should use a small data set, but emulating many modelers I do not)

Then usually several things happen

Oh no, the

1. Darn thing is infeasible
2. Darn thing is unbounded
3. Solver says optimal, but on closer look the answer makes no sense

Fixing Misbehaving Models
Why do I have such problems?

Generally models are infeasible or unbounded because they have small components within them that interact to make an infeasible or unbounded solution. Consider the following infeasible example.

$$
\begin{array}{llllll}
Max & 50X_1 & + & 50X_2 & & \\
s.t. & X_1 & + & X_2 & \leq & 50 \\
 & 50X_1 & + & X_2 & \leq & 65 \\
 & X_1 & & & \geq & 20 \\
 & X_1 & , & X_2 & \geq & 0
\end{array}
$$

The second and third constraints cannot be simultaneously satisfied. The direct cause of the infeasibility is that the lower bound on X_1 cannot be satisfied given the second constraint. The underlying cause may be that

the 20 is too large for a lower bound on X_1,
the coefficient of 50 on X_1 in the second constraint is too large and should have been 0.5,
the X_2 coefficient in the second constraint should have been large and negative or
the 65 in the third constraint right hand side is too small;
as well as some combination of the above which the modeler would have to sort out.

Note in this case the first constraint has little to do with it

Fixing Misbehaving Models
Why do I have such problems?

A simple case can be developed in an unbounded problem. Consider the following unbounded example

$$\begin{array}{llllr} Max & 3X_1 & -X_2 & +X_3 & \\ s.t. & X_1 & -X_2 & & =0 \\ & & & X_3 & \leq 20 \\ & X_1, & X_2, & X_3 & \geq 0 \end{array}$$

here the problem is unbounded because of the interaction between X_1 and X_2. Namely when X_1 and X_2 are set equal they can be raised to infinity while still making money.

The underlying cause of this may be the
omission of constraints on X_1 or X_2 or
omission of some sort of decreasing marginal revenue or increasing cost function that affects X_1 or X_2.

Again one characteristic of the problem is the constraint on X_3 and X_3 itself have nothing to do with the unboundedness.

Fixing Misbehaving Models
What do I look for to fix such problems?

Inherent in the two examples is an identification of the type of information we look for when trying to fix a model that it is either unbounded or infeasible.

Namely within the problem formulation, there's a small set of variables and constraints which causes the infeasibility or unbounded outcome.

We need to identify that set sorting it out from the elements of the model which do not cause infeasibility or unboundedness.

These notes cover ways to do that using solution information from GAMS.

Frequently such problems can be discovered using pre-solution analysis techniques such as covered in the pre-solution notes. Here we cover cases that would not be directly found by those procedures.

Fixing Misbehaving Models
Finding the cause of problems
(Newbook ch 17, fixmodel ch 9)

$Max\ 50X_1 + 50X_2 - 10^6 A\ s.t.\ X_1 + X_2 \le 50\ \ 50X_1 + X_2 \le 65$

When first learning about linear programming many are exposed to artificial variables. An artificial variable, when placed in a linear programming problem, allows feasibility at a very high cost. In the example above, adding an artificial variable (A) makes the problem look as follows

Note this variable will allow X_1 to be less than 20 but at the cost of 1 million dollars per unit. Given such a cost the A variable will only be in the optimum solution if absolutely required otherwise it will be driven to 0.

Suppose we set up and solve this problem (infeart.gms) , then we get the solution as follows:

Solution to Infeasible Example with Artificial Present

Objective Function = -18,699,935

Variable	Solution Value	Reduced Cost	Row	Slack Variable	Shadow Price
X_1	1.3	0	1	48.7	0
X_2	0	19950	2	0	20001
A	18.7	0	3	0	-1000000

Fixing Misbehaving Models
Finding the cause of problems
(Newbook ch 17, fixmodel ch 9)

This solution shows very high shadow prices for the second and third constraints. If we were to employ budgeting we would find out the reason for these very high shadow prices is the cost of A.

This shows a general mechanism for finding causes of feasibility. Namely

1. Modify the problem by adding artificial variables.
2. Solve the problem.
3. Look at the optimum solution and collect a list of the variables with high (in absolute value) reduced costs and constraints with very high shadow prices. The model components associated with those are the model components causing the infeasibility.

In the example, we find the infeasibility is caused by the interaction of constraints two and three along with the non-negativity condition on X_2. The X_2 identification indicates that it would be desirable to let this variable go negative.

Fixing Misbehaving Models
Finding the cause of problems
(Newbook ch 17, fixmodel ch 9)

Procedures to help find the set associated with infeasibility problems have been implemented in GAMSCHK under the nonopt and advisory options. In particular when running nonopt on a solved model the reduced costs and shadow prices which are larger in absolute value than a tolerance given by 10 to the margfilt parameter set in the option file.

In turn GAMSCHK generates output such as

```
----### THESE VARIABLE BOUNDS MAY PARTIALLY CAUSE
             INFEASIBLE MODEL
         Since their marginals are so large

   X2               marg    -19951.00


 ----### THESE EQUATIONS MAY PARTIALLY CAUSE
             INFEASIBLE MODEL
         Since their marginals are so large

   R2              marg     20001.00
   R3              marg    -1000000.
```

Generally margfilt should be three orders of magnitude smaller than the value of the artificial variable objective function.

Fixing Misbehaving Models
Finding infeasibility cause
(Newbook ch 17, fixmodel ch 9)

Step 1 Add artificial variables to constraints and bounds not feasible at X=0 The objective function entries are negative large numbers for maximization and positive for minimization. Artificials also have an entry in the constraints

> Minus one in $\leq$ constraints with negative rhs including negative upper bounds
>
> Plus one in $\geq$ constraints with positive rhs including positive lower bounds
>
> A plus or minus one in = constraints with nonzero right hand sides where the sign is the same as the rhs sign

GAMSCHK advisory or nonopt can list these.

Step 2 Solve

Step 3 If nonzero artificial variables are found, then find equations and variables with large marginals including nonnegativity conditions and bounds. Use the nonopt option in GAMSCHK.

Step 4 Examine that set and Repair model

Fixing Misbehaving Models
Infeasible Example (farminf.gms)

			Profit	Feed Cattle		Move Crops				Grow Crops				Sell Crops				Rent Land		Artificial Cattle			RHS	Slack	Shadow Price
				Farm 1	Farm 2	Farm 1 to Farm 2		Farm 2 to Farm 1		Farm 1		Farm 2		Farm 1		Farm 2									
						Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Farm 1	Farm 2	Farm 1	Farm 2				
Profit Accounting			1	-185	-153	0.11	4	0.11	4	250	220	240	195	-2.4	-55	-2.05	-50	100	100	1000000	1000000	=	0	0	1
Crop on Hand	Farm 1	Corn		39		1		-1		-130				1								≤	0	0	2.77
		Hay		0.75			1		-1		-5.5				1							≤	0	0	65.46
	Farm 2	Corn			38.5	-1		1				-128				1						≤	0	0	2.66
		Hay			0.74		-1		1				-4.8				1					≤	0	0	61.46
Land	Farm 1			10						1	1							-1				≤	100	0	10^5
	Farm 2				10							1	1						-1			≤	100	0	100
Min. Cattle Sold	Farm 1			1																**1**		≥	50	0	-10^6
	Farm 2				1																**1**	≥	50	0	-944.7
Max Rented Land	Farm 1																	1				≤	200	0	99903
	Farm 2																		1			≤	700	263.4	0
Optimal Level			-20058897	30	50	0	0	1170	22.5	0	0	24.2	12.4	0	0	0	0	200	436.6	20	0				
Reduced Cost			0	0	0	-0.22	-8	0	0	-10^5	-10^5	0	0	-0.37	-10.5	-0.61	-11.5	0	0	0	-10^6				

Fixing Misbehaving Models
Infeasible Example (farminf.gms)

Assistance from GAMSCHK

```
----#### Executing NONOPT
 ----### THESE VARIABLES ARE POTENTIALLY UNBOUNDED
             To find the cause of unboundedness
             bound them or the objective function
             variable at a large value. Then solve and
             manually or through GAMSCHK find
             large levels for variables in solution
     FEEDCATTLE(farm1)
     FEEDCATTLE(farm2)
     SELLCROPS(farm1,corn)
     SELLCROPS(farm1,hay)
     SELLCROPS(farm2,corn)
     SELLCROPS(farm2,hay)

 ----### THESE EQUATIONS ARE POTENTIALLY INFEASIBLE
             To find the cause of infeasibility
             enter artificial variables with an
             entry in these eqns and a penalty in the
             objective function. Then solve the model
             and manually or through GAMSCHK find
             large marginals for equations or bounds
     MINCATTLE(farm1)
     MINCATTLE(farm2)
---### THESE VARIABLE BOUNDS MAY PARTIALLY CAUSE INFEASIBLE          Since their
marginals are so large
   GROWCROPS(farm1,corn)                    marg    -99892.92
   GROWCROPS(farm1,hay)                     marg    -99862.77
   ARTCATTLE(farm2)                         marg    -999005.5

 ----### THESE EQUATIONS MAY PARTIALLY CAUSE INFEASIBLE MODEL
         Since their marginals are so large
   LAND(farm1)                              marg     100002.8
   MINCATTLE(farm1)                         marg    -1000000.
   RENTALLAND(farm1)                        marg     99902.79
```

Fixing Misbehaving Models

Infeasible Example (farminf2.gms)

CPLEX IIS

When using CPLEX one can use the irreducible infeasible set (IIS) finder to discover infeasibility. This involves an option file (cplex.op2) which invokes IIS and may require turning off presolve (see solve for options file discussion)

    iis yes
    presolve 0

In turn in the example with artificials dropped one gets

```
Starting infeasibility finder algorithm...
An IIS is a set equations and variables (ie a submodel) which is
infeasible but becomes feasible if any one equation or variable
bound is dropped. A problem may contain several independent IISs
but only one will be found per run.
Number of equations in the IIS:  3.
     upper: Land(farm1) ≤ 100
     lower: mincattle(farm1) ≥ 50
     upper: rentalLand(farm1) ≤ 200
Number of variables in the IIS:  2.
     lower: growcrops(farm1,corn) ≥ 0
     lower: growcrops(farm1,hay) ≥ 0
```

This indicates that the infeasible set involves the same items as found in our GAMSCHK supported exercise

Fixing Misbehaving Models
Unbounded

The basic approach to unbounded models is similar to that used for infeasibilities. We bound the model variables at very large values so that the model is no longer unbounded. In turn, then the solution may have certain variables at very large values which are those contributing to the unbounded case. Consider the implementation of this in the case of the example (unbdbnd.gms) we used above

$$\begin{array}{lllll} Max & 3X_1 & -X_2 & +X_3 & \\ s.t & X_1 & -X_2 & & =0 \\ & & & X_3 & \leq 20 \\ & X_1 & & & \leq 100000 \\ & & & X_3 & \leq 100000 \\ & X_1, & X_2, & X_3 & \geq 0 \end{array}$$

Note this formulation will stop the unboundedness but X_1 and X_2 will take on a very large value. Suppose we set up and solve this problem (unbdbnd.gms), than we get the solution as follows:

x1=x2=100000, x3=20

Fixing Misbehaving Models
Unbounded

Basic Approach — bound the unbounded variables with very large numbers

1 Add large bounds to all variables which improve the objective and if maximize

   a. Non neg var with positive obj need large upper bound;
   b. Non pos var with neg obj need large neg lower bound;
   c. Unrestrict var with pos obj need a large upper bound;
   d. Unrestrict var with neg obj need large neg low bound.

GAMSCHK will list these items when using advisory or nonopt.

2 Solve the resultant model.

3 If imposed large bounds are binding, then find set of all variables with solution levels which are unrealistically large in absolute value. GAMSCHK will list these items when using non-opt

4 Look over that set and find problem then Repair the model and go back to step 1.

Fixing Misbehaving Models

Unbounded Example (farmunb.gms)

			Profit	Feed Cattle		Move Crops				Grow Crops				Sell Crops				Rent Land		RHS		Slack	Shadow Price
				Farm 1	Farm 2	Farm 1 to Farm 2		Farm 2 to Farm 1		Farm 1		Farm 2		Farm 1		Farm 2							
						Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Farm 1	Farm 2				
Profit Accounting			1	**-76415**	-153	0.11	4	0.11	4	250	220	240	195	-2.4	-55	-2.1	-50	100	100	=	0	0	1
Crop on Hand	Farm 1	Corn		39		1		-1		-130				1						≤	0	0	2.69
		Hay		0.75			1		-1		-5.5				1					≤	0	0	58.18
	Farm 2	Corn			38.5	-1		1				-128				1				≤	0	0	2.58
		Hay			0.74		-1		1				-4.8				1			≤	0	0	59.43
Land	Farm 1			0.5						1	1							-1		≤	100	0	100
	Farm 2				0.5							1	1						-1	≤	100	0	90.28
Min. Cattle Sold	Farm 1			1																≥	50	1e+6	0
	Farm 2				1															≥	50	0	-35.21
Max Rented Land	Farm 1																	1		≤	200		
	Farm 2																		1	≤	700		
Upper Bounds			--	1e+06	1000000	--	--	--	--	--	--	--	--	1000000	1000000	1000000	1000000	--	--				

	Profit	Feed Cattle Farm 1	Feed Cattle Farm 2	Move F1→F2 Corn	Move F1→F2 Hay	Move F2→F1 Corn	Move F2→F1 Hay	Grow F1 Corn	Grow F1 Hay	Grow F2 Corn	Grow F2 Hay	Sell F1 Corn	Sell F1 Hay	Sell F2 Corn	Sell F2 Hay	Rent Farm 1	Rent Farm 2
Optimal Level	7e+10	1e+06	50	0	0	6689	0	300000	100000	67	8	0	0	0	0	900000	0
Reduced Cost	0	0	0	-0.2	-2.8	0	-5.3	0	0	0	0	-0.3	-3.2	-0.5	-9.4	0	-9.7

Model made unbounded by removing rental land limits. Also by entering cattle price in cents, not dollars, on farm 1

Fixing Misbehaving Models
Assistance from GAMSCHK

GAMSCHK will tell you variables and constraints to bound and list out variables and slacks with large values

```
---#### Executing NONOPT
 ----### THESE VARIABLES ARE POTENTIALLY UNBOUNDED
             To find the cause of unboundedness
             bound them or the objective function
             variable at a large value. Then solve and
             manually or through GAMSCHK find
             large levels for variables in solution
     FEEDCATTLE(farm1)
     FEEDCATTLE(farm2)
     SELLCROPS(farm1,corn)
     SELLCROPS(farm1,hay)
     SELLCROPS(farm2,corn)
     SELLCROPS(farm2,hay)

 ----### THESE EQUATIONS ARE POTENTIALLY INFEASIBLE
             To find the cause of infeasibility
             enter artificial variables with an
             entry in these eqns and a penalty in the
             objective function. Then solve the model
             and manually or through GAMSCHK find
             large marginals for equations or bounds
     MINCATTLE(farm1)
     MINCATTLE(farm2)

----### THESE VARIABLES MAY BE UNBOUNDED
         Since their levels are so large
   FEEDCATTLE(farm1)                level    1000000.
   GROWCROPS(farm1,corn)            level    299948.5
   GROWCROPS(farm1,hay)             level    136363.6
   LANDRENT(farm1)                  level    936212.2

 ----### THESE EQUATIONS MAY BE UNBOUNDED
         Since their levels are so large

   MINCATTLE(farm1)                 level    1000000.
```

Fixing Misbehaving Models
A Simpler Bound Approach

There is also one quick and dirty way of finding an unbounded solution that should be mentioned.

One can just add a bound to just the variable being maximized. For example in the case of a model where the solve statement says the model is maximizing profit, one can just bound the profit variable at one billion and proceed as above. For an example implementation of this procedure can be found in the file unbbd2.gms

```
objmax.up=1000000000;
```

converting the problem to

```
Maximize    objmax
            objmax  =e= CX
                        AX≤b;
            objmax.up=1000000000;
```

The drawback to this approach is that it will find the most unbounded case in each run and will not identify all the unbounded cases at one time.

Fixing Misbehaving Models
Permanent Model Additions

One may wish to keep the large upper bound and artificial variable model modifications in the model at all times

In a project I did years ago we had a model which we used with a lot of farmers. In the model, farmers could specify a minimum amount of each crop to be planted and also could specify a wage rate at which they could sell their family labor. This caused both infeasibility and unboundedness problems. Infeasibility problems occurred when the farmers specified the quantity of acres that needed to be planted which exceeded the acres available or implied usage of more resources than were available. The unboundedness occurred because when the sale wage rate was above the hired labor wage rate and an infinite amount of hired labor could be purchased, the model just went into the temporary employment brokerage business.

We fixed this by permanently leaving the artificials and large bounds in the model. Then when the farmer got a model solution they might find it cost a great deal of money to meet a particular acreage constraint and could identify the resources that were the problem. Similarly they might find the solution buying and selling a tremendous amount of labor. In either case they could diagnose and fix the problem. Often these model features let non-technical users fix their own problems.

Fixing Misbehaving Models

Theory -- Sensitivity Information

Two Equations

$$X_B = B^{-1}b - B^{-1}A_{NB}X_{NB}$$

$$Z = C_B B^{-1}b - (C_B B^{-1}A_{NB} - C_{NB})X_{NB}$$

Sensitivity Results

$$\frac{\partial Z}{\partial b} = C_B B^{-1} = u \quad \textit{Shadow Price or Equation Marginal}$$

$$\frac{\partial Z}{\partial X_j} = -(C_B B^{-1}A_{NB} - C_j) = -(\sum_i u_i a_{ij} - c_j) \quad \textit{Reduced Cost or Variable Marginal}$$

So

reduced costs or variable marginals are a function of u_i, a_{ij}, c_j and shadow prices

shadow prices or row marginals u are determined by a_{ij} and c_j terms for basic variables

So to find out why variable marginals or reduced costs are large enough to make variables non basic look at u along with A and c for non basic variables

So to find out why shadow prices or equation marginals are large look at variables are non basic look at u along with A and c for basic variables. They arise from setting reduced costs = 0 for basic variables.

Fixing Misbehaving Models – Budgeting

Reduced Cost or GAMS variable marginal reproduction

$$\frac{\partial Z}{\partial X_j} = -(C_B B^{-1} A_{NB} - C_j) = -(\sum_i u_i a_{ij} - c_j)$$

Budget for Variable X_j

Item	a_{ij} from model data	**Shadow Price from solution**	**Product your calculation**
Name for equation 1	a_{1j}	u_1	$u_1\ a_{ij}$
Name for equation 2	a_{2j}	u_2	$u_2\ a_{2j}$
	⋮	⋮	⋮
Name for equation m	a_{mj}	u_m	$u_m\ a_{mj}$
Indirect cost sum	--	--	$\sum_i u_i a_{ij}$
Objective function	--	--	$-c_j$
Reduced cost	--	--	$\sum_i u_i a_{ij} - c_j$

Fixing Misbehaving Models

Base Example Model (Farmb.gms)

			Profit	Feed Cattle		Move Crops				Grow Crops				Sell Crops				Rent Land		RHS	
				Firm 1	Firm 2	Firm 1 to Firm 2		Firm 2 to Firm 1		Firm 1		Firm 2		Firm 1		Firm 2					
						Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Firm 1	Firm 2		
Profit Accounting			1	-185	-153	.11	4	.11	4	250	220	240	195	-2.4	-55	-2.05	-50	100	100	=	0
Crop on Hand	Firm 1	Corn		39		1		-1		-130				1						≤	0
		Hay		0.75			1		-1		-5.5				1					≤	0
	Firm 2	Corn			38.5	-1		1				-128				1				≤	0
		Hay			0.74		-1		1				-4.8				1			≤	0
Land	Firm 1			0.5						1	1							-1		≤	100
	Firm 2				0.5							1	1						-1	≤	100
Min. Cattle Sold	Firm 1			1																≥	50
	Firm 2				1															≥	50
Max Rented Land	Firm 1																	1		≤	200
	Firm 2																		1	≤	700

Fixing Misbehaving Models Unrealistic Optimal (Farmbud.gms,Firmrsm.gms)

			Profit	Feed Cattle		Move Crops				Grow Crops				Sell Crops				Rent Land		RHS		Slack	Shadow Price
				Firm 1	Firm 2	Firm 1 to Firm 2		Firm 2 to Firm 1		Firm 1		Firm 2		Firm 1		Firm 2							
						Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Corn	Hay	Firm 1	Firm 2				
Profit Accounting			1	-185	-153	.11	4	.11	4	250	220	240	195	-2.4	-55	-2.05	-50	100	100	=	0	0	1
Crop on Hand	Firm 1	Corn		39		1		-1		-130				1						≤	0	0	2.40
		Hay		0.75			1		-1		-5.5				1					≤	0	0	57
	Firm 2	Corn			38.5	-1		1				-7168				1				≤	0	0	2.29
		Hay			0.74		-1		1				-4.8				1			≤	0	0	61
Land	Firm 1			0.5						1	1							-1		≤	100	0	96.49
	Firm 2				0.5							1	1						-1	≤	100	0	16160
Min. Cattle Sold	Firm 1			1																≥	0	157	0
	Firm 2				1															≥	0	0	0
Max Rented Land	Firm 1																	1		≤	200	200	0
	Firm 2																		1	≤	700	0	16060
Optimal Level			1.3e+7	157	0	0	0	5.7e+6	0	0	21	800	0	5.7e+6	0	0	0	0	700				
Reduced Cost			0	0	-8060	-.22	0	0	-8	-34	0	0	-16060	0	-2.54	-.24	-11.5	-3.51	0				

Model made unrealistic by entering corn yield Firm 2 in lbs not bushels.
Also took off cattle minimum
This was done by altered lines 21 and 33 in Farmb.gms to get Firmrsm.gms
GCK file just
    postopt
        variables

Budgeting for Unrealistic Optimal Case (Farmbud.gms)

Panel a FEEDCATTLE(Firm2) Budget

```
##   FEEDCATTLE(Firm2)
   SOLUTION VALUE                      .000000E+00

   EQN                          Aij          Ui         Aij*Ui
 PROFITACCT                   -153.20       1.0000      -153.20
 CROPONHAND(Firm2,corn)        38.480       2.2880       88.042
 CROPONHAND(Firm2,hay)        0.74000       61.543       45.542
 LAND(Firm2)                  0.50000       16160.       8080.2
 MINCATTLE(Firm2)             1.0000     0.00000E+00      000
 TRUE REDUCED COST                                       8060.6
```

Land too valuable

Panel b GROWCROPS(Firm2,CORN) Budget

```
##   GROWCROPS(Firm2,corn)
   SOLUTION VALUE                          800.000

  EQN                          Aij          Ui         Aij*Ui
 PROFITACCT                    240.00       1.0000       240.00
 CROPONHAND(Firm2,corn)       -7168.0       2.2880      -16400.
 LAND(Firm2)                   1.0000       16160.       16160.
 TRUE REDUCED COST                                  0.00000E+00
```

I have found the faulty corn yield

Fixing Misbehaving Models
Steps to Using Budgets

1) Choose a variable to budget which exhibits a bad reduced cost in the solution information or which uses resources with bad shadow prices.

2) Make the above table

3) Examine the table results in the last column to find out how things balance out then examine rows where things look bad in terms of the contained shadow prices and a_{ij}'s to find either unrealistically high shadow prices or data errors.

4) If an excessively high shadow price has been found then budget other basic variables which use the resource involved

5) If an error in the a_{ij}'s is found which is causing the distortion then repair the model

Fixing Misbehaving Models

Row Summing

Allocation information comes from AX=b.

Row Sum layout for row i

Variable Names	Coefficients from data	Solution values	Calculated product
X_1	a_{i1}	X_1^*	$a_{i1}X_1^*$
X_2	a_{i2}	X_2^*	$a_{i2}X_2^*$
...	...	...	...
X_n	a_{in}	X_n^*	$a_{in}X_n^*$
Sum	- -	- -	$\sum_j a_{ij}X_j^*$
RHS	- -	- -	b_i
Slack	- -	- -	$b_i-\sum_j a_{ij}X_j^*$

Fixing Misbehaving Models

Row Summing for Unrealistic Optimal Case

(Firmrsm.gms)

```
        Panel a Objective Function -- PROFITACCT   Row  Sum
  VAR                                 Aij          Xj         Aij*Xj
 PROFIT                             1.0000      0.12868E+08 0.12868E+08
 FEEDCATTLE(Firm1)                 -185.00       157.14      -29071.
 FEEDCATTLE(Firm2)                 -153.20      0.00000E+00 0.00000E+00
 MOVECROPS(Firm1,Firm2,corn)       0.11200      0.00000E+00 0.00000E+00
 MOVECROPS(Firm1,Firm2,hay)         4.0000      0.00000E+00 0.00000E+00
 MOVECROPS(Firm2,Firm1,corn)       0.11200      0.57344E+07 0.64225E+06
 MOVECROPS(Firm2,Firm1,hay)         4.0000      0.00000E+00 0.00000E+00
 GROWCROPS(Firm1,corn)              250.00      0.00000E+00 0.00000E+00
 GROWCROPS(Firm1,hay)               220.50       21.429       4714.3
 GROWCROPS(Firm2,corn)              240.00       800.00      0.19200E+06
 GROWCROPS(Firm2,hay)               195.00      0.00000E+00 0.00000E+00
 SELLCROPS(Firm1,corn)             -2.4000      0.5728E+07  -0.1375E+08
 SELLCROPS(Firm1,hay)              -55.000      0.00000E+00 0.00000E+00
 SELLCROPS(Firm2,corn)             -2.0500      0.00000E+00 0.00000E+00
 SELLCROPS(Firm2,hay)              -50.000      0.00000E+00 0.00000E+00
 LANDRENT(Firm1)                    100.00      0.00000E+00 0.00000E+00
 LANDRENT(Firm2)                    100.00       700.00       70000.

    =E=                                                        =E=
 RHS COEFF                                                  0.00000E+00
```

Cause is money from Firm 1 corn sale

```
 ##  CROPONHAND(Firm1,corn)
           VAR                        Aij          Xj           Aij*Xj
 FEEDCATTLE(Firm1)                  39.000       157.14       6128.6
 MOVECROPS(Firm1,Firm2,corn)        1.0000      0.00000E+00 0.00000E+00
 MOVECROPS(Firm2,Firm1,corn)       -1.0000      0.57344E+07 -0.5734E+07
 GROWCROPS(Firm1,corn)             -130.00      0.00000E+00 0.00000E+00
 SELLCROPS(Firm1,corn)              1.0000      0.57283E+07 0.57283E+07
    =L=                                                         =L=
 RHS COEFF                                                  0.00000E+00
 SLACK EQUALS                                               0.00000E+00
```

Cause is corn moving from Firm 2 to Firm 1

```
 ##  CROPONHAND(Firm2,corn)
  VAR                                Aij          Xj         Aij*Xj
 FEEDCATTLE(Firm2)                 38.480      0.00000E+00 0.00000E+00
 MOVECROPS(Firm1,Firm2,corn)      -1.0000      0.00000E+00 0.00000E+00
 MOVECROPS(Firm2,Firm1,corn)       1.0000      0.57344E+07 0.57344E+07
 GROWCROPS(Firm2,corn)            -7168.0       800.00     -0.57344E+07
 SELLCROPS(Firm2,corn)             1.0000      0.00000E+00 0.00000E+00
    =L=                                                        =L=
 RHS COEFF                                                  0.00000E+00
 SLACK EQUALS                                               0.00000E+00
```

Again the faulty yield is found

Fixing Misbehaving Models
Steps to use Row Sum

1) Find a variable or slack with an unreasonable value

2) Choose a constraint where this variable or slack appears

3) Make the above table

4) Examine the allocation calculations to find other unrealistic variable level values or aij/rhs data errors which balance off allowing the unreasonable value for the originally sought item.

5) If no other bad variable or data are found then examine another constraint

6) If another variable is found to have a bad level, then examine another constraint into which it falls

7) If bad data are found repair the model

Fixing Misbehaving Models
Presolve Checking
chapter 8 of fixmodel.pdf
GAMSchk.pdf

Two Forms of Structural Examinations

Automatic -- Rule Based

Manual
    Schematic based
    Equation and variable display based

Within GAMSCHK these can either be at the block or individual variable/equation level

Fixing Misbehaving Models
Simple Structural Checking

Do you see anything wrong here with X2 (unbounded)

$$\begin{array}{llcrcl} Max & 50X_1 & + & 50X_2 & & \\ s.t. & X_1 & - & X_2 & \leq & 50 \\ & 50X_1 & - & X_2 & \leq & 65 \\ & X_1 & & & \geq & 20 \\ & X_1 & , & X_2 & \geq & 0 \end{array}$$

Do you see anything wrong here with X2 (zero)

$$\begin{array}{llcrcl} Max & 50X_1 & - & 50X_2 & & \\ s.t. & X_1 & + & X_2 & \leq & 50 \\ & 50X_1 & + & X_2 & \leq & 65 \\ & X_1 & & & \geq & 20 \\ & X_1 & , & X_2 & \geq & 0 \end{array}$$

Fixing Misbehaving Models
Simple Structural Checking

Anything wrong here with the first constraint (force zero)

$$\begin{array}{llcllcl} Max & 50X_1 & + & 50X_2 & & \\ s.t. & X_1 & + & X_2 & \le & 0 \\ & 50X_1 & + & X_2 & \le & 65 \\ & X_1 & & & \ge & 20 \\ & X_1 & , & X_2 & \ge & 0 \end{array}$$

Anything wrong here with the first constraint (infeasible)

$$\begin{array}{llcllcl} Max & 50X_1 & + & 50X_2 & & \\ s.t. & X_1 & + & X_2 & \le & -10 \\ & 50X_1 & + & X_2 & \le & 65 \\ & X_1 & & & \ge & 20 \\ & X_1 & , & X_2 & \ge & 0 \end{array}$$

Anything wrong here with the first constraint (redundant)

$$\begin{array}{llcllcl} Max & 50X_1 & + & 50X_2 & & \\ s.t. & X_1 & + & X_2 & \ge & -10 \\ & 50X_1 & + & X_2 & \le & 65 \\ & X_1 & & & \ge & 20 \\ & X_1 & , & X_2 & \ge & 0 \end{array}$$

Fixing Misbehaving Models
Simple Structural Checking

$$
\begin{array}{llccl}
Max & \sum_j c_j X_j & & & \\
s.t. & \sum_j a_{ij} X_j & \leq & b_i & for\ all\ i \\
 & \sum_j e_{nj} X_j & = & d_n & for\ all\ n \\
 & \sum_j f_{mj} X_j & \geq & g_m & for\ all\ m \\
 & X_j & \geq & 0 & for\ all\ j
\end{array}
$$

Cases Where the Model Must have an Infeasible Solution

$b_i < 0$ and $a_{ij} \geq 0$ for all j implies row i will not allow a feasible solution
$d_n < 0$ and $e_{nj} \geq 0$ for all j implies row n will not allow feasible solution
$d_n > 0$ and $e_{nj} \leq 0$ for all j implies row n will not allow feasible solution
$g_m > 0$ and $f_{mj} \leq 0$ for all j implies row m will not allow a feasible solution

Cases where certain variables in the model must equal zero

$b_i = 0$ and $a_{ij} \geq 0$ for all j implies all X_j's with $a_{ij} \neq 0$ in row i will be zero
$d_n = 0$ and $e_{nj} \geq 0$ for all j implies all X_j 's with $e_{nj} \neq 0$ in row n will be zero
$d_n = 0$ and $e_{nj} \leq 0$ for all j implies all X_j 's with $e_{nj} \neq 0$ in row n will be zero
$g_m = 0$ and $f_{mj} \leq 0$ for all j implies all X_j 's with $f_{mj} \neq 0$ in row m will be zero

Cases where certain constraints are obviously redundant

$b_i \geq 0$ and $a_{ij} \neq 0$ for all j means row i is redundant
$g_m \neq 0$ and $f_{mj} \geq 0$ for all j means row m is redundant

Cases where certain variables cause the model to be unbounded

$c_j > 0$ and $a_{ij} \leq 0$ or $e_{nj} = 0$ and $f_{mj} \geq 0$ for all i, m, and n means variable j is unbounded

Cases where certain variables will be zero at optimality

$c_j < 0$ and $a_{ij} \geq 0$ or $e_{nj} = 0$ and $f_{mj} \leq 0$ for all i, m, and n means variable j will never be nonzero m, and n implies variable j will be zero

Fixing Misbehaving Models

Analysis Example

(Firman.gms)

			Feed Cattle		Move Crops				Grow Crops				Sell Crops				Rent Land		RHS
			Firm1	Firm2	Firm1 to Firm2		Firm2 to Firm1		Firm1		Firm2		Firm1		Firm2				
					Corn	hay	Corn	hay	Corn	hay	Corn	Hay	Corn	hay	Corn	hay	Firm1	Firm2	
Profit Accounting			-185	-153	.11	4	.11	4	250	220	240	195	-2.4	5.5	-2.45	-5.6	-100	-100	Min
Crop on Hand	Firm1	Corn	39		1		1		130				1						≤ 0
		Soy	0.75			1		1		5.5				1					≤ 0
	Firm2	Corn		38.5	1		1				128				1				≤ 0
		Soy		0.74		1		1				4.8				1			≤ 0
Land	Firm1		0.5						1	1							-1		≤ 100
	Firm2			0.5							1	1						-1	≤ 100
Min. Cattle Sold	Firm1		1																≥ 50
	Firm2			1															≥ 50
Max. Rented Land	Firm1																1		≤ -200
	Firm2																	1	≤ -700

Fixing Misbehaving Models

Assistance from GAMSCHK

Analysis Results

```
----### Analysis of Variables ( nonlinear terms at current
**** Warning  These variables will equal zero
              because they have a zero lower bound
              an undesirable object function coefficient
              all 0 or - coefficients in the =G= rows
              all 0 or + coefficients in the =L= rows
              and no coefficients in the =E= rows
##   GROWCROPS(Firm1,corn)
     GROWCROPS(Firm2,corn)
     GROWCROPS(Firm2,hay)

----### Analysis of Equations ( nonlinear terms at current)
**** ERROR This =L= constr. causes an infeasible model
              since the nonnegative variables present
                        have only 0 or + coefficients
                    the nonpositive variables present
                        have only 0 or - coefficents
                    the unrestricted variables
                        have only zero coefficients
                    and the RHS is negative
##   RENTALLAND(Firm1)
     RENTALLAND(Firm2)

----#### Using DISPLAYCR to show ANALYSIS Problems
         Note only the first 5 problems found  under each
          Error or warning type will be displayed up to
          maximum of 200 variables or equations
```

Fixing Misbehaving Models
Assistance from GAMSCHK
Analysis Results
Displaycr Called by Analysis

```
----###  DISPLAYING VARIABLES
----## VAR GROWCROPS
##   GROWCROPS(Firm1,corn)
PROFITACCT                                              250.00
CROPONHAND(Firm1,corn)                                  130.00
LAND(Firm1)                                             1.0000
##   GROWCROPS(Firm2,corn)
PROFITACCT                                              240.00
CROPONHAND(Firm2,corn)                                  128.00
LAND(Firm2)                                             1.0000
##   GROWCROPS(Firm2,hay)
PROFITACCT                                              195.00
CROPONHAND(Firm2,hay)                                   4.8000
LAND(Firm2)                                             1.0000

----###    DISPLAYING EQUATIONS
----## EQU RENTALLAND
##   RENTALLAND(Firm1)
LANDRENT(Firm1)                                         1.0000
        =L=                                            -200.00
##   RENTALLAND(Firm2)
LANDRENT(Firm2)                                         1.0000
        =L=                                            -700.00
```

Fixing Misbehaving Models
Assistance from GAMSCHK
BLOCKPIC or BLOCKLIST
Analysis Output

```
----### Analysis of Variables(nonlinear terms at cur point)

       ### The variables pass all analysis tests

 ----### Analysis of Equations(nonlinear terms at cur point)

 **** ERROR This =L= constr. causes an infeasible model
                since the nonnegative variables present
                          have only 0 or + coefficients
                      the nonpositive variables present
                          have only 0 or - coefficents
                      the unrestricted variables
                          have only zero coefficients
                      and the RHS is negative

 ##  RENTALLAND
```

Fixing Misbehaving Models

SCALING

$$\begin{array}{llclcl}
\text{Max} & c_1X_1 & + & c_2X_2 & & \\
\text{s.t.} & a_{11}X_1 & + & a_{12}X_2 & \leq & b_1 \\
 & a_{21}X_1 & + & a_{22}X_2 & \leq & b_2 \\
 & X_1, & & X_2 & \geq & 0
\end{array}$$

Suppose one wished to change the units of a variable (for example, from pounds to thousand pounds). The homogeneity of units test requires like denominators in a column.

Thus implies every coefficient under that variable needs to be multiplied by the scaling factor

i.e., if X_j is in old units and X'_j is to be in a new unit, then.

$$X_j' = X_j / SC_j;$$

where SC_j equals the scaling coefficient

and

$$a_{ij}' = a_{ij} * (SC_j).$$

Fixing Misbehaving Models

SCALING

The scaling procedure can be demonstrated by multiplying and dividing each entry associated with the variable by the scaling factor.

Suppose we scale X_1 using SC_1

$$\begin{array}{llllll}
\text{Max} & SC_1\, c_1\, X_1\, / SC_1 & + c_2 X_2 & & \\
\text{s.t.} & SC_1\, a_{11}\, X_1\, / SC_1 & + a_{12} X_2 & \leq & b_1 \\
 & SC_1\, a_{21}\, X_1\, / SC_1 & + a_{22} X_2 & \leq & b_2 \\
 & X_1 & X_2 & \geq & 0
\end{array}$$

or substituting a new variable $X_1' = X_1/SC_1$ we get

$$\begin{array}{llllll}
\text{Max} & SC_1\, c_1\, X_1' & + c_2 X_2 & & \\
\text{s.t.} & SC_1\, a_{11}\, X_1' & + a_{12} X_2 & \leq & b_1 \\
 & SC_1\, a_{21}\, X_1' & + a_{22} X_2 & \leq & b_2 \\
 & X_1 & X_2 & \geq & 0
\end{array}$$

And at optimality

$X_1 = X_1' * SC_1$

Plus what happens to reduced cost?

Fixing Misbehaving Models
SCALING

Scaling can also be done on the constraints. When scaling constraints; e.g., transforming their units from hours to thousands of hours, every constraint coefficient is divided by the scaling factor (SR) as follows:

$$
\begin{array}{lrcrcr}
\text{Max} & c_1 X_1 & + & c_2 X_2 & & \\
 & a_{11} / SR\ X_1 & + & a_{12} / SR\ X_2 & \leq & b_1 / SR \\
 & a_{21} X_1 & + & a_{22} X_2 & \leq & b_2 \\
 & X_1, & & X_2 & \geq & 0
\end{array}
$$

where SR is the number of old units in a new unit and must be positive.

Constraint scaling affects :1) the slack variable solution value, which is divided by the scaling factor; 2) the reduced cost for that slack, which is multiplied by the scaling factor; and 3) the shadow price, which is multiplied by the scaling factor.

Scaling In GAMS

Theory of Scaling

Numerical stability gains are made when rows and columns are simultaneously scaled. Scaling all variables multiplying by SC_j , all rows dividing by SR_i and the objective dividing coefficients by SO then we get (newbook.pdf ch 17)

$$c'_j = c_j \times \frac{SC_j}{SO}, \quad a'_{ij} = a_{ij} \times \frac{SC_j}{SR_i}, \quad b'_i = \frac{b_i}{SR_i}$$

Relationships Between Items Before and After Scaling

Item	Symbol Before Scaling	Symbol After Scaling	Unscaled Value in Terms of Scaled Value	Scaled Value in Terms of Unscaled Value
Variables	X_i	X_i'	$X_i = X_i' * SC_i$	$X_i' = X_i / SC_i$
Slacks	S_i	S_i'	$S_i = S_i' * SR_i$	$S_i' = S_i / SRi$
Reduced Cost	$z_i - c_i$	$z_i' - c_i'$	$z_i - c_i = (z_i' - c_i') * (SO/SC_i)$	$z_i' - c_i' = (z_i - c_i) / (SO/SC_i)$
Shadow Price	U_i	U_i'	$U_i = U_i' * (SO/SR_i)$	$U_i' = U_i / (SO/SR_i)$
Obj. Func. Value	Z	Z '	Z = Z' * SO	Z '= Z / SO

Fortunately GAMS does this for us adjusting all solutions so they look as if they were never scaled. But this table does show the solutions are equivalent only differing by multiples of the scaling factors.

Scaling In GAMS

Example of Scaling

$$
\begin{array}{lccccccccc}
Max & 10X_1 & - & 5000X_2 & - & 4000X_3 & - & 50000X_4 & & \\
s.t. & X_1 & - & 10000X_2 & - & 8000X_3 & & & \le & 0 \\
 & & & 5X_2 & + & 4X_3 & - & 50X_4 & \le & 0 \\
 & & & 1500X_2 & + & 2000X_3 & & & \le & 6000 \\
 & & & 50X_2 & + & 45X_3 & & & \le & 300 \\
 & X_1 & , & X_2 & , & X_3 & , & X_4 & \ge & 0
\end{array}
$$

Divide first constraint by 10000 and multiply X_1 coefficients by 10000 while dividing third constraint by 1000 and in the fourth by 50. The resultant model is

$$
\begin{array}{lccccccccc}
Max & 100000X_1 & - & 5000X_2 & - & 4000X_3 & - & 50000X_4 & & \\
s.t. & X_1 & - & X_2 & - & 0.8X_3 & & & \le & 0 \\
 & & & 5X_2 & + & 4X_3 & - & 50X_4 & \le & 0 \\
 & & & 1.5X_2 & + & 2X_3 & & & \le & 6 \\
 & & & X_2 & + & 0.9X_3 & & & \le & 6 \\
 & X_1 & , & X_2 & , & X_3 & , & X_4 & \ge & 0
\end{array}
$$

Now divide X_4 column by 50 and objective function by 10000. The final scaled problem then becomes

$$
\begin{array}{lccccccccc}
Max & 10X_1 & - & 0.5X_2 & - & 0.4X_3 & - & 0.1X_4 & & \\
s.t. & X_1 & - & X_2 & - & 0.8X_3 & & & \le & 0 \\
 & & & 5X_2 & + & 4X_3 & - & X_4 & \le & 0 \\
 & & & 1.5X_2 & + & 2X_3 & & & \le & 6 \\
 & & & X_2 & + & 0.9X_3 & & & \le & 6 \\
 & X_1 & , & X_2 & , & X_3 & , & X_4 & \ge & 0
\end{array}
$$

The disparity in numbers is now much less.