The Math and Physics of…

Suicide Burns

Introduction

The main theme of this part of the series is landing. How to land, where to land, and how to do it efficiently.

For all the analysis, we will be doing this while flying around Mun so we will use these parameters. (We are also assuming Mun is a super awesome nice perfect sphere):

$$\mu = Gravitational\ Parameter\ of\ Mun = 6.5138398 \times 10^{10}\ m^3/s^2$$

$$R_{mun} = Surface\ Radius\ of\ Mun = 200,000\ m$$

Suicide Burns

IMPORTANT NOTE: There is a lot of speculation on what makes the most efficient way to land. Let me be the first to tell you that they are not a solution to use for all cases. Just for one specific case.

When to use a suicide burn? Suicide burns are made the most efficient when you are falling straight down and want to land on a body and use the least amount of fuel. Lets begin with a Free Body Diagram of a rocket to analyze the forces involved and to define our directions.

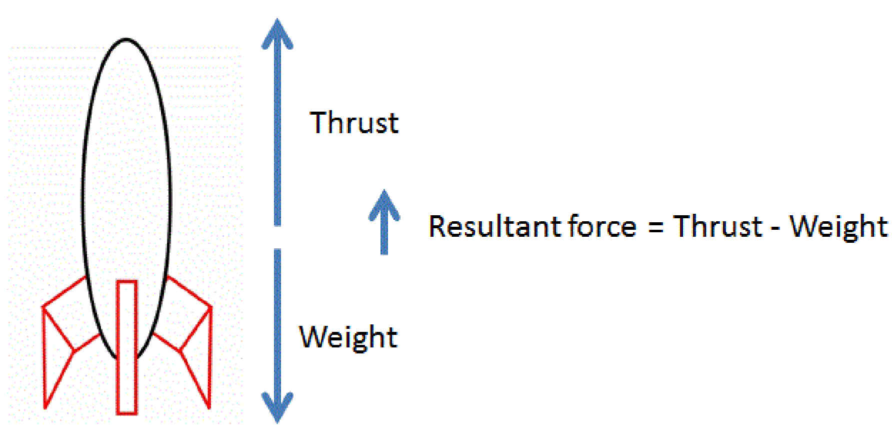

Figure 1

From Figure 1, we see its simply Gravity or Weight and Thrust acting on the rocket. This makes sense since there is no atmosphere on Mun, thus no drag and no other forces to consider. The resultant force is an important point that we will come to later.

Lets talk about how a Suicide Burn is performed.

1. The rocket is falling straight down
2. The rocket will gain speed and lose altitude
3. As it is falling, the distance, and time, it takes using maximum thrust to get its vertical speed to 0 will increase.
4. At some point along its falling, the distance it needs to slow down to 0 and the altitude it is currently at will match.
5. At this point you have to fire your engines to slow down or else by the time you reach the surface you will not be traveling slow enough and crash.

That is the basics of the maneuver, so with this knowledge we can derive some mathematical models of the rocket as its falling. The first and foremost is we need to determine the time it takes for the rocket to fall from a certain distance, as well as its speed along that path.

If you have ever taken Physics 101 in High School or College, you have most likely come across this equation:

$$Distance = V_i t + 1/2at^2 \quad \text{Eq. 1}$$

$$Distance = Distance\ traveled\ by\ an\ object\ in\ motion$$

$$V_i = The\ initial\ speed\ of\ an\ object$$

$$a = The\ acceleration\ experienced\ by\ an\ object$$

$$t = The\ time\ elapsed\ of\ the\ analysis$$

We can set all of these parameters into our analysis. For our derivation of the suicide burn, the rocket will have these starting parameters:

$$Initial\ Vertical\ Speed = 0\ m/s$$

$$Initial\ Altitude = 10\ km$$

$$a = g = surface\ acceleration\ due\ to\ gravity = 1.628\ m/s^2$$

Using the equation above, ‘Distance’ will be our starting altitude, ‘a’ will be ‘g’ or 1.628 m/s^2, and ‘V_i’ will be 0. Then we can rearrange this to solve for ‘t’ time.

$$(10,000\, meters) = (0)t + 1/2(g)t^2$$

$$\sqrt{2 \times 10,000 \times 1/g} = \sqrt{t^2}$$

$$t = 110.4\, seconds$$

Meaning, that if we just let the rocket fall, and do nothing about this, it will hit the ground about 2 minutes later. So HURRY WE HAVE 2 MINUTES TO SAVE JEB!!!

Just kidding this is fictional… I think.

But really is it all that dangerous? How fast would he really hit the ground? Let us solve for this. This brings the next important equation: How to determine speed with a constant acceleration.

$$V_f = V_i + at$$

$$V_f = Final\, Speed\, after\, acceleration$$

Thus plugging in our initial values and the time that we solved for we get a speed of 293 m/s.

I don’t know about your mods, but I know the stock parts are not rated to handle a 200+ m/s impact… So the lesson is we have to slow down.

We can do it gradually if you would like. Do several small burns all the way down so we don’t go too fast. However there is no excitement in that, and we lose out on some efficiency.

Let’s switch focus now and analyze the ship under power, that is to say throttled up. The equation will remain the same however now we must determine what the acceleration will be. For that, we need the free body diagram in Figure 1.

It is a simple summation of the forces involved in the vertical direction. Or in a simpler case the summation of the *accelerations* produced by each of the forces involved. Let’s set some parameters for our ship:

$$Mass = 25\, tonnes$$

$$TWR = Thrust\, to\, Weight\, Ratio = 3$$

Therefore…

$$Max\ Acceleration\ =\ a_{max}\ =\ TWR\ *\ g\ =\ \ 4.884m/s^2$$

If we then do a summation of the accelerations in the vertical direction (since we are only falling down with no horizontal speed) we get:

$$a_{max}\ -\ g\ =\ 4.884\ -\ 1.628\ =\ 3.256\ m/s^2$$

Thus we see that when we engage full throttle on the craft, in its current state, it will accelerate vertically 3.256 m/s^2. So… if you think about this, this is also the fastest that the ship can slow down. At some point, the rocket will turn on its engines until the ship reaches a speed of 0 m/s. How long will this take? And how far will the rocket travel? Let's say our rocket is traveling at 100 m/s and we want to slow it down to 0 m/s. Let's determine the distance and time:

$$t_{burn}\ =\ Time\ to\ 0\ m/s\ =\ V_i/a_{max}\ =\ 100/3.256\ =\ 30.712\ seconds$$

$$Distance\ Traveled\ =\ \ -Vi\ *\ t_{burn}\ +\ 1/2\ *\ a\ *\ t_{burn}^2\ =\ \ -943.227\ meters$$

Here I have to make sure one thing is understood: UP is positive and DOWN is negative. This is a very important aspect of analysis of any kind, understanding the coordinate system and which way is positive and negative. In this case the ship is falling down, and the rocket is accelerating up. And the negative answer means it has fallen 943.227 meters while it was slowing down.

We now have all the understanding needed to perform the full analysis. How do we know when to start the burn? We can calculate how much distance we will travel given a certain speed and the max acceleration of the rocket. Thus we can put a loop into kOS to continually check for the condition when the Altitude or Height the rocket at is equal to the Distance that the rocket needs to Slow Down to 0 m/s. If you calculate it correctly, you will slow down and stop exactly when your ship touches the ground.

This next part requires looking at the Google Sheets calculation page I have created

Link

In it I show with graphs how a rocket's speed changes as it falls straight down, and as it free falls the distance needed to stop the ship. You will notice on the first graph that the two points meet at around 90 seconds. This is the point where if the ship fires its engines it will hit the ground right when it stops. If the rocket passes this point and starts its engines too late, it can possibly hit the ground going too fast. If it does this too early, it will stop above the ground and depending on how early, this could mean it could fall dangerously.

HOWEVER… This is considering real life engines that are not able to be restarted as frequently as KSP. So if the last part does happen, no big deal just glide down slowly until you have landed.

Here is the equation you will need to calculate constantly to determine when to fire your engines

$$t_{burn} = Vertical\ Speed / a_{max}$$

$$0 \geq Current\ Altitude - Current\ Vertical\ Speed * t_{burn} + 1/2 * a_{max} * t_{burn}^{2}$$

IMPORTANT NOTES ABOUT SUICIDE BURNS:

The first note is that this analysis was done with a *constant* acceleration. Now we all know that as rockets fire their engines, their mass goes down. Thus increasing their acceleration. Now I *could* have tried to do some explanations and derivations with that in mind but the math would become very complicated. This does however give us a sort of Safety Feature.

If you were to do a full throttle burn, your acceleration would increase as you burn fuel and thus your distance to 0 m/s is actually shorter! This can be a good thing because terrain is quite uneven on some planets/satellites and it could be dangerous to try and hit the ground exactly when you stop.

The second note is about the Coriolis effect. You can look it up on wikipedia but essentially what it means is as you move to and from the center of a spinning reference frame (in this case the body you are trying to land on) you will begin to increase or decrease in horizontal speed.

I do not want to get into too much detail, but just remember that you will have to keep an eye out for and slow down your horizontal speed when touching down. Otherwise you might tip over!

Final Words

I hope you have learned a bit about how they work, and how to implement them. The equation is mainly for use with programming since this type of analysis would require constant calculation and checking.

There are equations that I used in my Sheets file that I derived based on time and such. It would solve for the burn altitude or burn time. I can give you these equations if anyone is interested, but I figured they weren't too important since they relied on the initial speed to be 0. I can do some derivations with equations that have initial speeds that are not 0 as well. If anyone needs them I can supplement them here or just send them to you.

What’s Next

I have planned next to do Impact Locations, I promised I would do that this time but I didn't get around to it. It's not particularly difficult conceptually and them math is a bit more graphical instead of numerical (if that's a thing) but I should be done by next week.

After that… I do not know… I will check my list and find out what to do next, let me know if anything peaks your interest that I can cover. If not I will just continue down my own personal list!