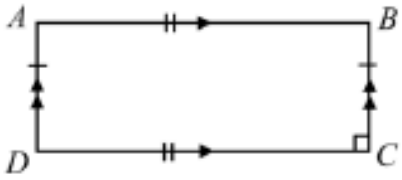
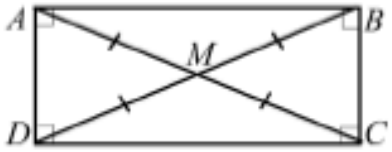
Proving that a Quadrilateral is a Rectangle

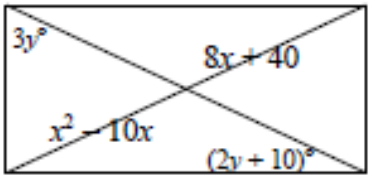
SOL G.9 (2016)

Definition of a Rectangle	
A quadrilateral with four right angles	

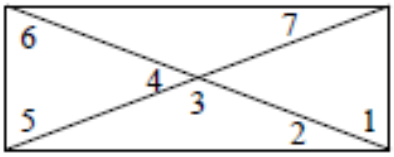
Properties of Rectangles	
If a parallelogram has one right angle, then the remaining three angles will also be right angles	
If the diagonals are congruent and they bisect one another, then the parallelogram is a rectangle. $\overline{AC} \cong \overline{BD}$ $\overline{AM} \cong \overline{BM} \cong \overline{CM} \cong \overline{DM}$	

Practice

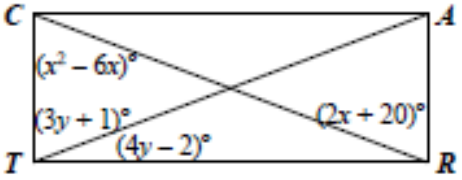
Find x and y so that the quadrilateral is a rectangle.

1. 

2. Find each angle in the rectangle if $m\angle 1 = 52$.



Example 1: Find x and y so that the quadrilateral is a rectangle.

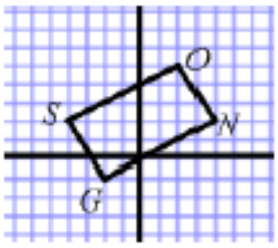


$x^2 - 6x = 2x + 20$
$x^2 - 6x - 2x - 20 = 0$
$x^2 - 8x - 20 = 0$
$(x + 2)(x - 10) = 0$
$x = -2$ or $x = 10$

$(3y + 1) + (4y - 2) = 90$
$7y - 1 = 90$
$7y = 91$
$y = 13$

1. Angles *TCR* and *ARC* are congruent because they are alternate interior angles. Write an equation setting the two expressions equal to each other.
2. Write the equation as a trinomial equal to zero.
3. Factor the trinomial. (A product-sum table may be helpful.)
4. Eliminate any "impossible" solutions.
5. Angles *CTA* and *RTA* are complementary, so write an equation show that their sum is 90.
6. Solve for y.

Example 2: Quadrilateral *SONG* has vertices *S*(-4, 2), *O*(2, 5), *N*(4, 2), and *G*(-2, -1). Determine whether *SONG* is a rectangle. Justify your answer.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{\overline{SO}} = \frac{5-2}{2-(-4)} = \frac{3}{6} = \frac{1}{2}$$

$$m_{\overline{ON}} = \frac{2-5}{4-2} = \frac{-3}{2}$$

If *SONG* is a rectangle, then its angles will be right angles. The slope formula can be used to check for right angles. If the slopes of two adjacent sides are opposite reciprocals, then the lines are perpendicular and for a right angle.

SONG is *not* a rectangle, because its sides do not form right angles.

*To prove that set of vertices forms a rectangle, a *pair of opposite angles* must be proven congruent. It is not enough to simply prove that one pair of sides form a right angle. If angle *O* was 90°, then the next step would be to show that angle *G* was also 90°.

Determine whether the given set of vertices forms a rectangle. Justify your answer.

3. Quadrilateral *PINK* *P*(-3, 7), *I*(3, 4), *N*(1, 1), *K*(-5, 4)