

2021-22 NCTS Undergraduate Research Project

Title: On K3 surfaces: moduli spaces, automorphism groups and fibrations

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Students:

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Background and Motivation:

Algebraic varieties are main objects in the study of algebraic geometry. From the point of view of classification theory in algebraic geometry, varieties with Kodaira dimension 0, Fano varieties, and varieties of general type are considered to be the three main blocks of varieties in general. Typical examples of varieties of Kodaira dimension 0 are abelian varieties and K3 surface. The beautiful theory of Riemann surfaces initiated by Riemann and Abel was built on abelian varieties and their moduli spaces. The purpose of this project is to investigate theories on K3 surfaces.

We will introduce the basic theory of K3 surface, and study their moduli spaces as well as the automorphism groups of K3 surfaces. We expect the students can have some hand-on experience working on these concrete but very interesting algebra-geometric objects.

Plan:

2021/10-2021/12, Stage 1: Basic training

The students will mainly study some basic material on K3 surfaces. The focus will be on examples of K3 surfaces, Hodge structure on K3 surfaces, period maps and period domain, Torelli-type theorem.

2022/01-2022/02, Stage 2: Advanced training and experiments

Some more advanced topics including lattice theory, moduli space of vector bundles on K3 surfaces, automorphism of K3 surfaces will be introduced. The students are expected to be able to construct examples of Calabi-Yau threefolds fibered by K3 surfaces by using the product-quotient construction to determine various geometric invariants of Calabi-Yau threefolds.

2022/03-2022/06, Stage 3: Project-oriented learning and research

We aim to explore the monodromy theory of K3-fibered Calabi-Yau threefolds. This kind of study is similar to Kodaira's theory on elliptic surfaces. In viewing the canonical bundle formula, special fibers will contribute to the "boundary part" of the canonical bundle of the total space while the "moduli part" can be computed via the period map. We hope that certain explicit canonical bundle formula can be obtained for some type of K3-fibered threefolds.

Reference:

- [1] S. Kondo , *K3 Surfaces*, EMS Tracts in Mathematics
 - [2] D. Huybrechts, *Lectures on K3 Surfaces*, Cambridge Studies in Advanced Mathematics, 158
 - [3] C. Doran, A. Harder, A. Novoseltsev, A. Thompson, Families of lattice polarized K3 surfaces with monodromy. *Int. Math. Res. Notices* 23, 12265–12318 (2015)
 - [4] C. Doran, A. Harder, A. Novoseltsev, A. Thompson, Calabi–Yau threefolds fibred by mirror quartic K3 surfaces. *Adv. Math.* 298, 369–392 (2016)
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