Math - Grade 6

Course Overview

In sixth grade, instruction is focused on the following areas: (1) connecting ratio and rate to whole-number multiplication and division, and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; (4) developing understanding of statistical thinking; and (5) reasoning about geometric shapes and their measurements.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from and extending pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication, division, and fractions from previous grades with ratios, rates, and percents. As students solve a wide variety of problems involving ratios and rates, they create a foundation for proportional reasoning and future work in algebra and geometry.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems, as well as demonstrate fluency in operations with whole numbers and decimals. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular, negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students build on their understanding of an unknown quantity from previous grades to know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $y = 3x$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

5. Students in grade six also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose areas they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths.

Adopted Materials

- BSD Curriculum, Grade 6
  - Illustrative Mathematics, Grade 6 (materials curated by Amplify Desmos)
- Exemplars Resource Library, Grade 6
- Idaho State Content Standards and the Standards for Mathematical Practice

First Semester		Second Semester	
Quarter 1	Quarter 2	Quarter 3	Quarter 4
Unit 1:Area and Surface Area Unit 2: Introducing Ratios	Unit 2: Introducing Ratios Unit 3: Unit Rates and Percentages	Unit 4: Dividing Fractions Unit 5: Decimal Arithmetic Unit 8: Data Sets and Distributions	Unit 6:Expressions and Equations Unit 7: Positive and Negative Numbers

Scales

Geometry (Reported in Q1)
Ratio (Reported in Q2)
Unit Rates and Percents (Reported in Q2)
Base Ten: Whole Numbers and Decimals (Reported in Q2, Q3)
Fraction Division (Reported in Q3)
Understanding Data (Reported in Q3)

Expressions and Equations (Reported in Q4)
Reasoning about Equations through Word Problems (Reported in Q4)
Rational Numbers (Reported in Q4)

Quarter 1
Learning Intentions ● We are deepening our understanding of area, volume, and the coordinate plane so we can apply them to more complex shapes. ○ Geometry Scale
Idaho State Standards ● **6.G.A.1**- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. ● **6.G.A.4** - Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Quarter 2
Learning Intentions ● We are learning to identify relationships of quantities so we can better explain ratios and rates. ○ Ratio Scale ● We are exploring concepts of ratio so we can explain measurement conversions, unit rates, and percentage concepts. ○ Unit Rates and Percents Scale ○ Base Ten: Whole Numbers and Decimals Scale
Idaho State Standards ● **6.RP.A.1** - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. ● **6.RP.A.2**- Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. ● **6.RP.A.3** - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. d. Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. ● **6.NS.B.4** - Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Quarter 3

Learning Intentions

- We are learning how to connect contexts, equations, and models so we can explain how to divide fraction by a fraction.
  - Fraction Division Scale
- We are learning to look for patterns so we can extend understanding of base ten operations.
  - Base Ten: Whole Numbers and Decimals Scale
- We are learning to explain and determine the values that show statistical variability so we can describe a set of data.
- We are learning to create and use displays of data to tell the story of the distribution of data.
  - Understanding Data Scale

Idaho State Standards

- **6.NS.A.1** - Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
- **6.NS.B.2** - Fluently divide multi-digit numbers using the standard algorithm.
- **6.NS.B.3** - Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation
- **6.EE.A**- Apply and extend previous understandings of arithmetic to algebraic expressions
- **6.SP.A.1**- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- **6.SP.A.2** - Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median and/or mean), spread (range, interquartile range, and/or mean absolute deviation), and overall shape. The focus of mean absolute deviation (MAD) is visualizing deviations from the mean as a measure of variability as opposed to a focus on calculating MAD.
- **6.SP.A.3** - Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- **6.SP.B.4** - Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- **6.SP.B.5** - Summarize numerical data sets in relation to their context, such as by:
  a. Reporting the number of observations.
  b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  c. Giving quantitative measures of center (median, and/or mean) and variability (range, interquartile range, and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Quarter 4

Learning Intentions

- We are learning to reason about equations so we can identify parts of equations and situations to use them in.
  - Expressions and Equations
- We are learning to understand and use equations and expressions so we can represent and model contexts.
  - Reasoning about Equations through Word Problems
- We are looking for structures in numbers so we can expand our understanding of numbers to those below zero and between whole numbers both positive and negative.
  - Rational Numbers

Idaho State Standards

- **6.EE.A.2** - Write, read, and evaluate expressions in which letters stand for numbers.
  a. Write expressions that record operations with numbers and with letters standing for numbers.
  b. Identify parts of an expression using mathematical terms (e.g. sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
  c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- **6.EE.A.3 -** Apply the properties of operations to generate equivalent expressions.
- **6.EE.A.4** - Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).
- **6.EE.B.5** - Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.6** - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.
- **6.EE.B.7** - Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.
- **6.EE.C.9** - Use variables to represent two quantities in a real-world problem that change in relationship to one another; write equations to represent the relationship between the two quantities. Analyze the relationship using graphs and tables and relate these to the equations. Include an understanding of independent and dependent variables.
- **6.NS.C.5** - Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers (including fractions and decimals) to represent quantities in real-world contexts, explaining the meaning of zero in each situation.
- **6.NS.C.6** - Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
  c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram

- **6.NS.C.7** - Understand ordering and absolute value of rational numbers.
  a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
  b. Write, interpret, and explain statements of order for rational numbers in real world contexts.
  c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
  d. Distinguish comparisons of absolute value from statements about order.

6th Grade Standards for Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. In grade six, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". Students can explain the relationships between equations, verbal descriptions, and tables and graphs. Mathematically proficient students check their answers to problems using a different method.
MP.2 Reason abstractly and quantitatively. In grade six, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations or other meaningful moves. To reinforce students' reasoning and understanding, teachers might ask, "How do you know?" or "What is the relationship of the quantities?".
MP.3 Construct viable arguments and critique the reasoning of others. In grade six, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, and histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?", and "Does that always work?". They explain their thinking to others and respond to others' thinking.
MP.4 Model with mathematics. In grade six, students model problem situations symbolically, graphically, in tables, contextually, and visually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students begin to represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (e.g. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate

and apply them to a problem context. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “What formula might apply in this situation?”

MP.5 Use appropriate tools strategically.
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade six may decide to represent figures on the coordinate plane to calculate area. Number lines are used to create dot plots, histograms, and box plots to visually compare the center and variability of the data. Visual fraction models can be used to represent situations involving division of fractions. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. Students should be encouraged to answer questions such as “What approach did you try first?” or “Why was it helpful to use?”

MP.6 Attend to precision.
In grade six, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities. When using ratio reasoning in solving problems, students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. Students also learn to express numerical answers with an appropriate degree of precision when working with rational numbers in a situational problem. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain ___?”

MP.7 Look for and make use of structure.
Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables, recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (e.g. $6 + 2n = 2(3 + n)$ by the distributive property) and solve equations (e.g. $2c + 3 = 15$, $2c = 12$ by the subtraction property of equality; $c = 6$ by the division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume. Teachers might ask, “What do you notice when ___?” or “What parts of the problem might you eliminate, simplify, or ___?”

MP.8 Look for and express regularity in repeated reasoning.
In grade six, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = a$d/bc and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between rates and representations showing the relationships between quantities. Students should be encouraged to answer questions such as “How would we prove that ___?” or “How is this situation like and different from other situations?”