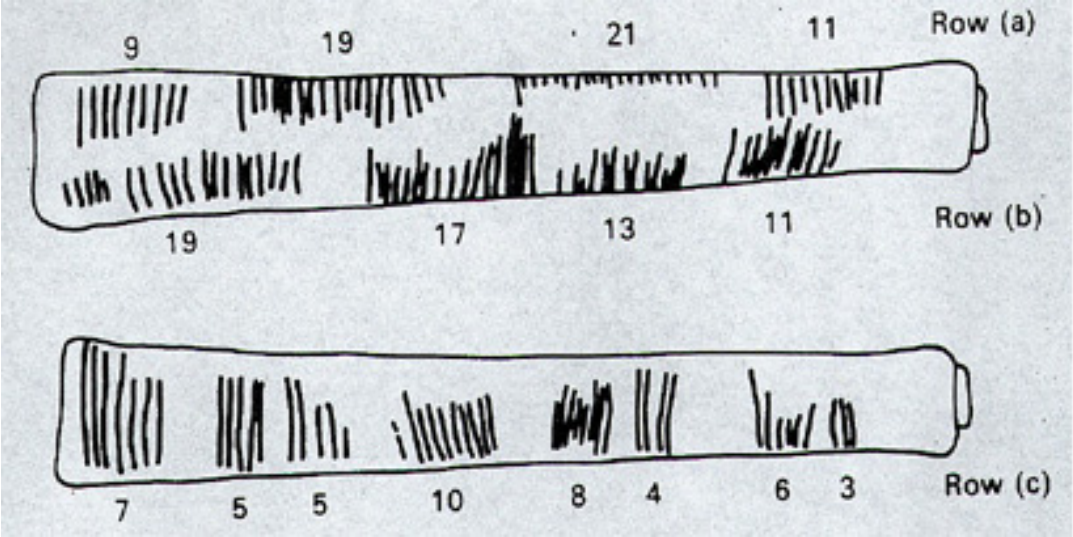
NAME_____ Date_____

Examine this image of the Ishango Bone and think carefully:



What do you notice?

What do you wonder?

Why might this be relevant for mathematics?

Watch this Video. Take notes below and be prepared to discuss:
https://www.youtube.com/watch?v=VTX3kfo6l5I

Homework: Read pages (30-35) from *The Crest of the Peacock: Non-European Roots of Mathematics,* Third Edition, by George Gheverghese Joseph, 2011 Princeton University Press. Answer the questions below and be prepared to discuss in our next class.

1) Gheverghese writes: "As far as we know there has never been a society without some form of counting or tallying…." Respond….

2) What is "Protomathematics"?

3) What are at least 3 possible ways the markings on the Ishango Bone have been interpreted?

4) It is likely African women were using the Ishango Bone. Why do we think this?

5) Who holds information and who gets credit for mathematical information?

6) Can you think of other possible ways the Ishango bone was used?

7) Is pattern recognition mathematics?

8) Where should the Ishango bones be kept? (in space? The Moon? Brussels? …..?) Why?

Chapter Two

Mathematics from Bones, Strings, and Standing Stones

It is taking an unnecessarily restrictive view of the history of mathematics to confine our study to written evidence. Mathematics initially arose from a need to count and record numbers. As far as we know there has never been a society without some form of counting or tallying (i.e., matching a collection of objects with some easily handled set of markers, whether it be stones, knots, or inscriptions such as notches on wood or bone). If we define mathematics as any activity that arises out of, or directly generates, concepts relating to numbers or spatial configurations together with some form of logic, we can then legitimately include in our study protomathematics, which existed when no written records were available.

Beginnings: The Ishango Bone

High in the mountains of central equatorial Africa, on the borders of Uganda and Congo, lies Lake Rutanzige (Edward), one of the furthest sources of the river Nile. It is a small lake by African standards, about eighty kilometers long and fifty wide. Though the area is remote and sparsely populated today, about twenty-five thousand years ago by the shores of the lake lived a small community that fished, gathered food, or grew crops, depending on the season of the year. The settlement had a relatively short life span of a few hundred years before being buried in a volcanic eruption. These Neolithic people have come to be known as the Ishango, after the place where their remains were found. There exists today a small village by that name.

Archaeological excavations at Ishango have unearthed human remains and tools for fishing, hunting, and food production (including grinding and pounding stones for grain). Harpoon heads made from bone may have

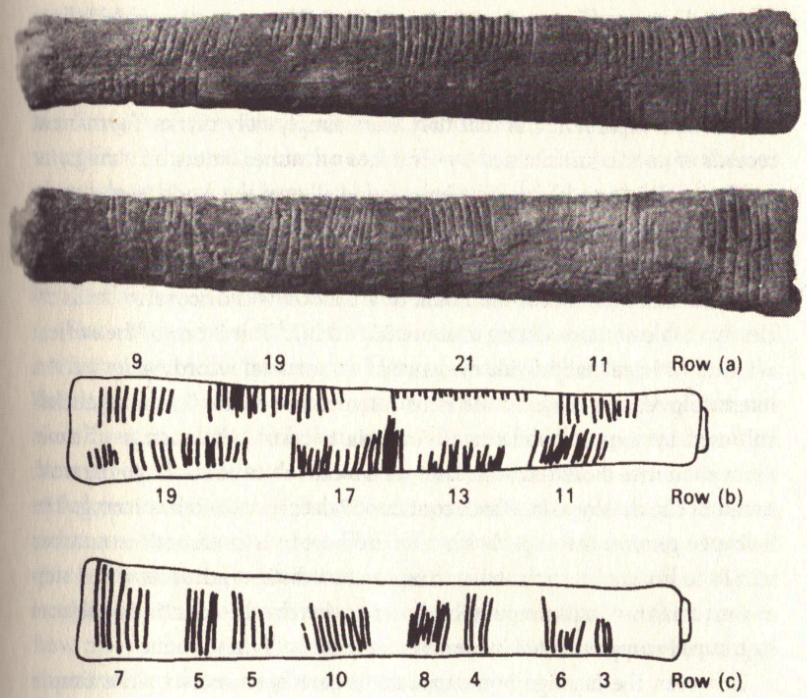


FIGURE 2.1: The Ishango bone (Courtesy of Dr. J. de Heinzelin)

served as prototypes for tools discovered as far away as northern Sudan and West Africa. However, the most interesting find, from our point of view, is a bone tool handle (figure 2.1) which is now at the Musée d'Histoire Naturelle in Brussels.[1] The original bone may have petrified or undergone chemical change through the action of water and other elements. What remains is a dark brown object on which some markings are clearly visible. At one end is a sharp, firmly fixed piece of quartz which may have been used for engraving, tattooing, or even writing of some kind.

The markings on the Ishango bone, as it is called, consist of series of notches arranged in three distinct rows. The asymmetrical grouping of these notches, as shown in figure 2.1, would make it unlikely that they were put there merely for decorative purposes. Row (a) contains four groups of notches with 9, 19, 21, and 11 markings. In row (b) there are also four groups, of 19, 17, 13, and 11 markings. Row (c) has eight groups of notches in the following order: 7, 5, 5, 10, 8, 4, 6, 3. The last two groups (6, 3) are

spaced closer together, as are (8, 4) and (5, 5, 10), suggesting a deliberate arrangement in distinct subgroups.

If these groups of notches were not decorative, why were they put there? An obvious explanation is that they were simply tally marks. Permanent records of counts maintained by scratches on stones, knots on strings, or notches on sticks or bones have been found all over the world, some going back to the very early history of human habitation. During an excavation of a cave in the Lebembo Mountains on the borders of Swaziland in southern Africa, a small section of the fibula of a baboon was discovered, with 29 clearly visible notches, dating to about 35,000 BC.[2] This is one of the earliest artifacts we have that provide evidence of a numerical recording device. An interesting feature of this bone is its resemblance to the "calendar sticks" still used by some inhabitants of Namibia to record the passage of time. From about five thousand years later we have the shin-bone of a young wolf, found in Czechoslovakia, which contains 57 deeply cut notches arranged in S-shaped groups. It was probably a record kept by a hunter of the number of kills to his credit. Such artifacts represent a distinct advance, a first step toward constructing a numeration system, whereby the counting of objects in groups is supplemented by permanent records of these counts.

However, the Ishango bone appears to have been more than a simple tally. Certain underlying numerical patterns may be observed within each of the rows marked (a) to (c) in figure 2.1. The markings on rows (a) and (b) each add up to 60: $9+19+21+11=60$, and $19+17+13+11=60$, respectively. Row (b) contains the prime numbers between 10 and 20. Row (a) is quite consistent with a numeration system based on 10, since the notches are grouped as $20+1$, $20-1$, $10+1$, and $10-1$. Finally, row (c), where subgroups (5, 5, 10), (8, 4), and (6, 3) are clearly demarcated, has been interpreted as showing some appreciation of the concept of duplication or multiplying by 2.

De Heinzelin (1962), the archaeologist who helped to excavate the Ishango bone, wrote that it "may represent an arithmetical game of some sort, devised by a people who had a number system based on 10 as well as a knowledge of duplication and of prime numbers" (p. 111). Further, from the existing evidence of the transmission of Ishango tools, notably harpoon heads, northward up to the frontiers of Egypt, de Heinzelin considered the possibility that the Ishango numeration system may have traveled

The African origins of Egyptian civilization are well attested to by archaeological and early written evidence. Herodotus wrote of the Egyptian people and culture having strong African roots, coming from the lands of the "long-lived Ethiopians," which meant in those days the vast tract of inner Africa inhabited by black people. However, de Heinzelin's speculations about the state of mathematical knowledge of the Ishango, based as they are on the evidence of a single bone, seem far-fetched. A single bone with suggestive markings raises interesting possibilities of a highly developed sense of arithmetical awareness; it does not provide conclusive evidence.

There is, however, another answer, more firmly rooted in the cultural environment, to the puzzle of the Ishango bone. Rather than attribute the development of a numeration system to a small group of Neolithic settlers living in relative isolation on the shores of a lake, apparently cut off from other traceable settlements of any size and permanence, a more plausible hypothesis is that the bone markings constitute a system of sequential notation—for example, a record of different phases of the moon. Whether this is a convincing explanation would depend in part on establishing the importance of lunar observations in the Ishango culture, and in part on how closely the series of notches on the bone matches the number of days contained in successive phases of the moon.

Archaeological evidence of seasonal changes in the habitat and activity of the Ishango highlights how important it was to maintain an accurate lunar calendar. At the beginning of the dry season, the Ishango moved down to the lake from the hills and valleys that formed their habitat during the rains. For those who were permanently settled along the shores of the lake, the onset of the dry season brought animals and birds to the lake in search of water. Now assume, for the sake of argument, that migration took place around the full moon or a few days before the full moon. About six months later the rainy season would begin, and the water levels of the lake would rise. Between the beginning of the dry season and the onset of the rainy season, there might be festivities that coincided with particular phases of the moon. And such events might very well be what is recorded notationally on the bone. Activities such as gathering and processing of nuts and seeds, or hunting, both of which archaeological evidence suggests were important in the Ishango economy, could be incorporated sequentially into the lunar calendar represented by the Ishango bone. Similarly, religious

know of present-day peoples who still follow the hunter-gatherer lifestyle of the Ishango.

A cursory examination of the pattern of notches on the Ishango bone shows no obvious regularity that one can associate with lunar phenomena. Two of the rows add up to 60, so that each of these rows may be said to represent two lunar months. The third row contains only 48 notches, which would account for only a month and a half. But a mere count of the notches would ignore the possible significance of the different sizes and shapes of the markings as well as the sequencing of the subgroups demarcated on the bone.

Marshack (1972) carried out a detailed microscopic examination of the Ishango bone and found markings of different indentations, shapes, and sizes. He concluded that there was evidence of a close fit between different phases of the moon and the sequential notation contained on the bone, once the additional markings—visible only through the microscope—were taken into account. Also, the different engravings represented by markings of various shapes and sizes may have been a calendar of events of a ceremonial or ritual nature.

These conjectures about the Ishango bone highlight three important aspects of protomathematics. First, the close link between mathematics and astronomy has a long history and is tied up with the need felt even by early humans to record the passage of time, out of curiosity as well as practical necessity. Second, there is no reason to believe that early humans' capacity to reason and conceptualize was any different from that of their modern counterparts. What has changed dramatically over the years is the nature of the facts and relationships with which human beings have had to operate. Thus the creation of a complicated system of sequential notation based on a lunar calendar was well within the capacity of prehistoric humans, whose desire to keep track of the passage of time and changes in seasons was translated into observations of the changing aspect of the moon. Finally, in the absence of records, conjectures about the mathematical pursuits of early human beings have to be examined in the light of their plausibility, the existence of convincing alternative explanations, and the quality of evidence available. A single bone may well collapse under the heavy weight of conjectures piled upon it.[3]

The notches of the Ishango bone open up other interesting conjectures. The epoch of the Ishango bone was around the same period when women were supposedly the temporal and spiritual leaders of their clans. Since a woman's menstrual cycle mimics the phases of the moon, would it be too fanciful to argue that the markings on the bone represent an early calendar of events of a ceremonial or ritual nature superimposed on a record of a lunar/menstrual cycle constructed by a woman? After all, among the Siaui of the Solomon Islands in the Pacific, a menstruating female is described as "going to the moon."[4]

There is yet another interpretation.[5] The Ishango bone may have been a precursor of writing. In that case, writing originated not in drawing figures or in attempting to record speech but in storing numerical information. The rows of notches became "graphically isomorphic" to the recorder's counting numbers. In this and subsequent chapters, there are illustrations of counting systems with forms of recording in which the iconic origin of the dash or stroke is the human digit (i.e., the finger or toe). These strokes are graphically isomorphic with the corresponding words used for counting. It makes little difference whether we "read" the sign pictorially, as standing for so many fingers held up, or in "script," as standing for a certain numeral.

Counting Systems and Numeration: The Pacific Dimension

The study of worldwide systems of numeration as they occur in natural languages has had a checkered career. A rich source of information on "exotic" languages and customs from the published literature of the explorers, administrators, and missionaries forms the core of the data now available on non-European counting systems. Initiated originally by those interested in linguistics and anthropology, the subject is now of only marginal interest to these groups. Yet no growth of interest on the part of historians or philosophers of mathematics has matched the waning interest on the part of the linguist or the ethnographer.

There are various reasons for this lack of interest. For the Western philosopher, the study of natural-language number systems seems to have little or no relevance to understanding the nature of number—an abstract philosophical concept derived from ancient Greeks and independent of linguistics and cultural vagaries. Historians of mathematics tend to concentrate on the origins of written numbers because of their uneasiness about straying into territories where culture and language interpose. In any case, they prefer to work with written records, even if these records are mainly confined to those in European languages. The occurrence of ideas relating to numbers as existing among "primitive" tribes, if considered at all, is mentioned in a dismissive fashion.