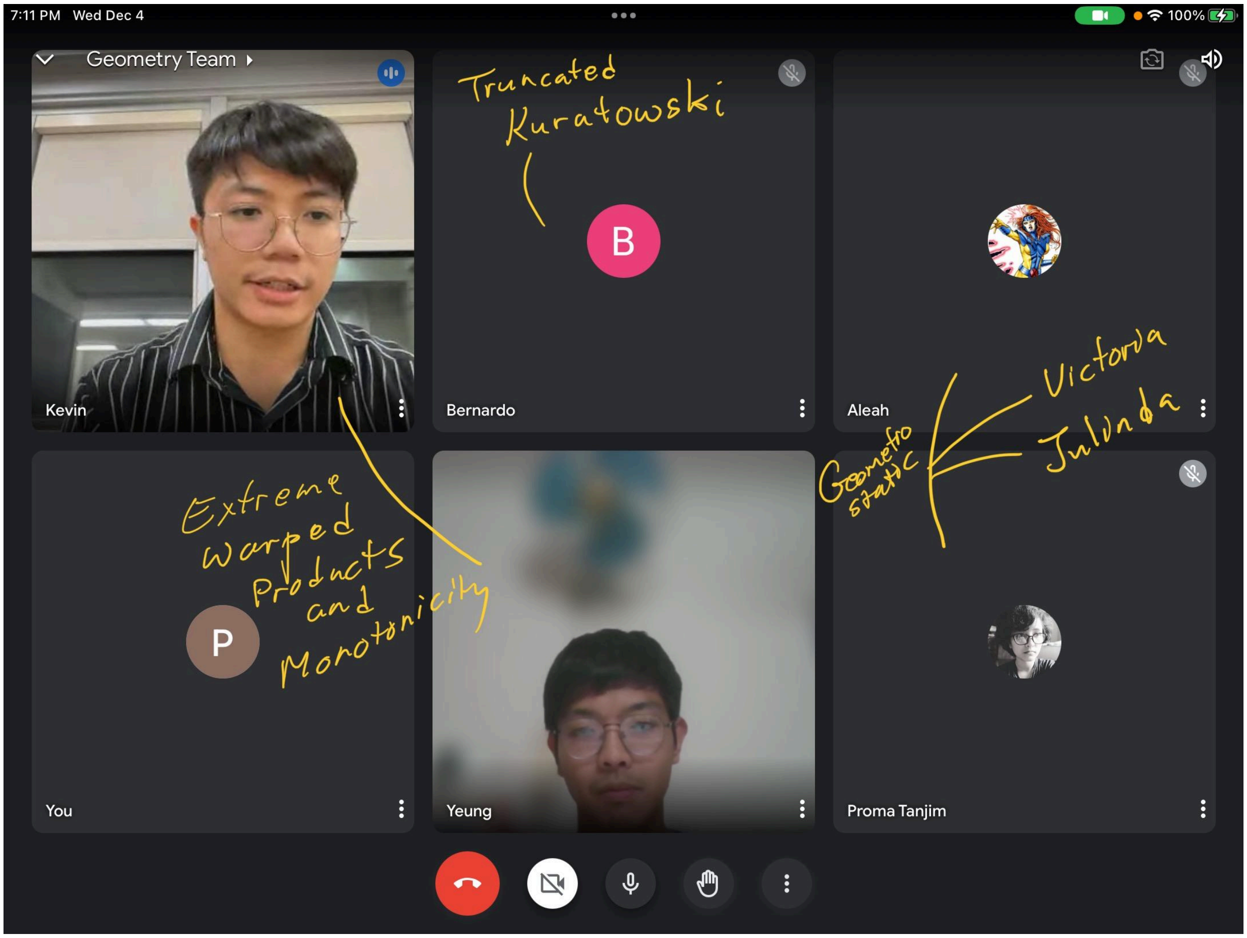
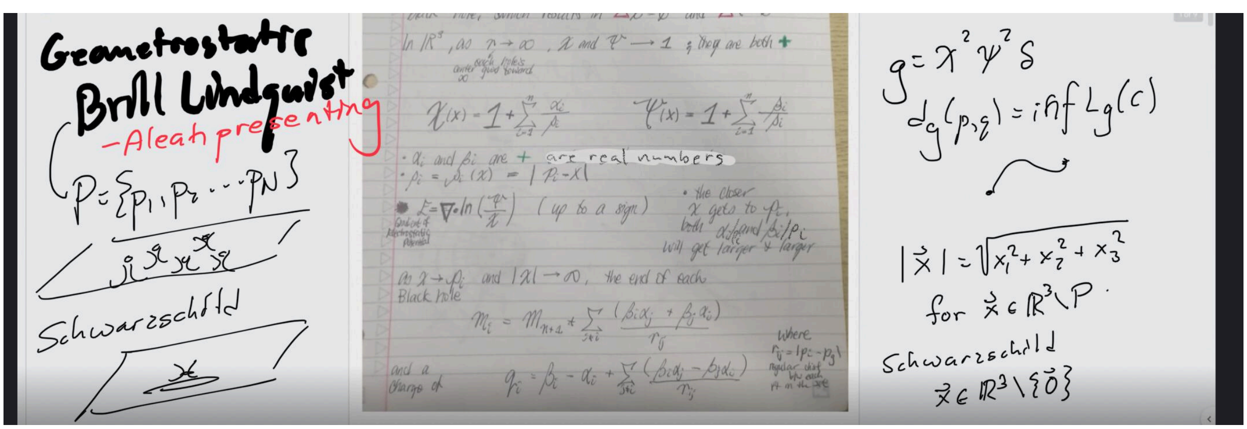
December 4, 2024

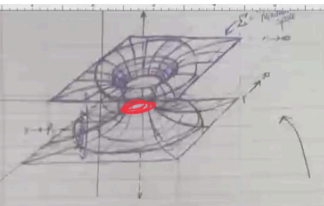


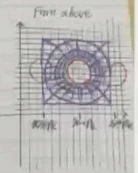
Aleah presents Geometrostatic Project:

Schwarzschild inversion map is its own inverse



Only one pi is called Schwarzschild





event horizon

$$g = \left(1 + \sum \frac{m_i}{r_i}\right)^4 \delta$$

defined on

$\mathbb{R}^3 \setminus \{p_1 \dots p_n\}$ (which represents the center of each Black hole)

The smallest circle region in the neck of the black hole is known as the Event Horizon.

At each level along the neck, a circle represents this region getting closer and closer to the Event horizon and spread out the closer they get to Σ, which is represented as a square surface.

Micah Lydiatte-Hepburn
Geometry Research Team, Fall 2024
?/19/2024

Session 5
Homework #Q1

Check to see if $F(y) = \left(\frac{m}{2}\right)^2 \frac{y}{|y|^2}$ and $F^{-1}(x) = \left(\frac{m}{2}\right)^2 \frac{x}{|x|^2}$ are inverses of each other.

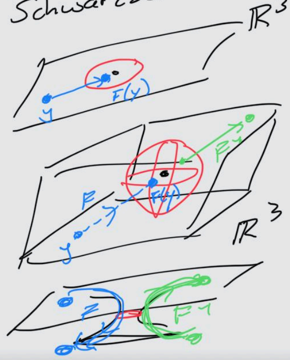
(I) Need the defn. of a Function

A Function, $\underset{\text{Domain}}{f: A} \to \underset{\text{Range}}{B}$, $\forall x \in A$ "for every pt in the domain", $\exists y = f(x) \in B$ "there exists a corresponding result for the function in the range"

(II) Need the defn. of Inverse Function

$f^{-1}: B \to A$ s.t. $f^{-1}(f(x)) = x$, $\forall x \in A$
$f(f^{-1}(y)) = y$, $\forall y \in B$

Schwarzschild



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(1) $F(y) = \left(\frac{m}{2}\right)^2 \frac{y}{|y|^2}$ — by given

Show: $\forall y \in A$, $\exists x = F(y) \in B$ s.t. $\boxed{f^{-1}(f(y)) = y}$, $\forall y \in A$ — by defns of a function and its inverse

(2) $f^{-1}(f(y)) = f^{-1}\left(\left(\frac{m}{2}\right)^2 \frac{y}{|y|^2}\right)$ — by defn of $F(y)$

(3) $= \left(\frac{m}{2}\right)^2 \dfrac{\left(\frac{m}{2}\right)^2 \frac{y}{|y|^2}}{\left|\left(\frac{m}{2}\right)^2 \frac{y}{|y|^2}\right|^2}$ — by defn of $f^{-1}(f(y))$

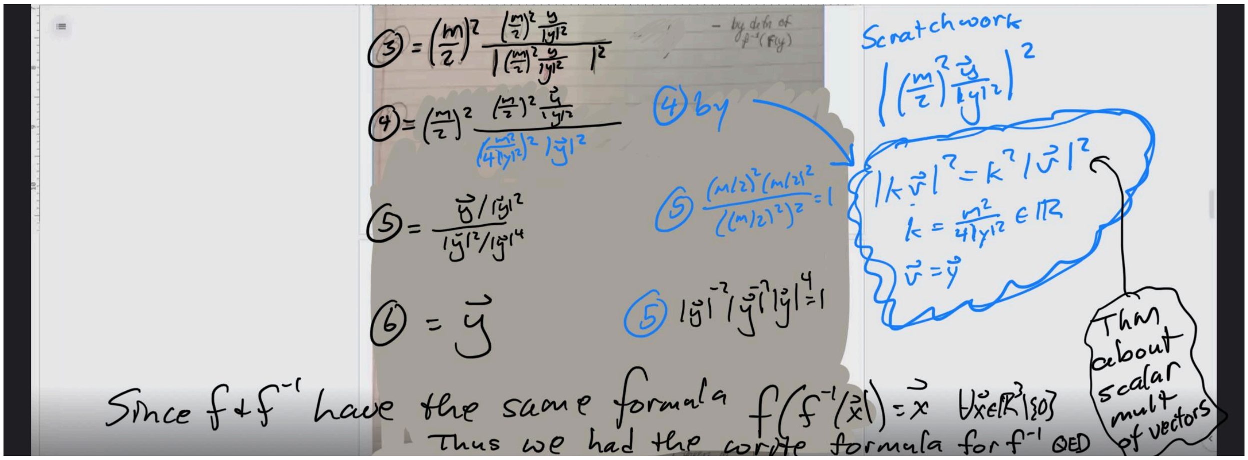
Here $A = \mathbb{R}^3 \setminus \{\vec{0}\}$
and $B = \mathbb{R}^3 \setminus \{\vec{0}\}$

Example

$$F\left(\begin{pmatrix}4\\3\\0\end{pmatrix}\right) = \left(\frac{m}{2}\right)^2 \frac{\begin{pmatrix}4\\3\\0\end{pmatrix}}{\left|\begin{pmatrix}4\\3\\0\end{pmatrix}\right|^2} = \left(\frac{m}{2}\right)^2 \begin{pmatrix}4/25\\3/25\\0\end{pmatrix}$$

$$\begin{pmatrix}4\\3\\0\end{pmatrix} \overset{F}{\longmapsto} \left(\frac{m}{2}\right)^2 \begin{pmatrix}4/25\\3/25\\0\end{pmatrix}$$

$$\left|\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}\right|^2 = v_1^2 + v_2^2 + v_3^2$$



Next step show Schwarzschild inversion map is distance preserving.
Proma did not wish to present. Victoria sick. Julinda in lab.

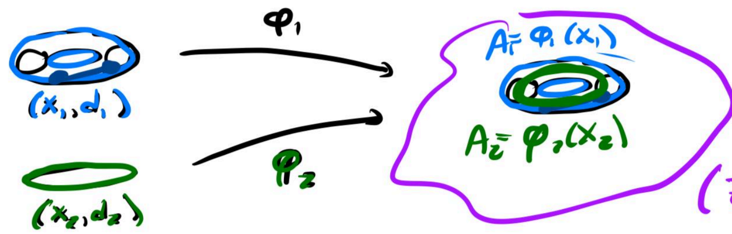
Thomas question:

Gromov-Hausdorff Distance and almost isometries:

(Useful for all teams)

Gromov Hausdorff distance between two compact metric spaces (X_1, d_1) and (X_2, d_2)

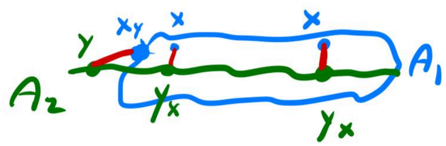
$$d_{GH}((X_1,d_1),(X_2,d_2)) = \inf d_H^Z(\varphi_1(X_1), \varphi_2(X_2))$$



φ_i are distance preserving

where the Hausdorff distance in Z

$$d_H^Z(A_1,A_2) = \inf \left\{ R \;\middle|\; \begin{array}{l} \forall x \in A_1\ \exists y_x \in A_2 \text{ s.t. } d_Z(x,y_x) < R \\ \forall y \in A_2\ \exists x_y \in A_1 \text{ s.t. } d_Z(y,x_y) < R \end{array} \right\}$$



To show the inf is actually a min, is ~~a~~ serious thm by Gromov using compactness.

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<u>Defn</u> ε - Almost Isometries: $f: X_1 \to X_2$ is both:

ε-almost ~~dist~~ pres: $|d_2(f(p),f(q)) - d_1(p,q)| < \varepsilon \ \forall p,q \in X_1$

ε-almost onto $\forall y \in X_2\ \exists x \in X_1$ s.t. $d(y, f(x)) < \varepsilon$

<u>Thm</u> [Gromov]
If $\exists\ \varepsilon$ almost isom $f: X_1 \to X_2$
then $d_{GH}(X_1, X_2) < 2\varepsilon$

<u>Thm</u> [Gromov]
If $d_{GH}(X_1,X_2) < R$ then
$\exists\ 3R$ almost isom $F: X_1 \to X_2$.

← Lets prove this

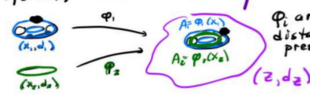
Given: $d_{GH}(X_1, X_2) < R$ Show $\exists F: X_1 \to X_2$

Given: $d_{GH}(X_1, X_2) < R$ Show $\exists F: X_1 \to X_2$ that is a $3R$ almost isom.

Proof:

① $d_{GH}(X_1, X_2) < R$ ① Given

② $\inf d_H^Z(\varphi_1(X_1), \varphi_2(X_2)) < R$ ② by defn of GH

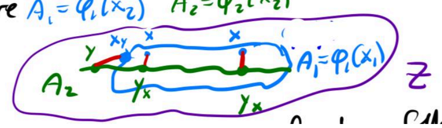


φ_i are distance preserving

③ $\exists$ metric space (Z, d_Z) and dist pres maps $\varphi_i : X_i \to Z$ $d_H^Z(\varphi_1(X_1), \varphi_2(X_2)) < R$ ③ by defn of inf.

④ $\forall x \in A_1\ \exists y_x \in A_2$ s.t. $d_Z(x, y_x) < R$
$\forall y \in A_2\ \exists x_y \in A_1$ s.t. $d_Z(y, x_y) < R$ ④ by defn of d_H^Z

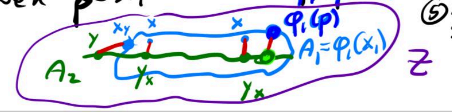
where $A_1 = \varphi_1(X_2)$ $A_2 = \varphi_2(X_2)$



* Let $F: X_1 \to X_2$ be defined as follows

⑤ given $p \in X_1$ there is $\varphi_1(p) \in \varphi_1(X_1) \subset Z$

⑤ by step ③



⑥ $\exists\, y_{\varphi_1(p)} \in A_2$ s.t. $d_Z(\varphi_1(p), y_{\varphi_1(p)}) < R$

⑥ by step ④

⑦ $A_2 = \varphi_2(X_2)$ so $\exists\, q \in X_2$ s.t.

$\varphi_2(q) = y_{\varphi_1(p)}$.

⑦ defn of image and step ⑥

Let $F(p) = q$

* Show F is an $3R$-almost isom.

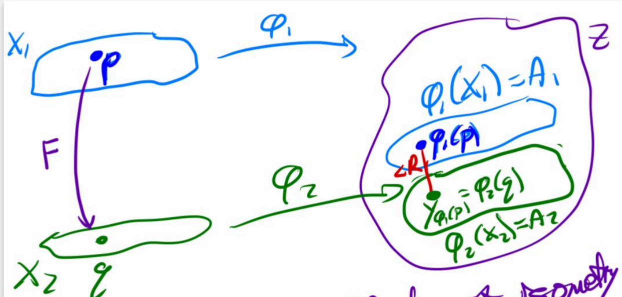
Try for homework

Hint use ⑤, ⑥, ⑦

and use φ_1 and φ_2 are distance preserving

and use triangle inequality on (Z, d_Z)

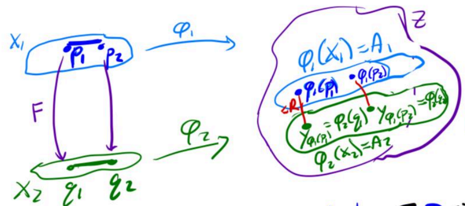
What $F: X_1 \to X_2$ is:



Now show F is $3R$ almost isometry

X_2 q

Now show F is $3R$ almost isometry



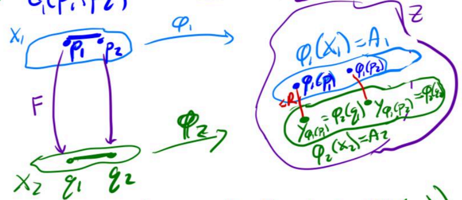
picture for HW1

Show $|d_2(q_1, q_2) - d_1(p_1, p_2)| < 2R < 3R$ $\boxed{HW1}$

Show $\exists q \in X_2$ $\exists p' \in X_1$ $d_{X_2}(F(p'), q) < 3R$ $\boxed{HW2}$

Work towards HW1

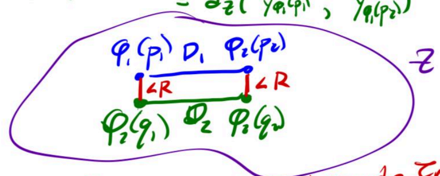
(8) $d_1(p_1, p_2) = d_Z(\varphi_1(p_1), \varphi_1(p_2))$ because φ_1 is dist pres



(9) $d_2(q_1, q_2) = d_Z(\varphi_2(q_1), \varphi_2(q_2))$ because φ_2 is dist pres

$= d_Z(y_{\varphi_1(p_1)}, y_{\varphi_1(p_2)})$

(10)



(11) $D_1 \leq R + D_2 + R$ by triangle Ineq

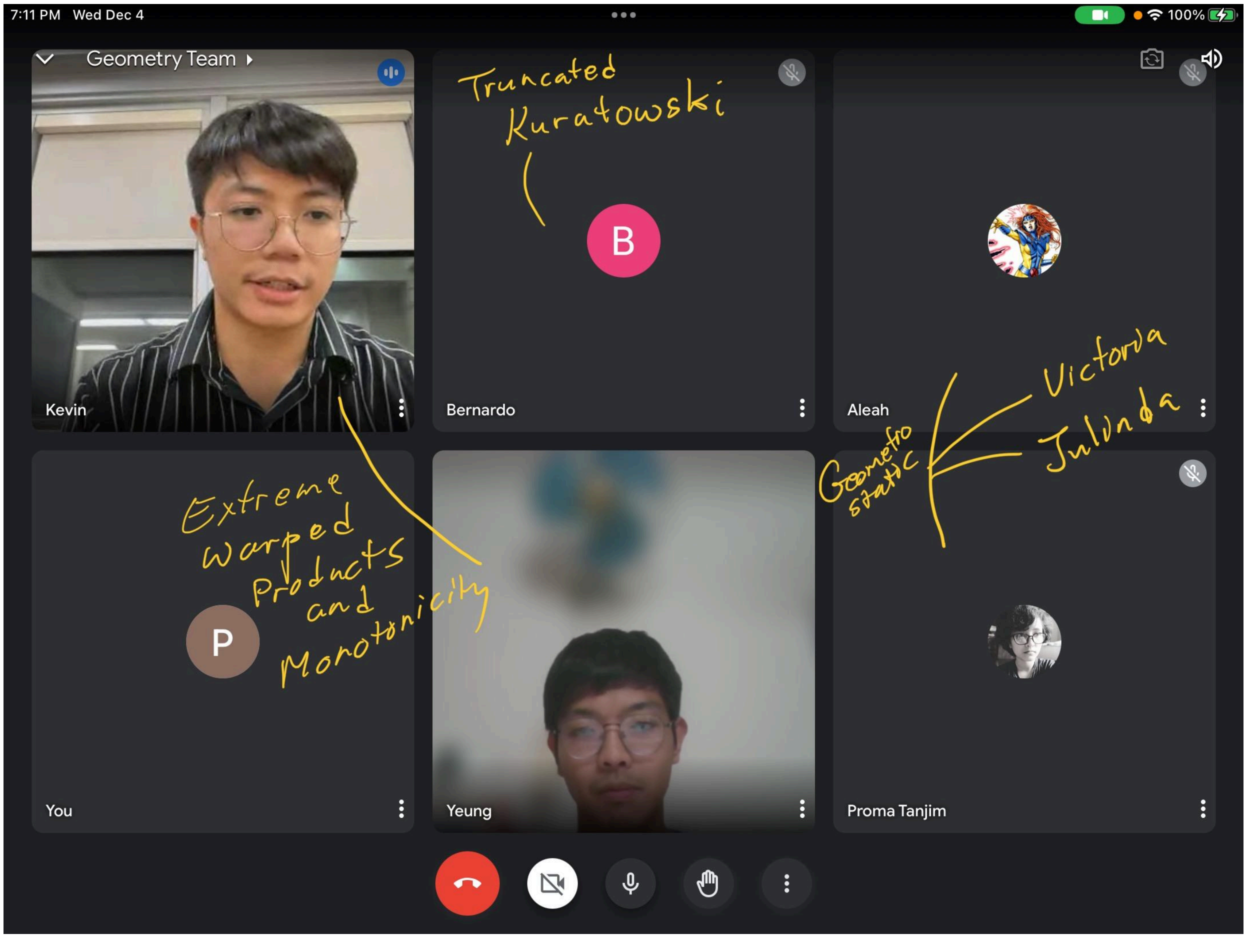
$D_2 \leq R + D_1 + R$

(12) $D_1 - D_2 \leq 2R$ by algebra

$D_2 - D_1 \leq 2R$

(13) $|D_1 - D_2| \leq 2R$

HW1 basically done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Geometry Team
Truncated Kuratowski
Kevin
Bernardo
Aleah
Victoria
Julinda
Geometro static
Extreme warped Products and Monotonicity
You
Yeung
Proma Tanjim