

Similarity in Right Triangles

G.7 (2016)

Practice

Geometric Mean	
The geometric mean of positive numbers a and b is x , the positive number that makes $\frac{a}{x} = \frac{x}{b}$ true.	$\frac{a}{x} = \frac{x}{b}$ $x^2 = ab$ $x = \sqrt{ab}$

Find the geometric mean between each pair of numbers.

1. $\sqrt{8}$ and $\sqrt{32}$

2. $\frac{1}{4}$ and 9

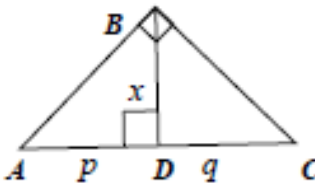
Example 1: Find the geometric mean of 12 and 9.

$$\frac{12}{x} = \frac{x}{9}$$

$$x^2 = 108$$

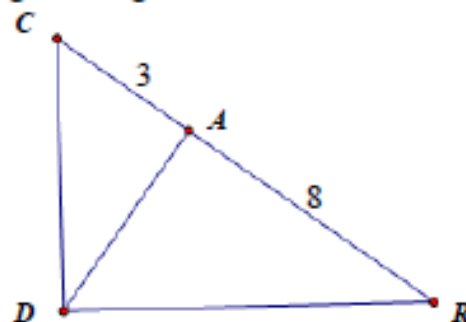
$$x = \sqrt{108} = 6\sqrt{3} \approx 10.4$$

1. Set up a proportion using the definition of geometric mean.
2. Cross multiply
3. Take the positive square root of each side.
4. Rationalize the square root or use a calculator.

The altitude drawn from the right angle of a right triangle to the hypotenuse forms two triangles that are similar to the original triangle and each other. $\triangle ABC \sim \triangle ADB \sim \triangle BDC$	
The measure of this altitude is the geometric mean of the lengths of the two segments of the hypotenuse.	$\frac{p}{x} = \frac{x}{q}$

3. $\triangle ABC$ is a right triangle and $\overline{AD}$ is an altitude. Write a similarity statement relating the three triangles. Then find AD if $BD = 4$ and $CD = 16$.

Example 2: $\triangle CRD$ is a right triangle with altitude $\overline{DA}$. Find DA .



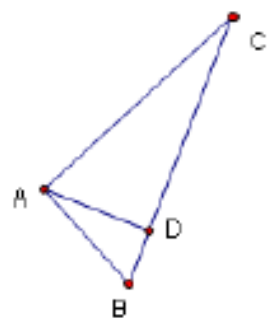
$$\frac{AC}{AD} = \frac{AD}{RA}$$

$$\frac{3}{x} = \frac{x}{8}$$

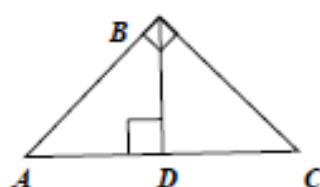
$$x^2 = 24$$

$$x = \sqrt{24} = 2\sqrt{6}$$

1. AD is the geometric mean of AC and RA . Set up a geometric mean proportion.
2. Replace known values.
3. Solve.
4. Rationalize the root.
(Remember, $\sqrt{24}$ is $\sqrt{4} \cdot \sqrt{6}$.)

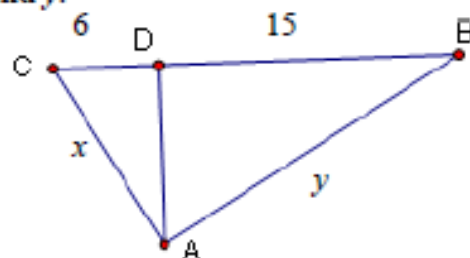


If an altitude drawn from the right angle of a right triangle to the hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measure of the hypotenuse and the segment of the hypotenuse adjacent to the leg.



$$\frac{AC}{BA} = \frac{BA}{AD} \text{ and } \frac{AC}{BC} = \frac{BC}{DC}$$

Example 3: Find x and y .



$$\frac{BC}{AC} = \frac{AC}{DC}$$

$$\frac{21}{x} = \frac{x}{6}$$

$$x^2 = 126$$

$$x = \sqrt{126} = 3\sqrt{14}$$

$$\frac{BC}{BA} = \frac{BA}{BD}$$

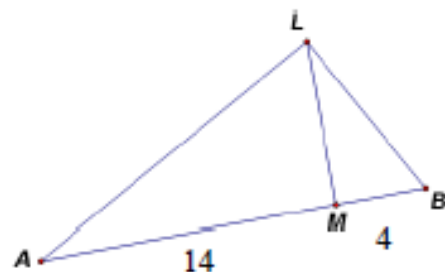
$$\frac{21}{y} = \frac{y}{15}$$

$$y^2 = 315$$

$$y = \sqrt{315} = 3\sqrt{35}$$

1. Write a proportion based on the theorem that the length of the leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to that leg.
2. Cross multiply and solve. (Rationalize radicals.)
3. Repeat the process to solve for the other variable. (Rationalize radicals.)

4. $\triangle LAB$ is a right triangle and $\overline{LM}$ is an altitude. Find LA and LB .



5. $\triangle CAB$ is a right triangle and $\overline{DA}$ is an altitude. Find x , y , and z .

