

[Vector Calculus MAT226 Fall 2021](#)

[Professor Sormani](#)

[Lesson 13: Level sets and Contour Maps \(13.1\)](#)

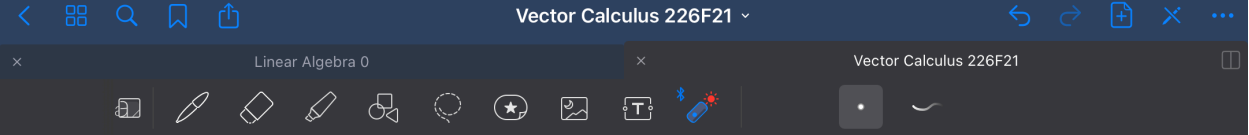
Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together. Please wear masks when meeting with classmates even if you meet off campus.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT226F21-lesson13-lastname-firstname

and share editing of that document with me sormanic@gmail.com and with our graders. If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT226 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

Watch [Playlist 226F21-13-1to12](#)



Contour Maps + Level Sets

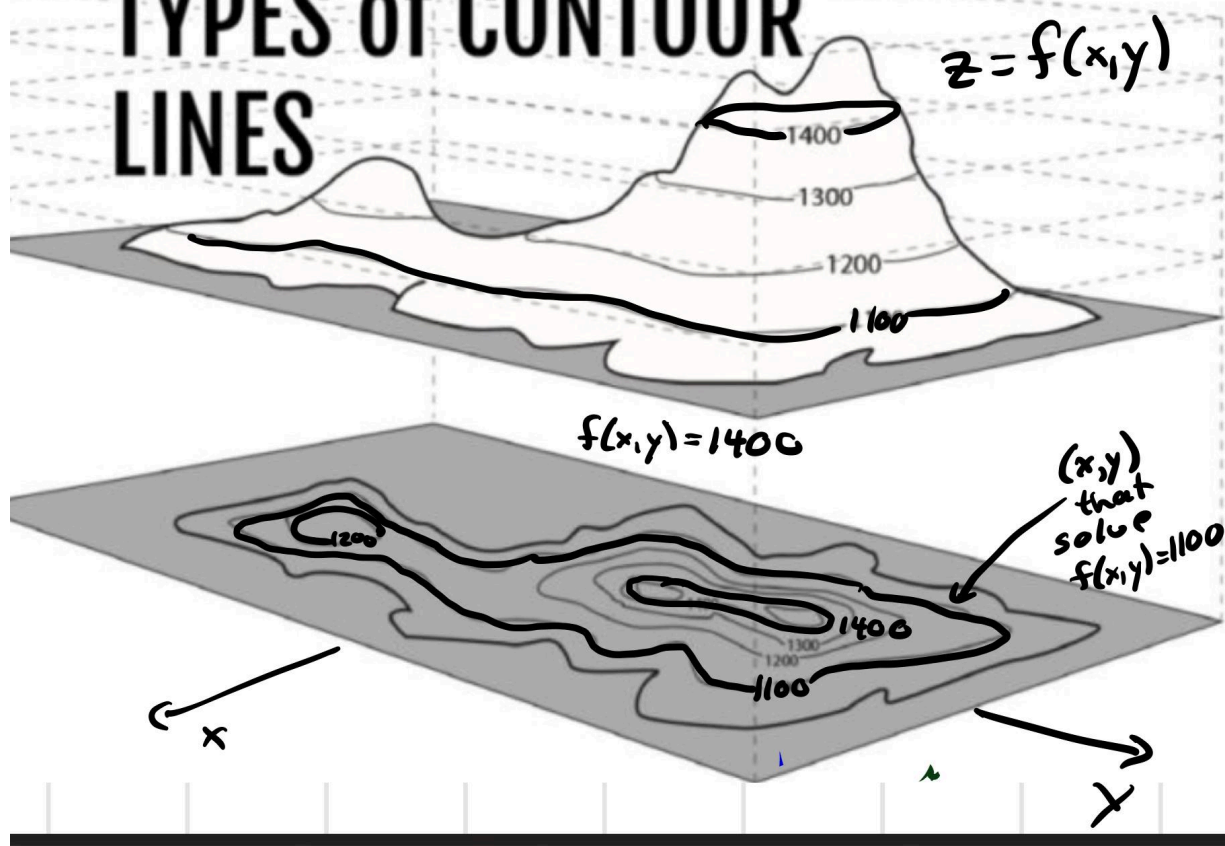
Bronx Land Surveyor
 We Provide Property Boundary Surveys, Engineering Surveys, and Architectural Surveys.

This chapter starts with tips on how to read the margins of a topographic map. Then it describes how to interpret contour lines. Finally, it covers how to estimate slope, aspect, acreage, distances, and percent contained using a topographic map.

TYPES of CONTOUR LINES

1200 (contour)
 places below 1100
 above 1500
 1300-1400
 1400-1500

TYPES of CONTOUR LINES



What Is Terrace Farming?



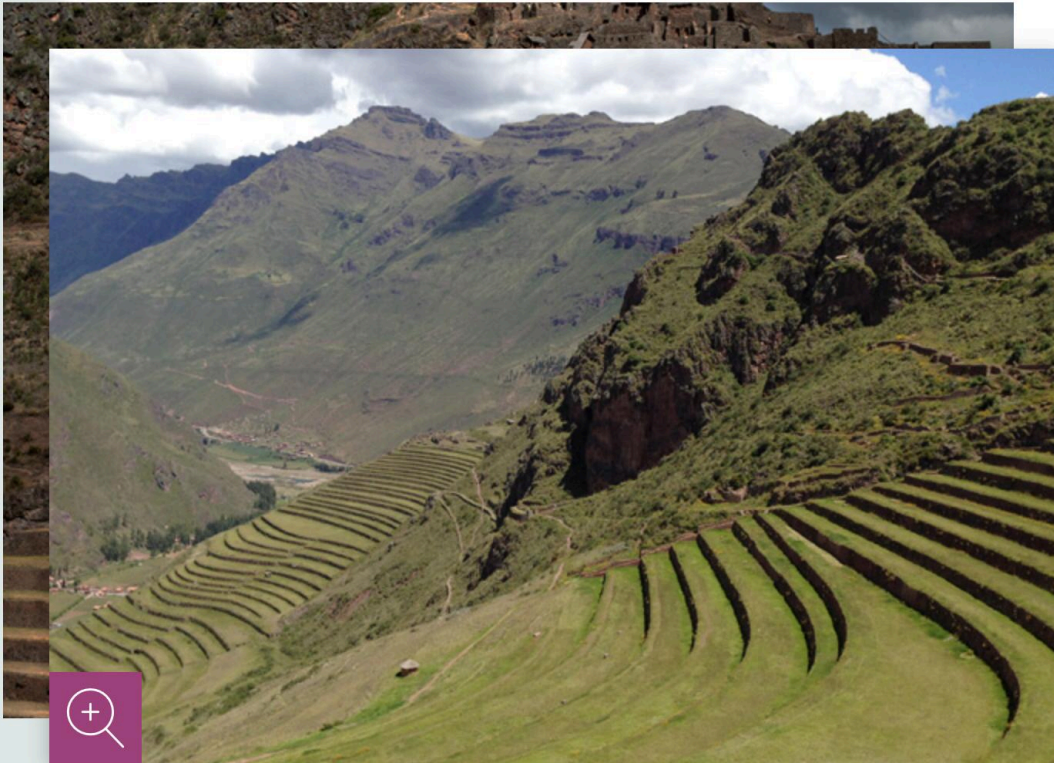
Terraced Rice Field in Chiangmai, Thailand. Image credit: Chatrawee Wiratgasem/Shutterstock.com

- **Terrace farming is a method of farming whereby “steps” known as terraces are built onto the slopes of hills and mountains to be used for crop cultivation.**
- **Terrace farming is commonly used in Asia by rice-growing countries such as Vietnam, Philippines, and Indonesia.**
- **Terrace farming prevents the washing away of soil nutrients by the rains. This leads to the growth of healthy crops.**
- **There are two types of terracing known as graded terracing and level terracing.**



2. Rice terraces in the late afternoon light in the mountainous district of Mu Cang Chai, in the northwestern Vietnamese province of Yen Bai on October 23, 2012. #

Hoang Dinh Nam / AFP / Getty



Many of the slopes in the Andes region of Peru have been reshaped by terracing, a technique perfected by the Inka to extend the agricultural area in this steep mountain range.

Scroll through images of Inka terracing in various ancestral and contemporary contexts in the Andes.

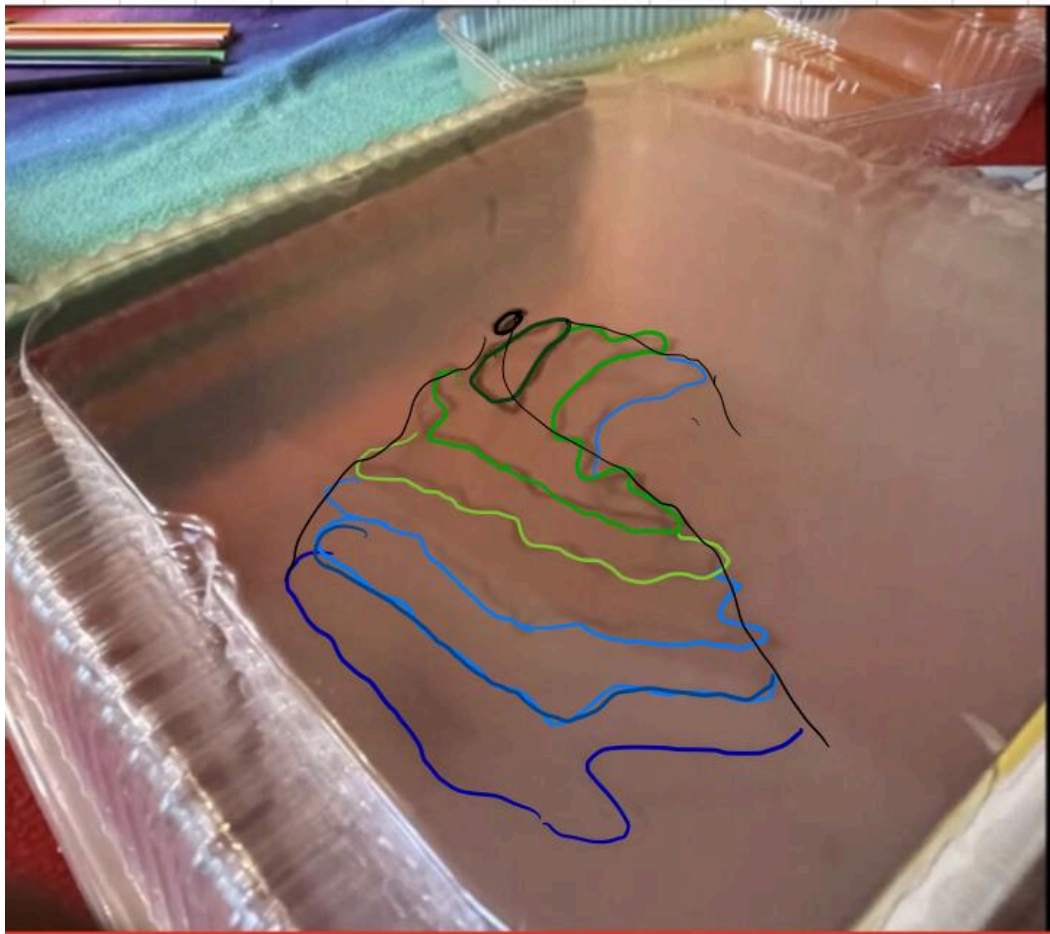
2/7

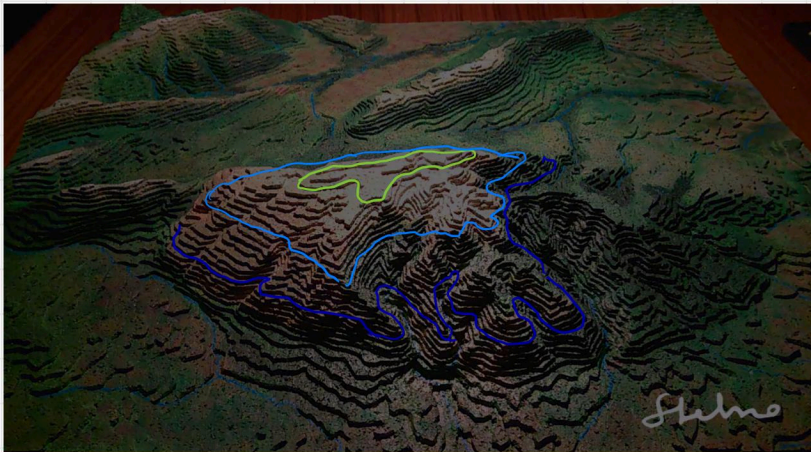


Detail of the terracing in Pisac, 2014.

Photo by Doug McMains, NMAI.







The cork was cut along the level sets to draw this 3D map.

3D TOPOGRAPHIC MAPPING AND PAINTING

333 views



9



0



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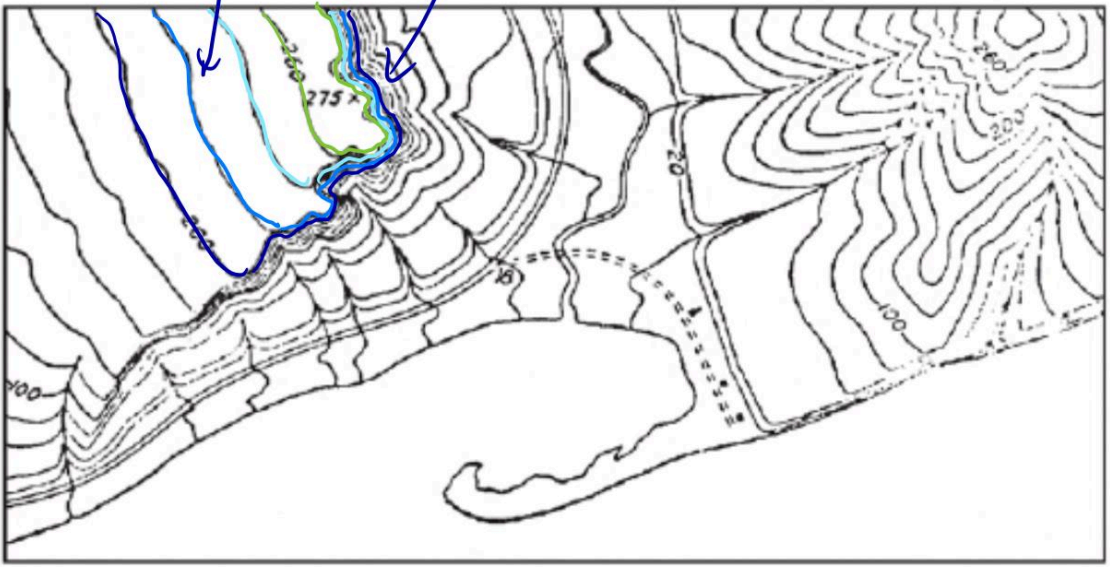
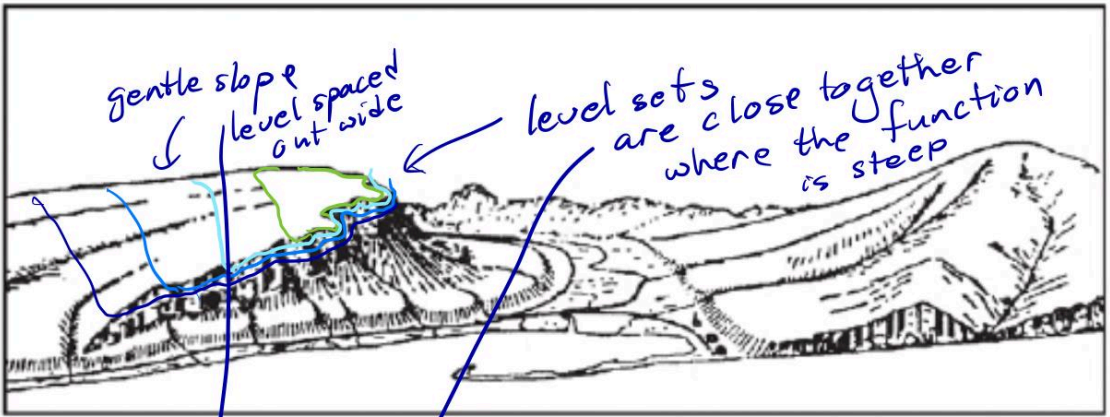


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Stelvio Staffieri
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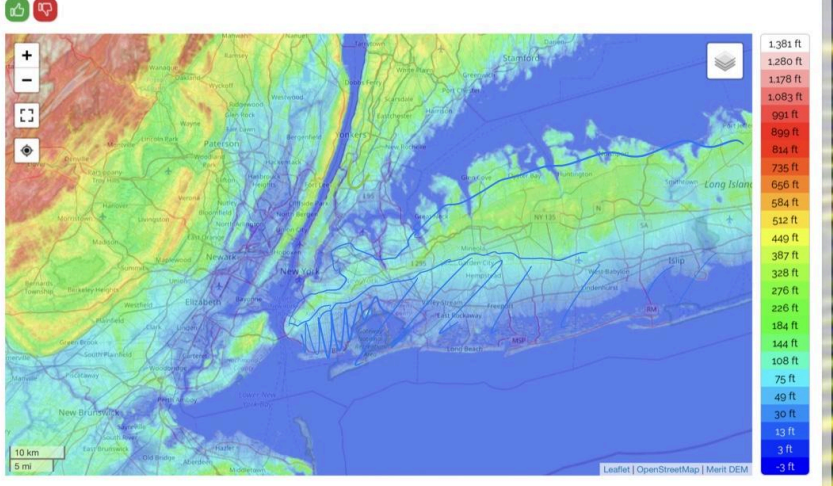
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New York

Topographic maps > United States > New York > New York > New York

Click on the map to display elevation.

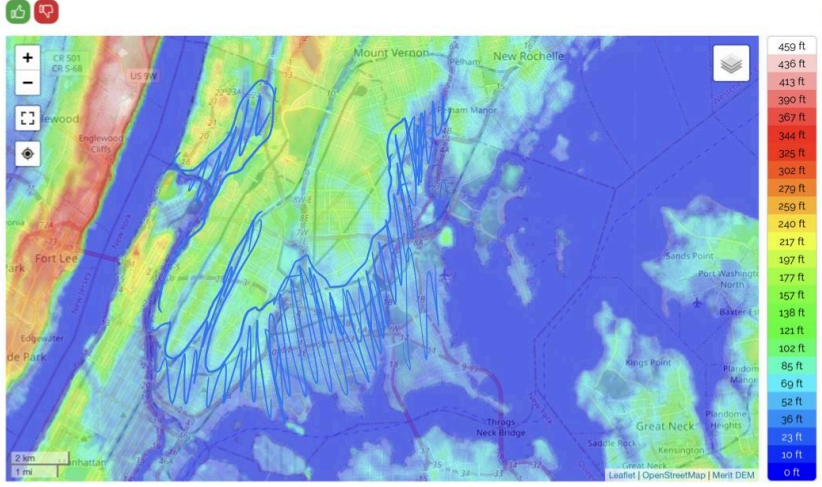


75 ft above sea level
3 ft above sea level
-3 ft below sea level

Bronx County

Topographic maps > United States > New York > New York > Bronx County

Click on the map to display elevation.

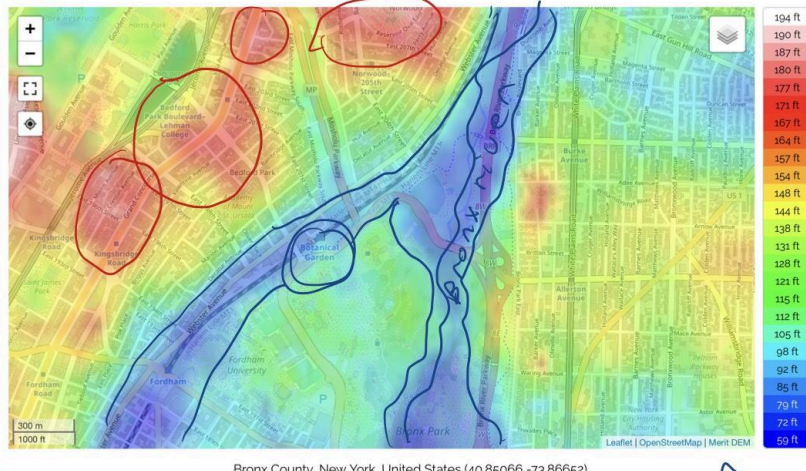


Bronx County, New York, United States (40.85066 -73.86652)

Bronx County

Topographic maps > United States > New York > New York > Bronx County

Click on the map to display elevation.



Lehman College

167 feet above sea level

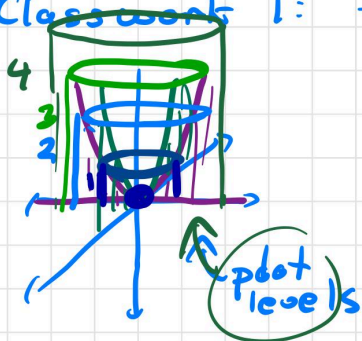
79 feet above sea level

Extra Credit Make a model in 3D carefully trace levels where colors change + build.

Google Earth has level set maps. if you wish to choose a different location.

Drawing the Contour Map of $f(x,y)$.

Classwork 1: $f(x,y) = x^2 + y^2$



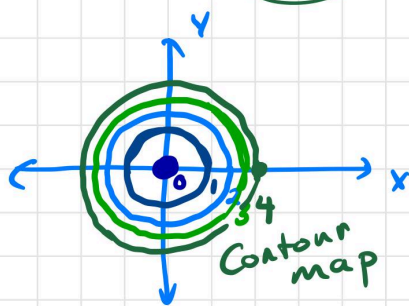
$$f(x,y) = 0 \Rightarrow x^2 + y^2 = 0 \Rightarrow (x,y) = (0,0)$$

$$f(x,y) = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow (x,y) \text{ on circle of radius } 1$$

$$f(x,y) = 2 \Rightarrow x^2 + y^2 = 2 \Rightarrow (x,y) \text{ on circle of radius } \sqrt{2}$$

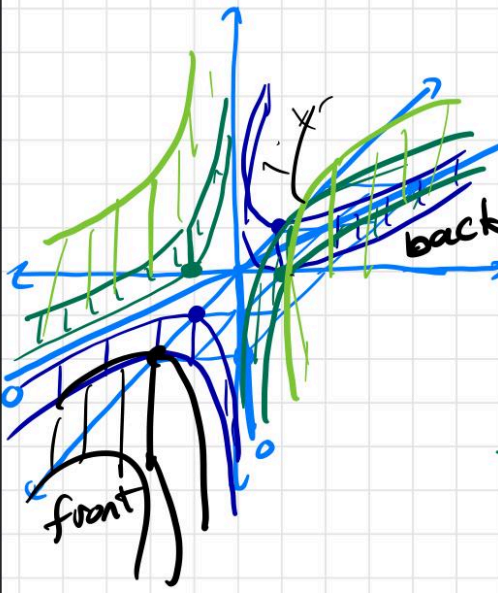
$$f(x,y) = 3 \Rightarrow x^2 + y^2 = 3 \Rightarrow (x,y) \text{ on circle of radius } \sqrt{3}$$

$$f(x,y) = 4 \Rightarrow x^2 + y^2 = 4 \Rightarrow (x,y) \text{ on circle of rad} = \sqrt{4} = 2$$



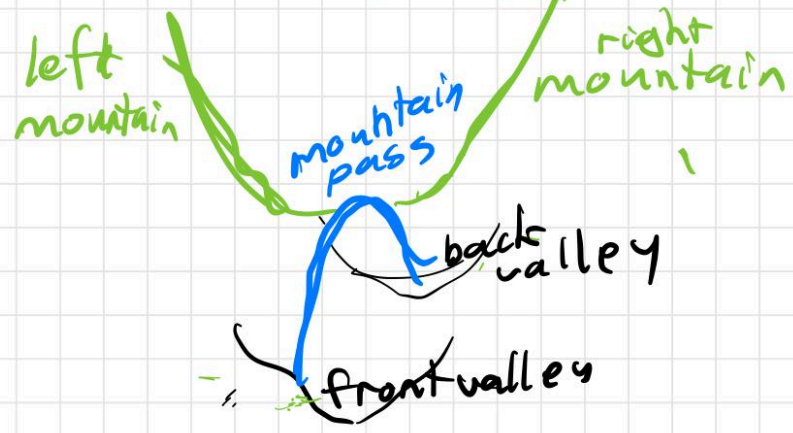
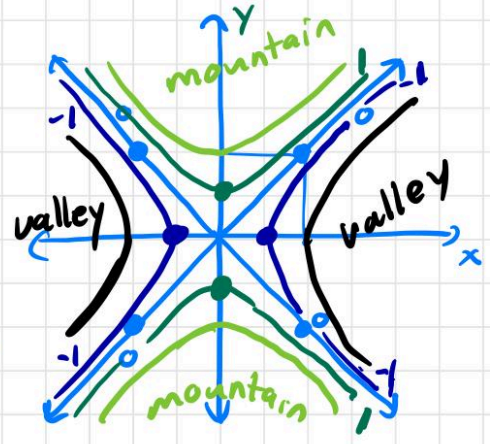
Contour Map is drawn in the xy plane and label the value of $f(x,y)$

Classwork 2: Draw Levels and Contour Map for $f(x,y) = y^2 - x^2$



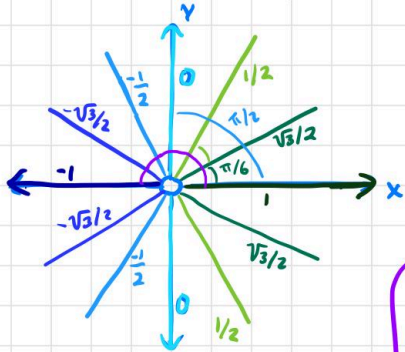
$f(x,y) = 0$
 $y^2 - x^2 = 0$
 $y^2 = x^2$
 $y = \pm x$
 $f(x,y) = -1$
 $y^2 - x^2 = -1$
 hyperbola
 x never 0
 $(1,0) + (-1,0)$

$f(x,y) = 1$
 $y^2 - x^2 = 1$
 hyperbola
 y never 0
 $(0,1) + (0,-1)$



Classwork 3: Start with the Contour Map and find the function.

Contour Map:



Notice the level sets are half lines that do not include $\vec{0}$

The half line at angle $\theta = 0^\circ$ which is the positive x axis has $f(r, 0^\circ) = 1$ for $r > 0$

polar coords

$$f(r, \frac{\pi}{6}) = \frac{\sqrt{3}}{2} \text{ for } r > 0$$

$$f(r, \frac{\pi}{2}) = \frac{1}{2} \text{ for } r > 0$$

$$f(r, \frac{\pi}{2}) = 0$$

$$f(r, \frac{2\pi}{3}) = -\frac{1}{2}$$

$$f(r, \frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$$

$$f(r, \pi) = -1$$

pause
+
try

use polar coords because the levels are radial

$$f(r, \theta) = \cos(\theta) \text{ for } r > 0$$

Want $f(x, y) = ?$

convert $\cos(\theta)$ to rect. coords

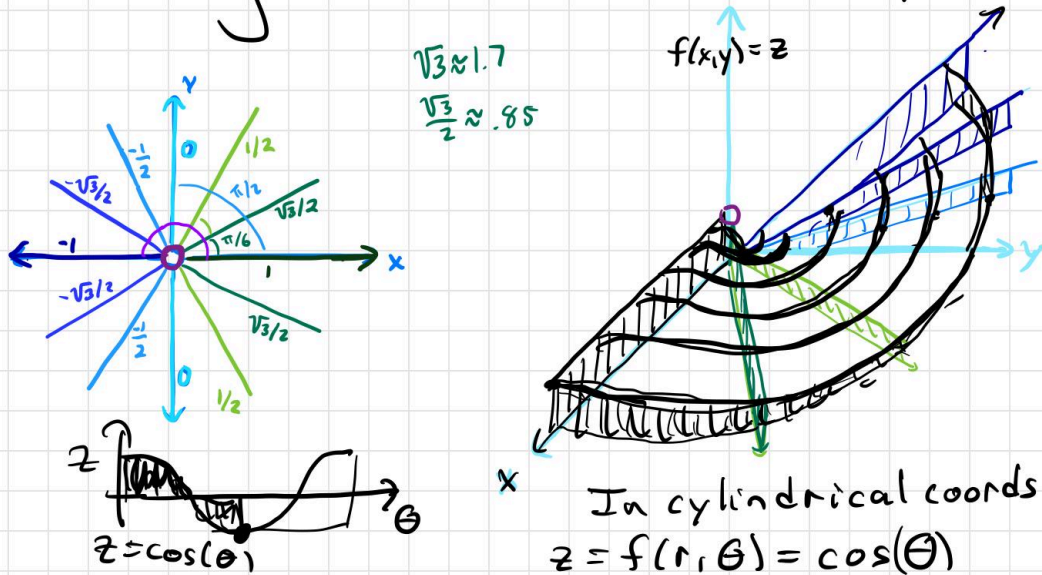
$x = r \cos \theta$ in polar coords

$$\text{solve for } \cos \theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$$

Notice the domain does not include $(0, 0)$.

Next graph this function using the contour map.



This cosine curve is being wrapped around the z axis so it goes from 1 to 0 to -1 quickly near the z axis (very steep) and goes slowly far from the z axis.

Extra Credit: Plot using MATLAB or Maple.

Classwork 4: Find $f(x,y)$

for the contour map:

Use polar coords because
the shapes are circles

$$f(0, \theta) = -3$$

$$f(1, \theta) = -2$$

$$f(2, \theta) = -1$$

$$f(3, \theta) = 0$$

$$f(4, \theta) = 1$$

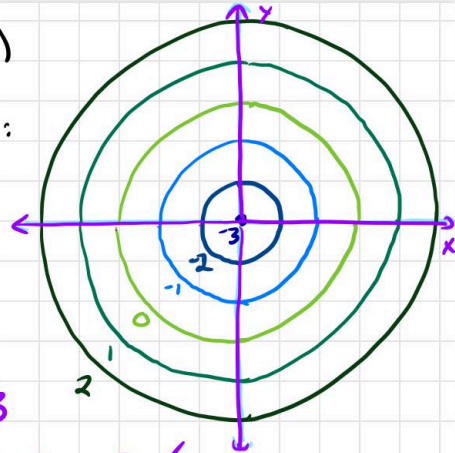
$$f(5, \theta) = 2$$

$$f(r, \theta) = r - 3$$

$$f(0, \theta) = 0 - 3 = -3 \checkmark$$

$$f(1, \theta) = 1 - 3 = -2 \checkmark$$

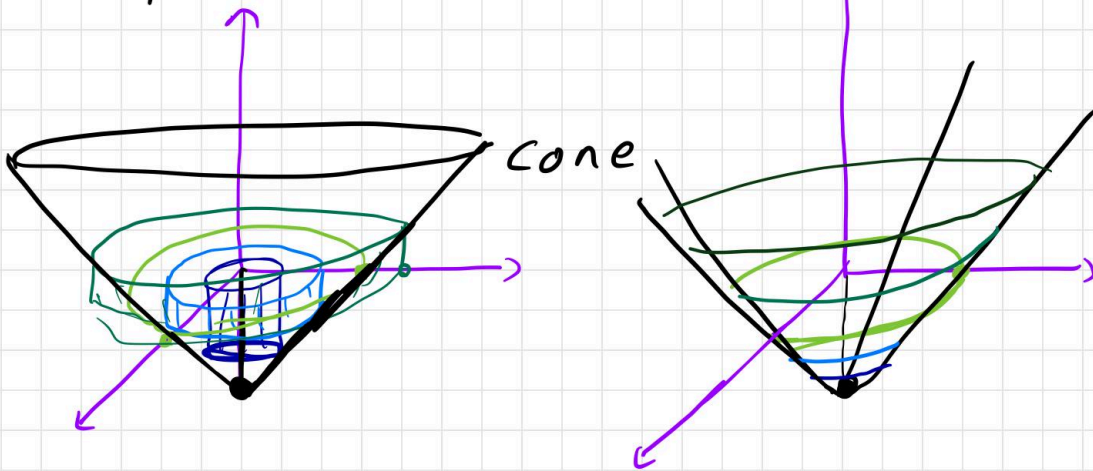
$$f(4, \theta) = 4 - 3 = 1 \checkmark$$



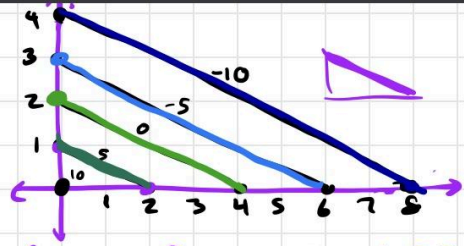
Convert to rectangular coords.

$$f(x,y) = \underbrace{\sqrt{x^2+y^2}}_r - 3$$

Graph the function:



Classwork 5: Find and
Plot the function with
this contour map →



This function $f(x,y)$ is defined for $x \geq 0$ and $y \geq 0$

$$\begin{aligned} f(0,0) &= 10 \\ f(2,0) &= 5 = f(0,1) \\ f(4,0) &= 0 = f(0,2) \\ f(6,0) &= -5 = f(0,3) \\ f(8,0) &= -10 = f(0,4) \end{aligned}$$

all our lines
have the same
slope $\frac{\Delta y}{\Delta x} = \frac{1-0}{0-2} = -\frac{1}{2}$
all are parallel
 $-\frac{a}{b} = -\frac{1}{2}$

$f(x,y) = ?$
pause + try
guess:

$$\begin{aligned} f(x,y) &= C - ax - by \\ \text{the levels of this} \\ &\text{function are lines} \\ k &= C - ax - by \\ by &= C - k - ax \\ y &= \frac{C-k}{b} - \frac{a}{b}x \\ &\text{line of slope } -\frac{a}{b} \end{aligned}$$

$f(x,y) = C - ax - by$
plug in some values to find
 $C, a,$ and b
pause + try

$$10 = f(0,0) = C - a \cdot 0 - b \cdot 0 = C \quad \text{so } C = 10$$

$$f(x,y) = 10 - ax - by$$

$$5 = f(2,0) = 10 - a \cdot 2 - b \cdot 0$$

$$\text{so } 5 = 10 - 2a \quad \text{so } 2a = 10 - 5 = 5 \\ \text{so } a = 5/2$$

$$5 = f(0, 1) = 10 - a \cdot 0 - b \cdot 1$$

$$\text{so } 5 = 10 - b \quad \text{so } b = 10 - 5 = 5$$

$$f(x, y) = 10 - \frac{5}{2}x - 5y$$

$$\text{Slope} = \frac{-a}{b} = \frac{-5/2}{5} = -\frac{1}{2} \checkmark$$

Also check the other levels

$$f(0, 0) = 10 \quad \checkmark$$

$$f(2, 0) = 5 = f(0, 1) \quad \checkmark$$

$$f(4, 0) = 0 = f(0, 2)$$

$$f(6, 0) = -5 = f(0, 3)$$

$$f(8, 0) = -10 = f(0, 4)$$

$$f(4, 0) = 10 - \frac{5}{2} \cdot 4 - 5 \cdot 0 = 0 \checkmark$$

$$= 10 - 10 - 0$$

$$f(0, 2) =$$

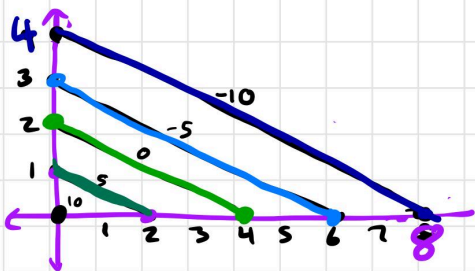
$$f(6, 0) =$$

$$f(0, 3) =$$

$$f(8, 0) =$$

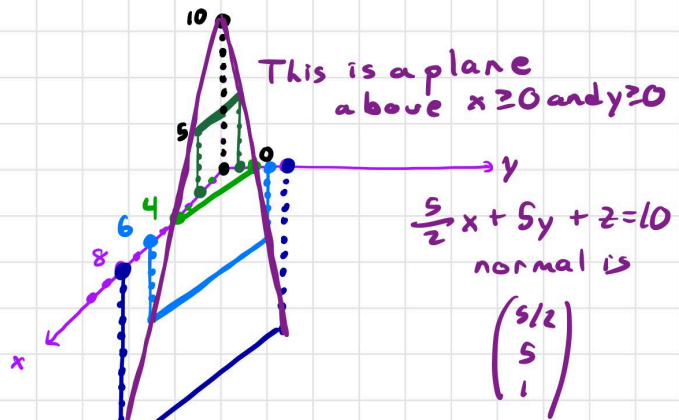
$$f(0, 4) =$$

check
all



Plot $f(x, y) = 10 - \frac{5}{2}x - 5y$

$$z = 10 - \frac{5}{2}x - 5y$$



HW (do before next lesson): 13.1/ two contour maps, three descriptions of level sets, three sketch graph of levels, three applications

Review HW: Dot products and planes 11.3-11.5