

## OVERVIEW

The focus this week will be on how to represent multiplication in a variety of ways. Students will learn to use equal groups to represent a multiplication problem.

Target A Standard <a href="#">3.OA.1</a>	Multiplication can be represented by making equal groups of items.		
Skills	<ul style="list-style-type: none"><li>use equal groups to begin understanding of multiplication</li></ul>		
Learning Intention	Students will <ul style="list-style-type: none"><li>use equal groups to model multiplication</li></ul>		
Success Criteria	I can <ul style="list-style-type: none"><li>Create a visual representation to model multiplication</li></ul>		
Item Specification	<a href="#">Strategy Bank</a> (No SBAC Stems)		
Vocabulary	equal groups, factor, product, multiplication, total, each, per		
Sentence Frames	If each dog has _____, then ____ dogs will have _____. There are _____ in all/total/altogether.		
Intro	<p><b>Opening: Addition - Landmark Numbers and Compensation</b> (Adapted from SFUSD)</p> <p><i>Landmark numbers</i> (sometimes called friendly numbers) are numbers that are easy to use in mental computation. The goal of <i>compensation</i> is to manipulate the numbers into easier, friendlier numbers to add. These two strategies are related because when compensating, you remove a specific amount from one addend to make a landmark number and give that exact amount to the other addend.</p> <p>Question/Prompt: <i>What is the answer and how do you know?</i></p> <table><tr><td><b>Category 1:</b> One addend is 1 away from a landmark number</td></tr><tr><td><div>9 + 8</div><div>19 + 5</div><div>9 + 26</div><div>16 + 19</div></td></tr></table> <p><b>Example - Anticipated Student Responses:</b> 16 + 19</p> <ul style="list-style-type: none"><li><b>Landmark numbers strategy:</b> <i>I made 19 into 20 by adding 1, then added 20 to 16 and got 26. Then I had to subtract the 1 that I added before and got 25.</i></li><li><b>Landmark numbers and compensation strategy:</b> <i>I made 19 into 20 by</i></li></ul>	<b>Category 1:</b> One addend is 1 away from a landmark number	<div>9 + 8</div> <div>19 + 5</div> <div>9 + 26</div> <div>16 + 19</div>
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taking 1 from 16 and adding it to 19. So 19 became 20 and 16 became 15. Then I added 20 and 15 and got 35.

- **Breaking into place value:** I added the 6 and the 9 and got 15. Then I added the ten from each number and got 20. Then I added 15 and 20 and got 35.

This will be done **daily**. Only select 1 -2 landmark numbers per day to practice. Spend 5-10 minutes on this, then do the lesson.

## Lesson

### Model/Think-Aloud (Adapted from SFUSD)

Show the video of [Amanda Bean's Amazing Dream by Cindy Neushwander](#) to connect counting with multiplication. In this story, Amanda Bean loves to count everything she sees, but gets overwhelmed when she sees so many things coming in groups because she can't count them one by one before they are gone. She uses this as motivation to use multiplication to help her count.

Ask students if anyone has dogs at home. Ask them how they would describe dogs in general, possibly to someone who has never seen a dog before. Notice what kind of math students mention (2 ears, 4 legs, 2 eyes, and so on).

Read [Dog Ears](#) with students. Answer any questions they have. Students can use [counters](#) if they need to solve the problem.

Provide 2–3 minutes for students to try the task on their own. Students may create a poster in Google Slides, use [counters](#), or on a piece of paper.

Students can share their thinking in a Google Meets, or on Flipgrid ([Teacher: How to Create a Flipgrid](#)).

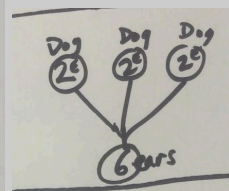
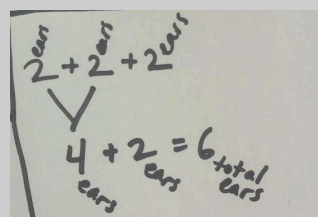
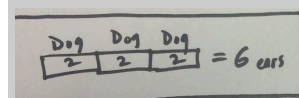
Some ways students might show their thinking and the kinds of calculations they suggest:

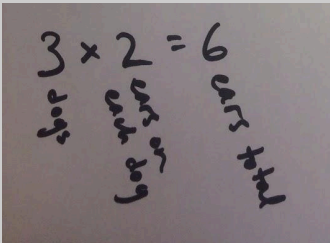
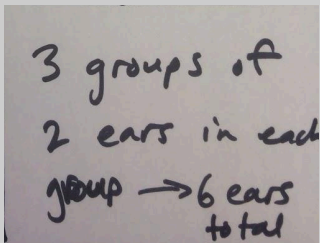
Counting all



Skip counting  
or double  
counting:

Dog 1	Dog 2	Dog 3	
2 ears	2 ears	2 ears	
2	4	6	6 ears



	<div data-bbox="342 52 592 342"> <p>Multiplication</p> </div> <div data-bbox="735 71 1062 312">  </div> <div data-bbox="1127 71 1445 312">  </div>
	<p><b>Equal groups:</b></p> <p>Focus discussion on the equal groups that were in the problem. Ask, <i>How did you know all the dogs had exactly 2 ears? What in the problem told you that?</i></p> <p>Some student responses may be:</p> <ul style="list-style-type: none"> <li>• <i>Each</i> indicates that there was more than one group that had the same thing.</li> <li>• It would not have made sense for only one dog to have 2 ears.</li> </ul> <p>To emphasize the importance of the groups being equal, tell students the following story:  <i>My neighbor has 2 dogs and 5 chickens. How many legs is that altogether?</i></p> <p>Emphasize that because some of the animals have 2 legs and some have 4, we cannot use the same strategies - we have to use addition rather than multiplication.</p> <p>In this problem, it was particularly important for students to pay attention to the word <i>each</i>. 3 dogs each had 2 ears, which equals 6 ears. You may also introduce the word <i>per</i> at this time; both <i>each</i> and <i>per</i> mean that the groups have an equal number of objects in them.</p>
<p><b>Closing</b></p>	<p><b>Check for Understanding:</b></p> <ul style="list-style-type: none"> <li>• If you had 2 dogs, how many ears would there be? Show how you know.</li> </ul>
<p><b>Resources</b></p>	<p><a href="#">Amanda Bean's Amazing Dream by Cindy Neushwander</a> (Teacher Resource)</p> <p><a href="#">Dog Ears</a> (Make a copy for each student)</p> <p><a href="#">Counters</a> (Student manipulative)</p>

Target A Standard 3.OA.1	Multiplication can be represented by making equal groups of items.		
Skills	<ul style="list-style-type: none"><li>• use equal groups to begin understanding of multiplication</li><li>• Write multiplication equations using equal groups model</li></ul>		
Learning Intention	Students will <ul style="list-style-type: none"><li><input type="checkbox"/> Use equal groups to model multiplication</li><li><input type="checkbox"/> Use equal groups to write a multiplication sentence</li></ul>		
Success Criteria	I can <ul style="list-style-type: none"><li><input type="checkbox"/> Create a visual representation to model multiplication</li><li><input type="checkbox"/> Write a multiplication sentence using equal groups</li></ul>		
Item Specification	Strategy Bank (No SBAC Stems)		
Vocabulary	Per, each, every, item, represent		
Sentence Frames	There are ____ per ____.		
Intro	<p><b>Opening: Addition - Landmark Numbers and Compensation</b> (Adapted from SFUSD)</p> <p>Students may use a number of strategies to solve these addition expressions, though these math talks lend themselves to using the strategy of landmark numbers and compensation. <i>Landmark numbers</i> (sometimes called friendly numbers) are numbers that are easy to use in mental computation. The goal of <i>compensation</i> is to manipulate the numbers into easier, friendlier numbers to add. These two strategies are related because when compensating, you remove a specific amount from one addend to make a landmark number and give that exact amount to the other addend.</p> <p><b>Suggested Math Talks:</b></p> <p>Question/Prompt: <i>What is the answer and how do you know?</i></p> <table><tr><td><b>Category 1:</b> One addend is 1 away from a landmark number</td></tr><tr><td><div>39 + 16</div><div>28 + 39</div><div>59 + 13</div><div>23 + 49</div></td></tr></table> <p><b>Example - Anticipated Student Responses:</b> 16 + 19</p> <ul style="list-style-type: none"><li>• <b>Landmark numbers strategy:</b> <i>I made 19 into 20 by adding 1, then added 20 to 16 and got 26. Then I had to subtract the 1 that I added before and got 25.</i></li><li>• <b>Landmark numbers and compensation strategy:</b> <i>I made 19 into 20 by taking 1 from 16 and adding it to 19. So 19 became 20 and 16 became 15.</i></li></ul>	<b>Category 1:</b> One addend is 1 away from a landmark number	<div>39 + 16</div> <div>28 + 39</div> <div>59 + 13</div> <div>23 + 49</div>
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- **Breaking into place value:** I added the 6 and the 9 and got 15. Then I added the ten from each number and got 20. Then I added 15 and 20 and got 35.

This will be done **daily**. Only select 1 -2 landmark numbers per day to practice. Spend 5-10 minutes on this, then do the lesson.

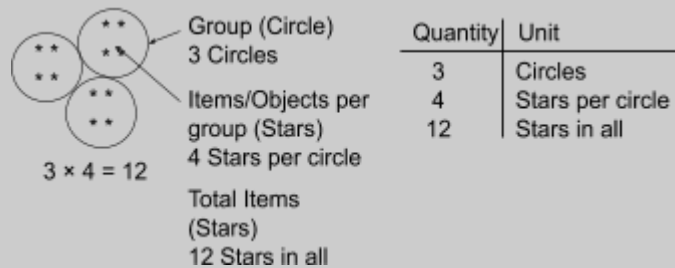
## Lesson

### Model/Think-Aloud (Adapted from SFUSD)

Begin with Video: [Making Equal Groups | BrainPop Jr.](#)

→ **Core Math to Emphasize**

Multiplication can be represented by making equal groups of items.



### Equal groups:

Use one of the turns to discuss the different parts of the representations: the group, the items (or objects/things being counted) in each group, and the total number of items. Each of these parts has a quantity (number) and a unit (thing being counted).

Group	Items per Group	Total Items
<u>3 CIRCLES</u>	<u>4 STARS</u> per <u>CIRCLE</u>	<u>12 STARS</u> in all

$$3 \times 4 = 12$$

**12 STARS IN ALL**

Elicit this information from students by asking:

- *What is the group?*
- *What are the items per group?*
- *What is the total number of items?*

Write this information as a multiplication diagram.

Emphasize the convention that we record the number of groups first and the number of items per group next: "3 groups of 4 items is 12 items total."

### Please see note below for more about this.

Discuss some other things that come in groups of items (bags of candy, classes of students, pages of homework per night) and the different parts of each situation. Discuss the use of words such as *per*, *each*, and *every* in the context of each example.

As time allows, start looking at **patterns** in the results. More work will be

	<p>devoted to this in later lessons and in Unit 3.5.</p> <p>Some patterns students might notice:</p> <ul style="list-style-type: none"> <li>• <i>Patterns when there is 1 group</i></li> <li>• <i>Patterns when there is 1 in each group</i></li> <li>• <i>Patterns in familiar numbers like 2 and 5</i></li> <li>• <i>Patterns in odd or even numbers</i></li> <li>• <i>Patterns that lead to discovery of the commutative property. E.g. <math>2 \times 4 = 8</math> &amp; <math>4 \times 2 = 8</math>.</i></li> </ul> <p>Show the digital lesson <a href="#">Represent Equal Groups using Multiplication   LearnZillion</a></p> <p>Students can practice making equal groups and finding the product in Google Slides using this resource: <a href="#">Equal Groups Practice</a></p> <p><a href="#">Anchor Charts</a> (slide 2) have been created for the students to reference the Ways to Show Multiplication.</p>
<b>Closing</b>	<p><b>Check for Understanding:</b></p> <ul style="list-style-type: none"> <li>• How could you represent the multiplication expression <math>3 \times 4</math>?</li> </ul>
<b>Resources</b>	<p><a href="#">Making Equal Groups   BrainPop Jr.</a> (Teacher Resource)</p> <p><a href="#">Represent Equal Groups using Multiplication   LearnZillion</a> (Teacher Resource)</p> <p><a href="#">Equal Groups Practice</a> (Make a copy for each student)</p> <p><a href="#">Anchor Charts</a> (Anchor Chart)</p>

Target A Standard <a href="#">3.OA.1</a>	Students interpret and represent word problems involving equal groups by using a tape diagram, and connecting repeated addition with multiplication.		
Skills	<ul style="list-style-type: none"><li>interpret and represent equal groups by using a tape diagram</li><li>Write multiplication sentences using their tape diagram</li></ul>		
Learning Intention	Students will <ul style="list-style-type: none"><li><input type="checkbox"/> Model multiplication with tape diagrams</li><li><input type="checkbox"/> Write multiplication sentences</li></ul>		
Success Criteria	I can <ul style="list-style-type: none"><li><input type="checkbox"/> Create a visual representation to model multiplication</li><li><input type="checkbox"/> Write a multiplication sentence to match a tape diagram</li></ul>		
Item Specification	<a href="#">Strategy Bank</a> (No SBAC Stems)		
Vocabulary	Tape diagram, strategy		
Sentence Frames	<ul style="list-style-type: none"><li><i>There are _____ per _____ .</i></li><li><i>_____’s strategy is similar to/different from _____’s strategy because _____ .</i></li></ul>		
Intro	<p><b>Opening: Addition - Landmark Numbers and Compensation</b> (Adapted from SFUSD)</p> <p>Students may use a number of strategies to solve these addition expressions, though these math talks lend themselves to using the strategy of landmark numbers and compensation. <i>Landmark numbers</i> (sometimes called friendly numbers) are numbers that are easy to use in mental computation. The goal of <i>compensation</i> is to manipulate the numbers into easier, friendlier numbers to add. These two strategies are related because when compensating, you remove a specific amount from one addend to make a landmark number and give that exact amount to the other addend.</p> <p><b>Suggested Math Talks:</b></p> <p>Question/Prompt: <i>What is the answer and how do you know?</i></p> <table><tr><td><b>Category 2:</b> One addend is 2 away from a landmark number.</td></tr><tr><td><math>8 + 4</math> <math>18 + 6</math> <math>28 + 17</math> <math>27 + 28</math></td></tr></table> <p><b>Example - Anticipated Student Responses:</b> <math>16 + 19</math></p> <ul style="list-style-type: none"><li><b>Landmark numbers strategy:</b> <i>I made 19 into 20 by adding 1, then added</i></li></ul>	<b>Category 2:</b> One addend is 2 away from a landmark number.	$8 + 4$ $18 + 6$ $28 + 17$ $27 + 28$
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20 to 16 and got 26. Then I had to subtract the 1 that I added before and got 25.

- **Landmark numbers and compensation strategy:** I made 19 into 20 by taking 1 from 16 and adding it to 19. So 19 became 20 and 16 became 15. Then I added 20 and 15 and got 35.
- **Breaking into place value:** I added the 6 and the 9 and got 15. Then I added the ten from each number and got 20. Then I added 15 and 20 and got 35.

This will be done **daily**. Only select 1 -2 landmark numbers per day to practice. Spend 5-10 minutes on this, then do the lesson.

## Lesson

### Model/Think-Aloud (Adapted from SFUSD)

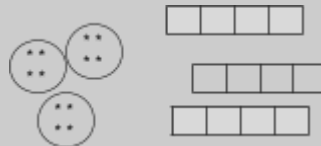
Ask students this question and discuss student responses:

*How are multiplication and addition similar? How are they different?*

*Tip:* This may be a good place to incorporate **math notebooks** in Google Classroom. You can do this by watching this video about online [Journal](#), or other similar videos.

**Teacher Note:** It is important not to lead students to the idea that multiplication and repeated addition are the same. Although at grade 3 multiplication can be represented easily with repeated addition, as students progress into later grades this interpretation breaks down. If students need more on repeated addition, you can show [Repeated Addition | BrainPop Jr.](#) or [Repeated Addition | LearnZillion](#).

Refer students back to [Dog Ears](#) in the previous lesson. Explain that they were using one way to represent multiplication in that lesson, which was to draw pictures of their groups and items. Tell students that another way to represent multiplication is by using a tape diagram.



Then say:

*What if each star was a linking cube?*

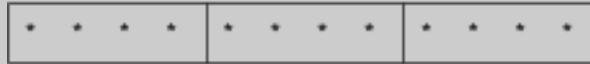
Make 3 trains of 4 cubes, using a different color cubes for each group using [Linking Cubes](#). Then, combine them into one long train.

Tape diagram showing groups of items represented by:

Cubes:



Symbols



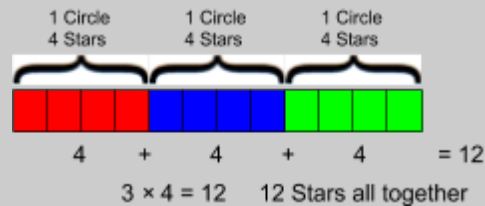
Numbers:



$$4 + 4 + 4$$

$$3 \times 4$$

"Three groups of four."



Help students see the correspondence between the 4 stars in each circle and the 4 cubes in each part of the train.

Discuss how to see both repeated addition and multiplication equations in the train.

This train is a concrete representation of a tape diagram.

Show students how they may draw this train of linker cubes. Students can either draw each cube as one discrete item, or draw each group containing symbols or numbers to represent how many cubes (items) are in each group.

Tell students that they will be reading word problems, figuring out what the groups and items are, and representing the problem using a tape diagram and a multiplication equation. Provide students with linker cubes ([Linking Cubes](#)) to use first, before drawing their representations.

#### → Core Math to Emphasize

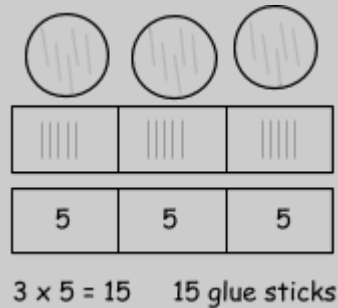
A tape diagram is a way to represent a multiplication situation by showing the relationship between the quantities and units.

Discuss the different ways that students solved the problems and how this may lead to the different strategies students used to find the total items in

each question. Students can explain their strategy using one of the problems on [Flipgrid](#).

**Select** and **sequence** student work that shows use of:

- Equal groups
- Tape Diagrams with concrete, abstract, or numeric representations



After each student shares a new strategy, **connect** it to the previous strategy by asking:

- *How is \_\_\_\_\_'s strategy similar to \_\_\_\_\_'s strategy? How is it different?*
- *What kind of strategy would be best for each kind of number?*
- *How would we record this?*

If there is not student evidence of a tape diagram choose one problem to model this strategy. Ask students to compare the tape diagram to the strategies that they used. “How is this tape diagram similar/different to \_\_\_\_\_ (picture, skip count, addition, etc...)”

“Tape Diagram” is added to your anchor chart of “Ways to Show Multiplication” in [Anchor Charts](#), Slide 3.

Finally, have students complete: [Day 3 Classwork](#). Students may use [Linking Cubes](#) if needed.

## Closing

### Check for Understanding:

- How could you represent the multiplication expression  $3 \times 4$  using a tape diagram?

## Resources

[Journal](#) (Teacher Resource)  
[Repeated Addition | BrainPop Jr.](#) (Video)  
[Repeated Addition | LearnZillion](#) (Video)  
[Dog Ears](#) (Teacher Resource)  
[Linking Cubes](#) (Student Manipulative)  
[Flipgrid](#) (Teacher Resource)  
[Anchor Charts](#) (Teacher Resource)  
[Day 3 Classwork](#) (Make a copy for each student)

Target A Standard 3.OA.1	Students interpret and represent word problems involving measurement in equal groups by using a number line with groups of 2.					
Skills	<ul style="list-style-type: none"><li>• interpret and represent equal groups by using a number line</li><li>• Write multiplication sentences</li></ul>					
Learning Intention	Students will <ul style="list-style-type: none"><li><input type="checkbox"/> Model multiplication with a number line</li><li><input type="checkbox"/> Write multiplication sentences</li></ul>					
Success Criteria	I can <ul style="list-style-type: none"><li><input type="checkbox"/> Create a visual representation to model multiplication</li><li><input type="checkbox"/> Write a multiplication sentence to match a number line</li></ul>					
Item Specification	Strategy Bank (No SBAC Stems)					
Vocabulary	Measure, number line					
Sentence Frames	_____ per _____.					
Intro	<p><b>Opening: Addition - Landmark Numbers and Compensation</b> (Adapted from SFUSD)</p> <p>Students may use a number of strategies to solve these addition expressions, though these math talks lend themselves to using the strategy of landmark numbers and compensation. <i>Landmark numbers</i> (sometimes called friendly numbers) are numbers that are easy to use in mental computation. The goal of <i>compensation</i> is to manipulate the numbers into easier, friendlier numbers to add. These two strategies are related because when compensating, you remove a specific amount from one addend to make a landmark number and give that exact amount to the other addend.</p> <p>Question/Prompt: <i>What is the answer and how do you know?</i></p> <table><tr><td><b>Category 2:</b> One addend is 2 away from a landmark number.</td></tr><tr><td>18 + 63</td></tr><tr><td>38 + 37</td></tr><tr><td>67 + 28</td></tr><tr><td>48 + 52</td></tr></table> <p><b>Example - Anticipated Student Responses:</b> 16 + 19</p> <ul style="list-style-type: none"><li>• <b>Landmark numbers strategy:</b> <i>I made 19 into 20 by adding 1, then added 20 to 16 and got 26. Then I had to subtract the 1 that I added before and got 25.</i></li><li>• <b>Landmark numbers and compensation strategy:</b> <i>I made 19 into 20 by taking 1 from 16 and adding it to 19. So 19 became 20 and 16 became 15. Then I added 20 and 15 and got 35.</i></li></ul>	<b>Category 2:</b> One addend is 2 away from a landmark number.	18 + 63	38 + 37	67 + 28	48 + 52
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18 + 63						
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- **Breaking into place value:** *I added the 6 and the 9 and got 15. Then I added the ten from each number and got 20. Then I added 15 and 20 and got 35.*

This will be done **daily**. Only select 1 -2 landmark numbers per day to practice. Spend 5-10 minutes on this, then do the lesson.

## Lesson

### Model/Think-Aloud (Adapted from SFUSD)

Ask students to think of things that come in groups. Discuss a few possibilities. Notice whether anyone mentions a group or item that you cannot “touch” (such as days, hours, minutes, inches, pounds). Most of these will be things that you measure, such as time, length, weight, and so on. The same measurable thing can represent the “group” (candy eaten per *day*) or the “item” (*days* per week). Money is also considered measurable, though it is easier to “touch” dollars than it is to “touch” days.

Read the first situation from [Measuring in Groups](#) (“I earned 2 dollars per day for 3 days”).

Gain information from them about the problem and write it in a multiplication diagram. Mark the unknown quantity with a question mark.

Group	Items per Group	Total Items
<u>3 DAYS</u>	<u>2 DOLLARS</u> per <u>DAY</u>	<u>? DOLLARS</u> in all

$$3 \times 2 = ?$$

Tell students that they will try to figure out the unknown amount using two models you have discussed (pictures of groups and tape diagram), and one you have not (number lines).

Make linking cubes ([Linking Cubes](#)) available to students to use as they work on [Measuring in Groups](#). Tell students to work on the first two representations (pictures of equal groups and tape diagrams) independently. Tell students to label each of their diagrams so it is clear where the dollars and the days are shown in each representation.

### → Core Math to Emphasize

Multiplication can be used to measure units rather than count objects. These units can be represented on a number line by using individual “jumps” and groups of “jumps.”

Show the video: [Multiplying on a Number Line | LearnZillion](#) or [Solve Multiplication by Skip Counting | LearnZillion](#), which are brief introductions to multiplying using a number line. Have students draw a number line for the first problem on [Measuring in Groups](#).

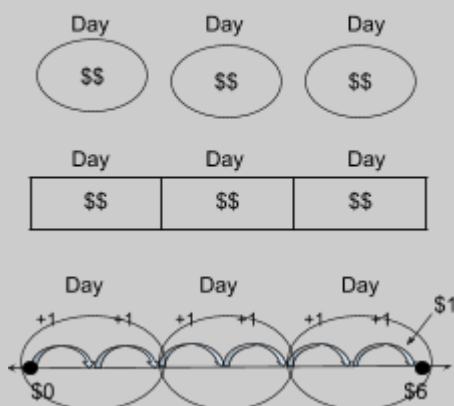
**If many students are still counting all, Sequence and connect** work that

shows:

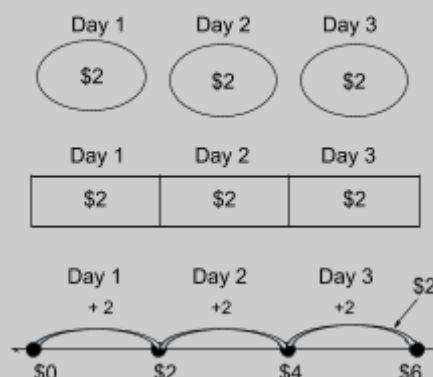
- Individual jumps on a number line (adding \$1 each day)
- Groups of jumps on a number line (adding \$2 each day)
- A solid connection between the three types of visual models

Help students make a **connection** between the uses of the number lines and the multiplication strategies by asking, *Where do you see counting all? Where do you see skip counting?*

Representations that encourage counting all



Representations that encourage skip counting or double counting



Help students **make connections** between each visual model by asking:

- *Where do you see the \$2 in each model?*
- *Where do you see 3 days in each model?*
- *Where do you see multiplication in each model?*

Add “Number Line” to your anchor chart of “Ways to Show Multiplication” found here - [Anchor Charts](#), slide 3.

Have students draw a number line for the bottom two problems on [Measuring in Groups](#). Additional/Supplemental Assignment: [3.OA.1 Day 4 Classwork](#).

## Closing

### Check for Understanding:

- How could you use a number line to represent the multiplication expression  $3 \times 4$ ?

## Resources

[Measuring in Groups](#) (Make a copy for each student)  
[Linking Cubes](#) (Student manipulative)  
[Multiplying on a Number Line | LearnZillion](#) (Video)  
[Solve Multiplication by Skip Counting | LearnZillion](#) (Video)  
[Anchor Charts](#) (Teacher Resource)  
[3.OA.1 Day 4 Classwork](#) (Make a copy for each student)