# Unit 4: Curve Sketching

Unit Objectives

- Create detailed sketches of graphs
- Determine key features of f, f', and f'' one of the three
- Determine differentiability

## Unit 4 Lesson 1: Sketch Curves Using Critical Points

Lesson Objectives

- Use the first derivative to locate local maxima and minima

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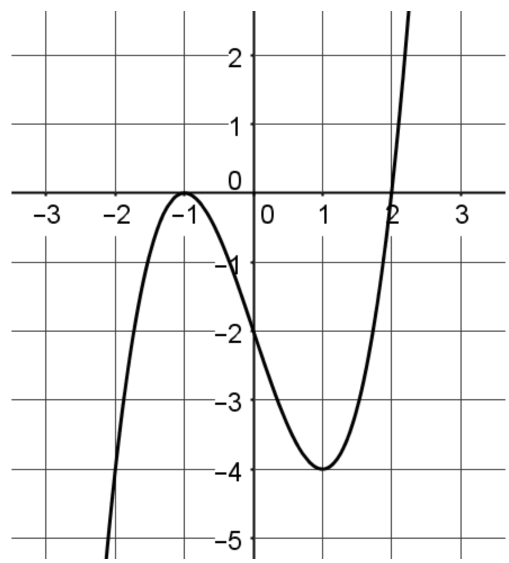
- Zeros (roots) are important, but so are ____________________




- _______________ occur when the _______________ of the

  ________________ ______________ equals _____________

- ______________ give the value of the ______________ of the

  _______________ _______________.

- Therefore, to find the _________________, find the points

  where the _____________________ equals ______________.

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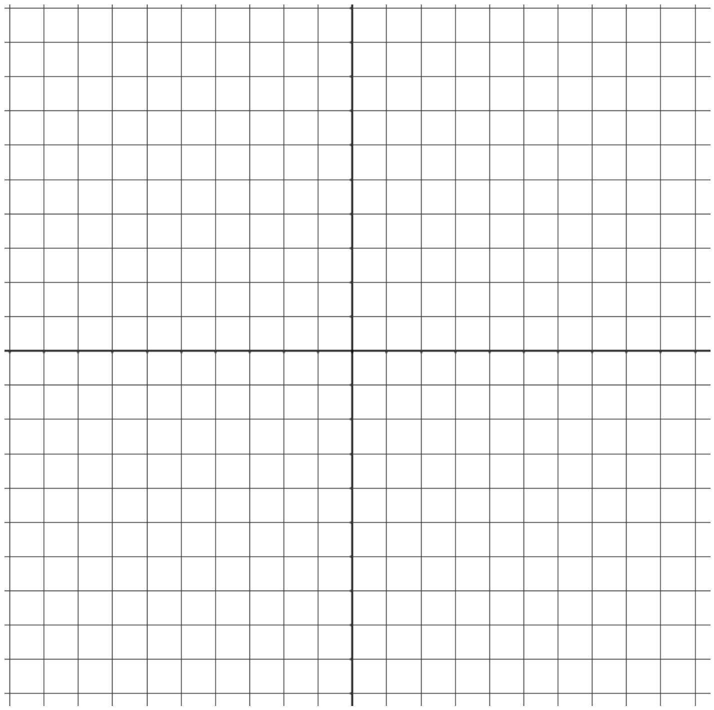
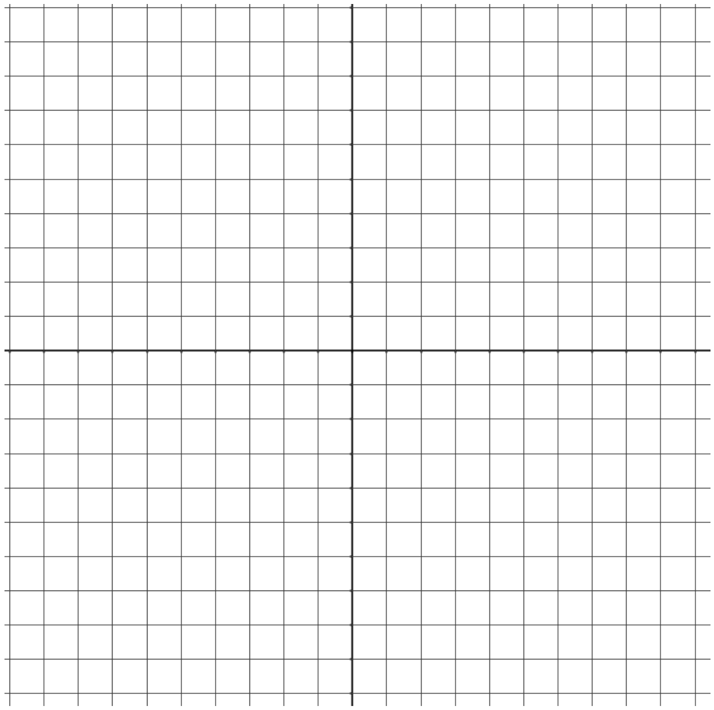
Example: The graph to the right shows the equation $y = x^3 - 3x - 2$. Show why calculus would predict this.

1. For the equation $f(x) = -2x^3 + 6x^2$, determine the roots and the extrema, and use them to sketch the graph.

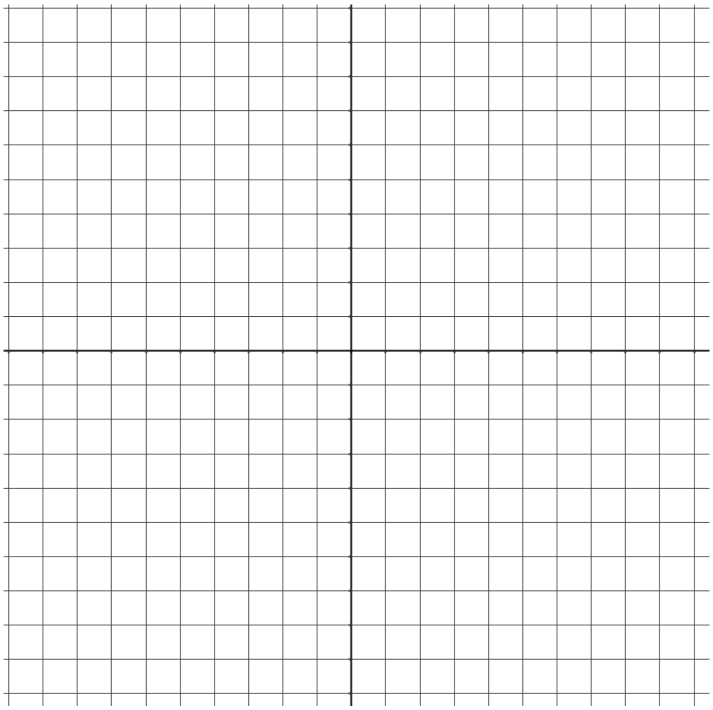


2. For the equation $y = \dfrac{-12x + 24}{x^2 + 12}$, determine the roots and the extrema, and use them to sketch the graph.

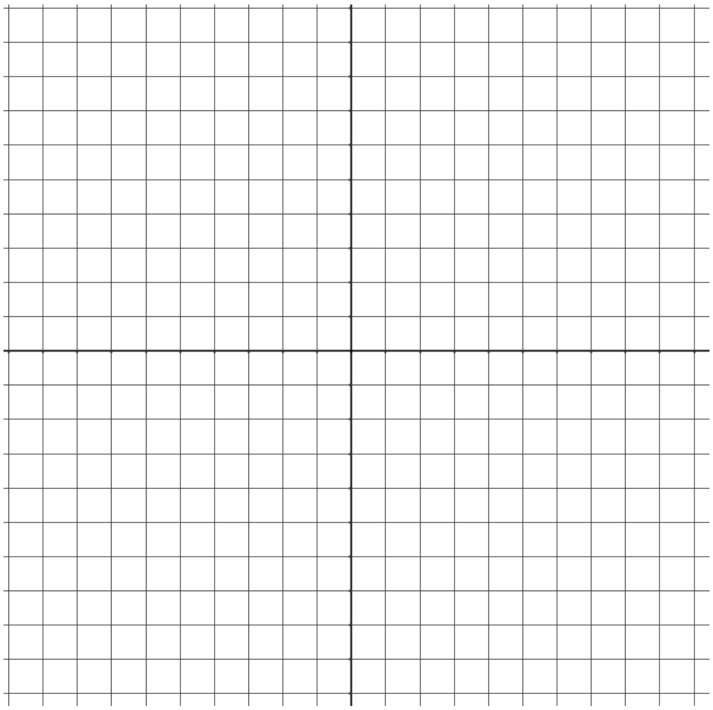


**~~~U4L1 Homework~~~** If $f(x) = x(x-3)^2$, determine the zeros, the extrema, and sketch the graph (nicely!)

1. For the equation $y = \dfrac{4x}{x^2 + 1}$, determine the roots and the extrema, and use them to sketch the graph.



2. For the equation $f(x) = (1/3)x^4 + (4/3)x^3$, determine the roots and the extrema, and use them to sketch the graph.



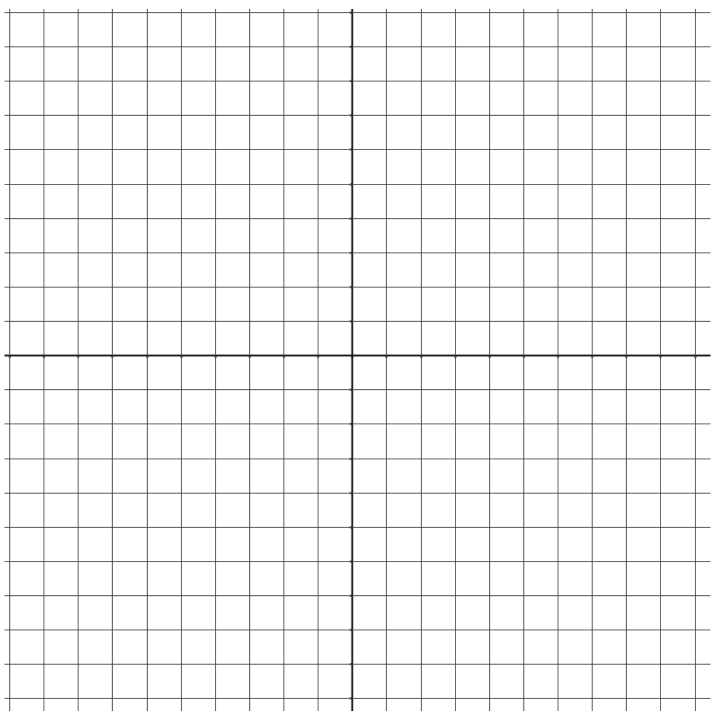
# Unit 4 Lesson 2: Sketch Curves Using Asymptotes

Lesson Objectives

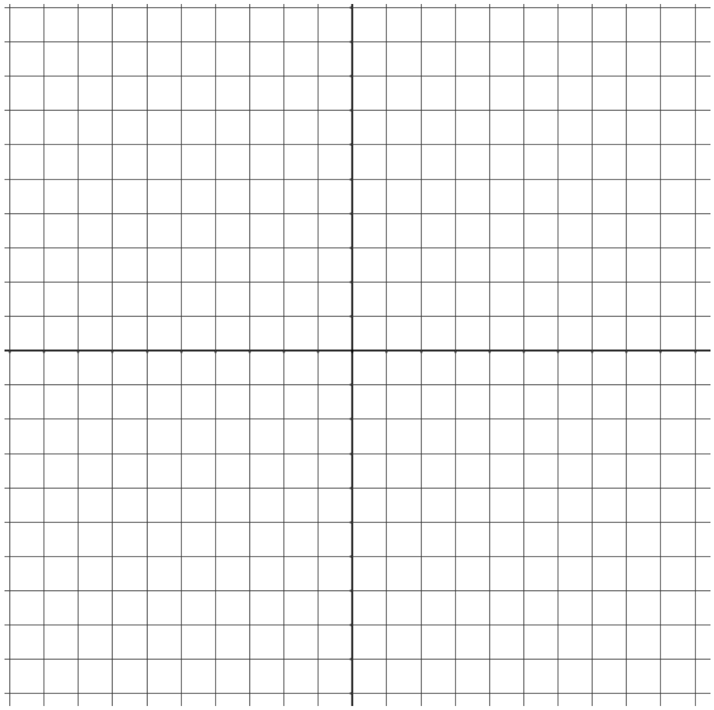
- Use limits to locate asymptotes

Asymptotes:

1. For the equation $y = \frac{(x-1)^2}{x-2}$, determine the asymptotes and the extrema, and use them to sketch the graph.

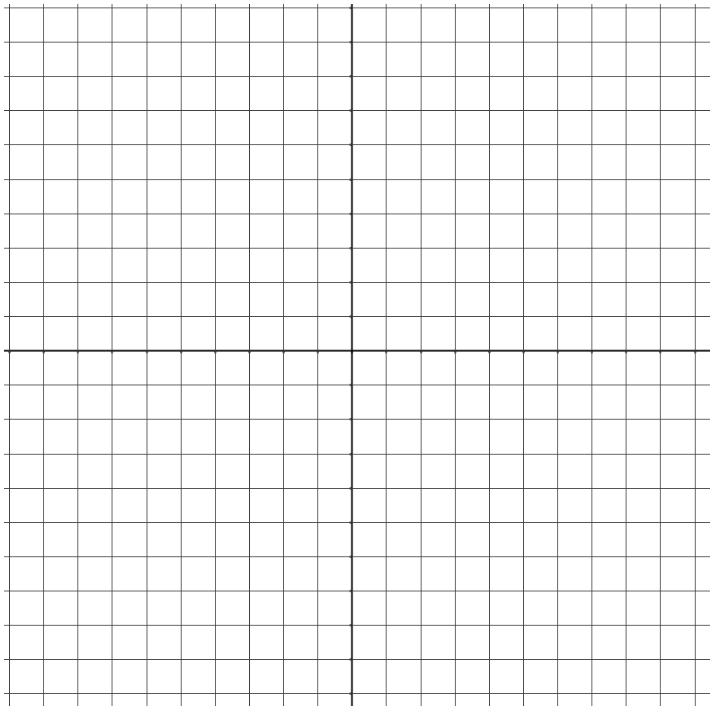


2. For the equation $y = \dfrac{x}{x-4}$, determine the asymptotes and the extrema, and use them to sketch the graph.

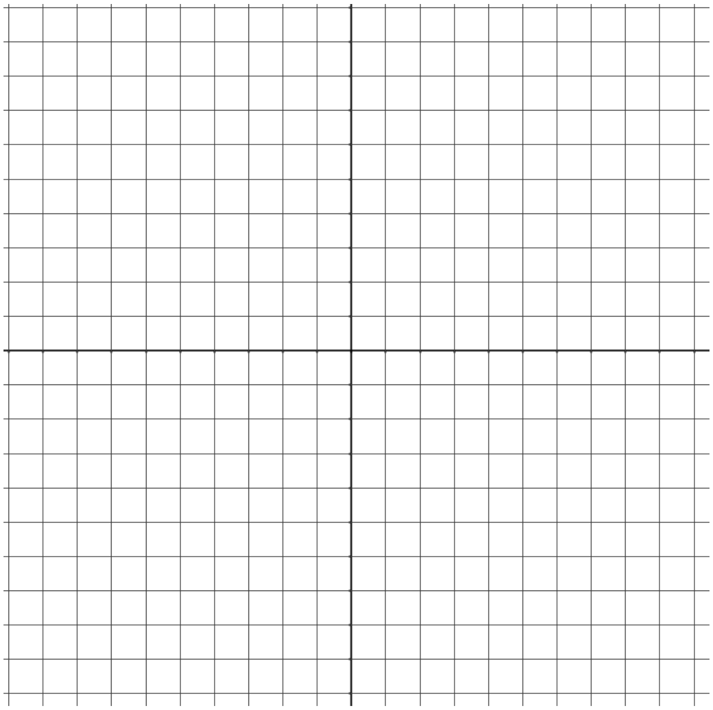


**~~~U4L2 Homework~~~** $y = \dfrac{16}{x^2 - 2x - 15} + 3$, determine the asymptotes and the extrema

1. For the equation $y = -\frac{8}{x^2 + 6x + 5} - 1$, determine the asymptotes and the extrema, and use them to sketch the graph.



2. For the equation $y = \dfrac{2x^2 - 3x}{x - 2}$, determine the asymptotes and the extrema, and use them to sketch the graph.



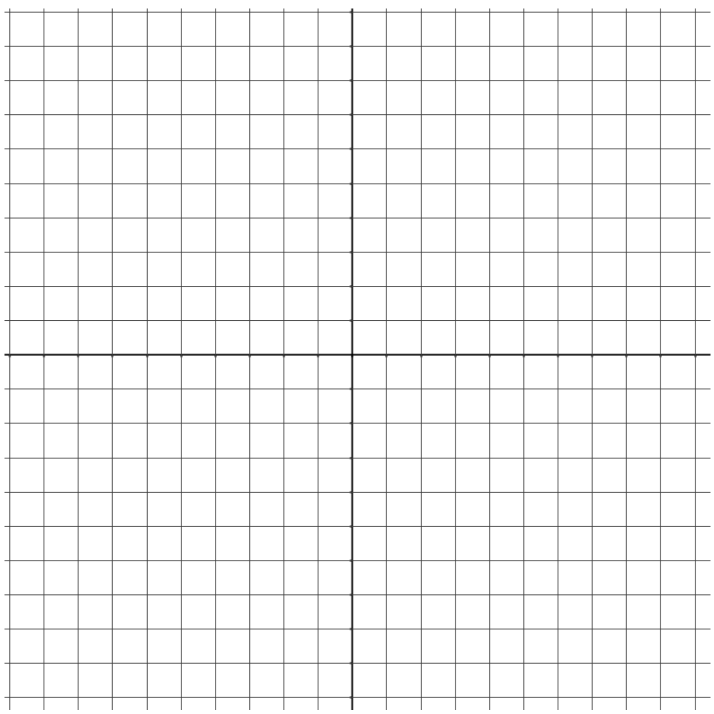
# Unit 4 Lesson 3: Sketch Curves Using Inflection Points

Lesson Objectives

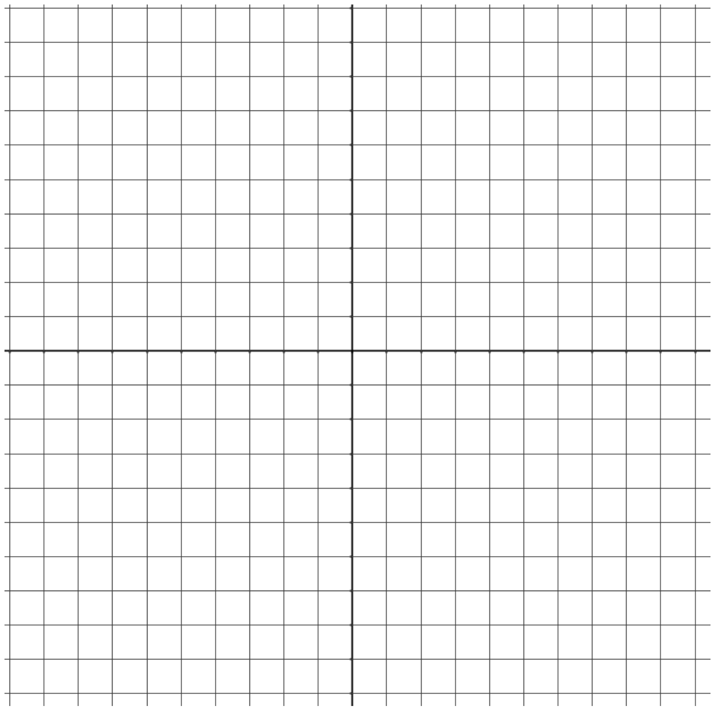
- Use inflection points to help sketch curves

Concavity:

1. For the equation $y = 5e^{-\frac{x^2}{2}}$, determine the asymptotes, extrema, and points of inflection and use them to sketch the graph.



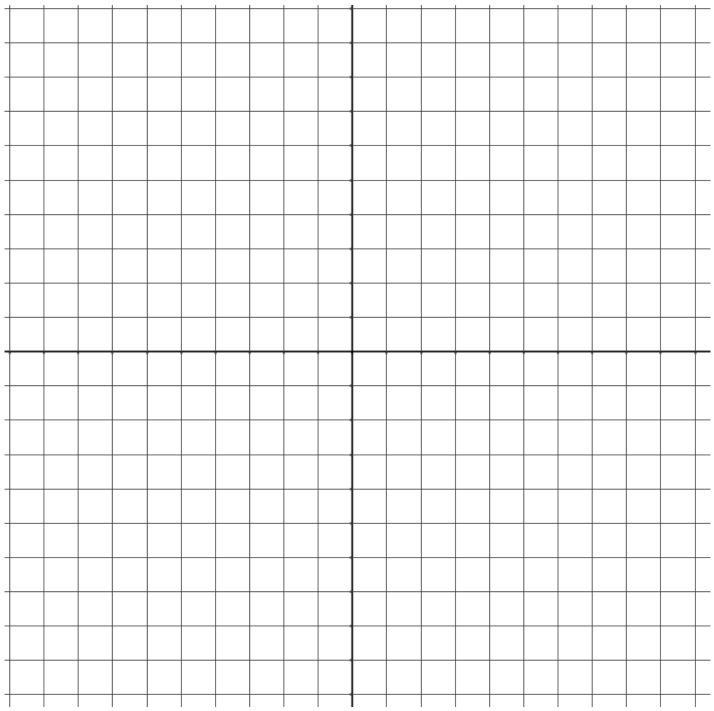
2. For the equation $x^3 - 12x^2 + 45x - 49$, determine the extrema and the points of inflection, and use them to sketch the graph.



**~~~U4L3 Homework~~~** $y = \frac{x^4}{12} - \frac{x^3}{6} - x^2$, determine the extrema and inflection points, and graph.

~~~U4L3 Classwork~~~

1. For the equation $f(x) = 2x^3 - 6x^2$, determine the following features and use them to sketch the graph.
   a. roots b. asymptotes c. critical points d. points of inflection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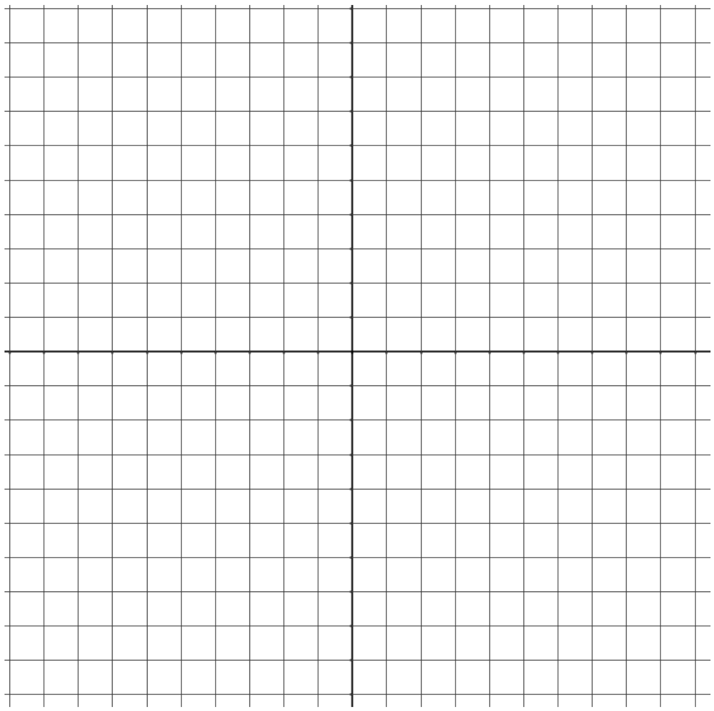
2. For the equation $y = 10\frac{lnx}{x}$, determine the following features and use them to sketch the graph.
    a. roots  b. asymptotes  c. critical points  d. points of inflection



# Unit 4 Lesson 4: Evaluate Limits Using L'Hospital's Rule (L'Hôpital's Rule): 0/0 and ∞/∞

Lesson Objectives

- Evaluate Limits Using L'Hospital's Rule
- Use this to help you find horizontal asymptotes

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L'Hospital's Rule

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1. If $f(x) = \frac{e^x}{x}$, determine the horizontal asymptotes

2. If $f(x) = \frac{x^4 - 3}{(x^3 - 2)(x + 1)}$, determine the horizontal asymptotes

3. $\lim_{x \to \infty} \frac{lnx}{2\sqrt{x}}$

4. $\lim_{x \to 0} \frac{sinx}{x}$

5. $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x - 1}$

6. $\lim_{x \to \infty} \frac{x}{e^x}$

## ~~~U4L4 Homework~~~

**~~~U4L4 Classwork~~~**

1. $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

2. If $f(x) = \frac{ln(-x)}{x}$, determine the horizontal asymptotes

3. If $f(x) = \frac{\sqrt{1+x}-1}{x}$, determine the horizontal asymptotes

4. $\lim_{x \to 0} \frac{sin x}{x^2}$

5. $\lim_{x \to \infty} \frac{ln x}{x}$

6. $\lim_{x \to 0} \frac{sin(5x)}{x}$

7. $\lim_{x \to \infty} \frac{x^{10}}{e^x}$

# Unit 4 Lesson 5: Determine Absolute Maxima and Minima

Lesson Objectives

- Determine the highest and lowest points on a graph

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Relative Maximum

Relative Minimum

Absolute Maximum

Absolute Minimum

Can occur at 2 possible places

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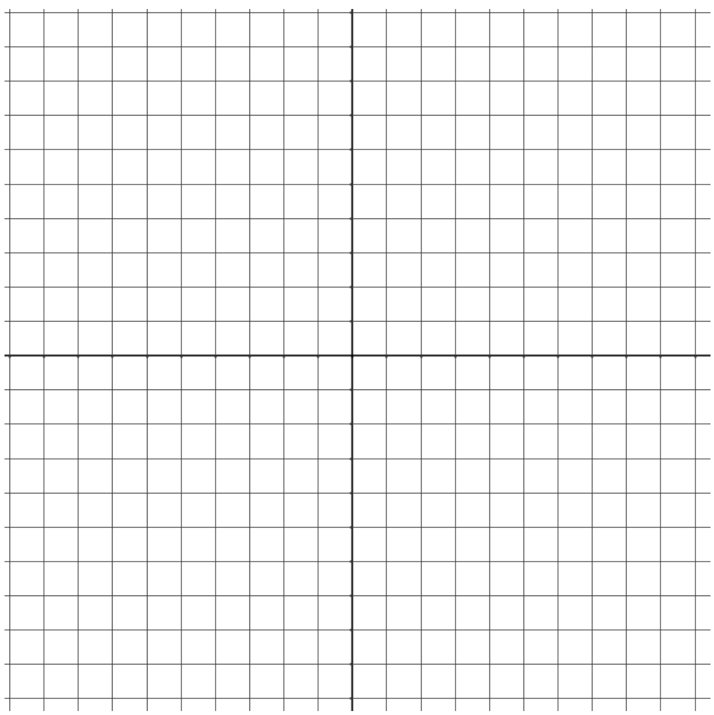
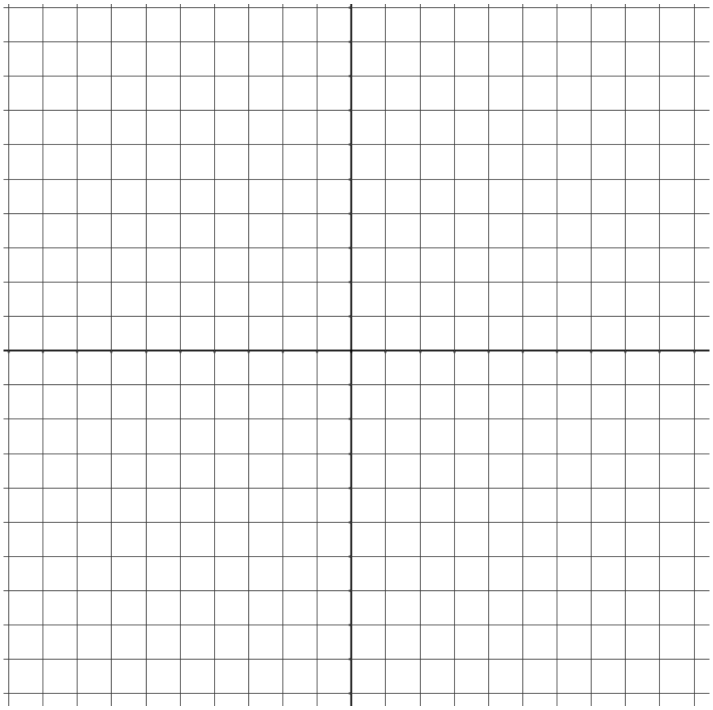
1. For the equation $g(x) = \frac{1}{8}x^3 - \frac{3}{2}x - 1$, determine the absolute maximum and absolute minimum on the interval [-3, 5]. Sketch a graph to verify your answer.



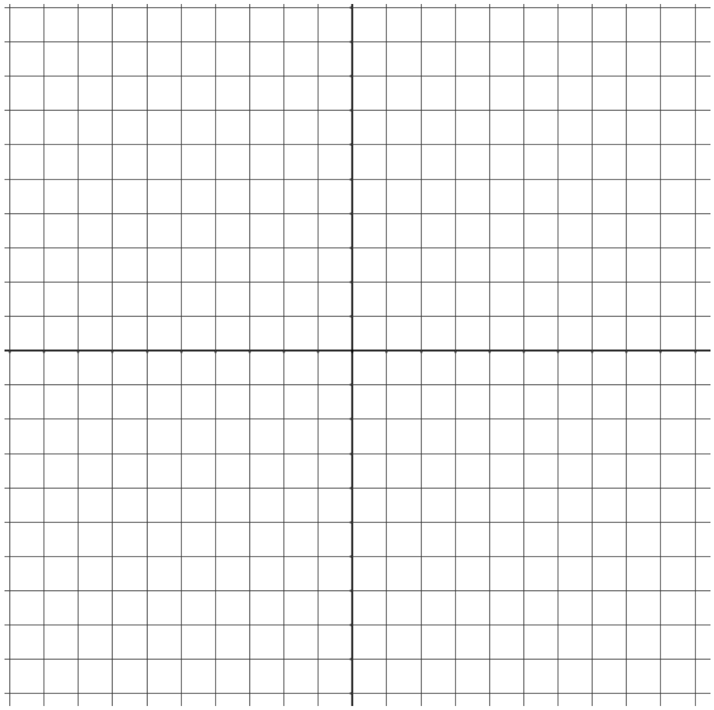
2. For the equation $f(x) = \dfrac{x^2}{(x-3)^2} - 2$, determine the absolute maximum and absolute minimum on the interval [-3, 6]. Sketch a graph to verify your answer.



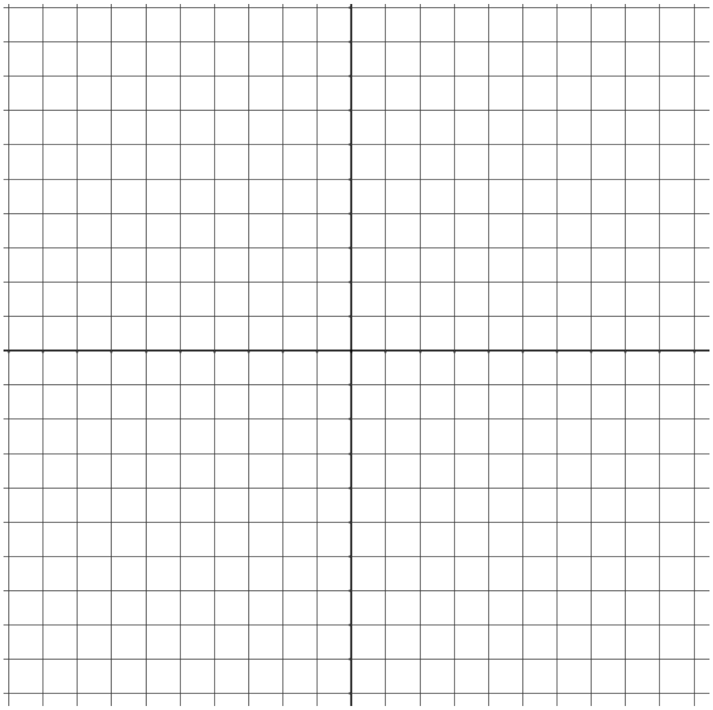
**~~~U4L5 Homework~~~ meh**

**~~~U4L5 Classwork~~~**

1. For the equation $f(x) = -\dfrac{2x^2}{(1-x)^2} + 2$, determine the absolute maximum and absolute minimum on the interval [-1, 2]. Sketch a graph to verify your answer.



2. For the equation $4x^4 - 20x^2 + 16$, determine the absolute maximum and absolute minimum on the interval [-2, 1]. Sketch a graph to verify your answer.



# Unit 4 Lesson 6: Sketch Curves Sans Sketching Curves--No Calculators

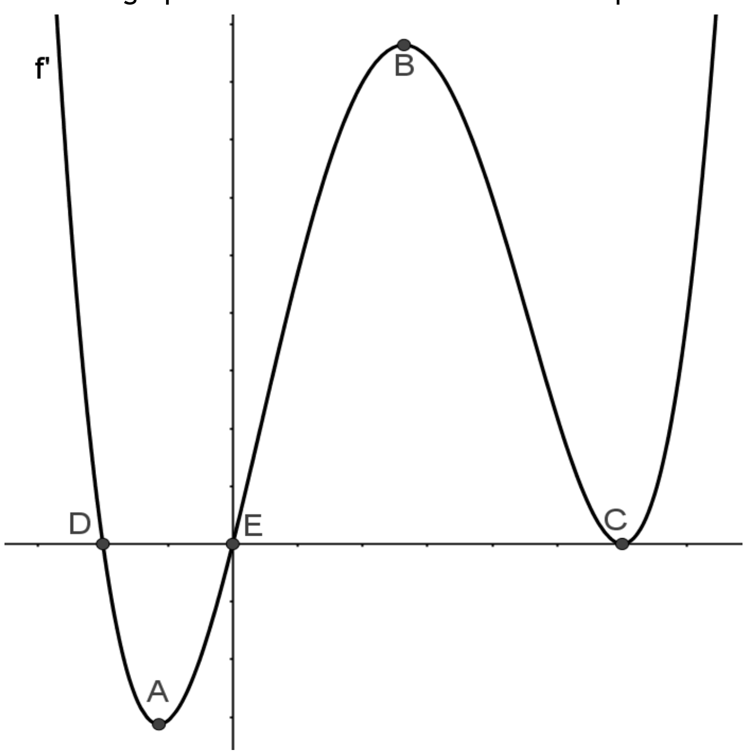
Lesson Objectives

- Answer questions about curve sketching that don't involve actually sketching a curve--no calculators

1. Let *g* be the function defined by $g(x) = 2x^4 + 8x^3$. How many relative extrema does g have?

2. The function *f* has a first derivative given by $f'(x) = x(x+3)^2(x-1)$. At what value(s) of x does *f* have a relative maximum?

3. A graph of f' is shown below. State the points which have x-values that correspond to the following.




   a. Critical points:

   b. Relative extrema:

   c. Relative minima:

   d. Relative maxima:

   e. Inflection points:

   f. Roots:

4. For what values of x does the graph of $y = 2x^6 + 6x^5$ have a point of inflection?

5. If *g* is the function given by $g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 70x - 8$
   a. On which interval(s) is *g* decreasing?
   b. On which interval(s) is *g* increasing?

6. Let *f* be the function defined by $f(x) = 2x^3 + 3x^2 - 12x + 2$. On which of the following intervals is the graph of *f* both decreasing and concave up?

**~~~U4L6 Homework~~~** *Ponder calculus deeply. Keep all revelations to yourself.*

## ~~~U4L6 Classwork~~

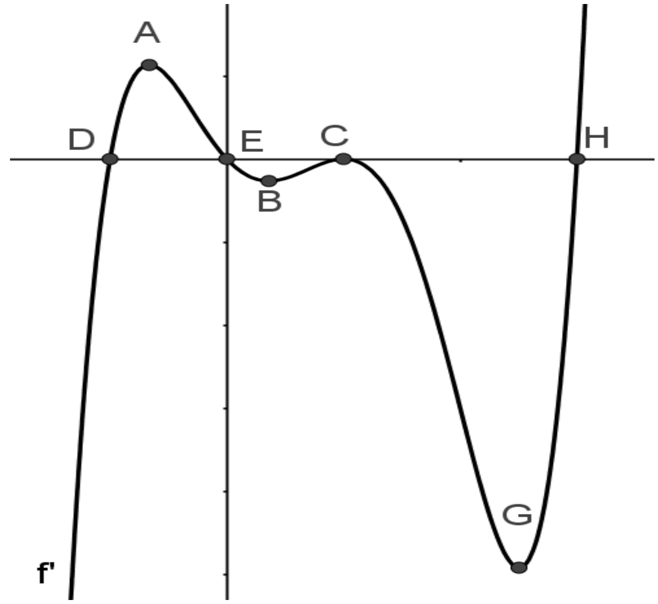
1. The graph below shows f'. State the points which have x-values that correspond to the following.
   a. Critical points:

   b. Relative maxima:

   c. Relative minima:

   d. Inflection points:

   e. Roots:




2. Let $g$ be the function defined by $g(x) = 2x^2 + 8x$. How many points of inflection does g have?

3. The function $f$ has a first derivative given by $f'(x) = x(x-4)^2(x-2)$. At what value(s) of x does $f$ have a relative minimum?

4. For what values of x does the graph of $y = 2x^6 + 6x^5$ have a relative maximum?

5. If *g* is the function given by $g(x) = -x^3 - 6x^2 + 180x + 5$
   a. On which interval(s) is *g* decreasing?

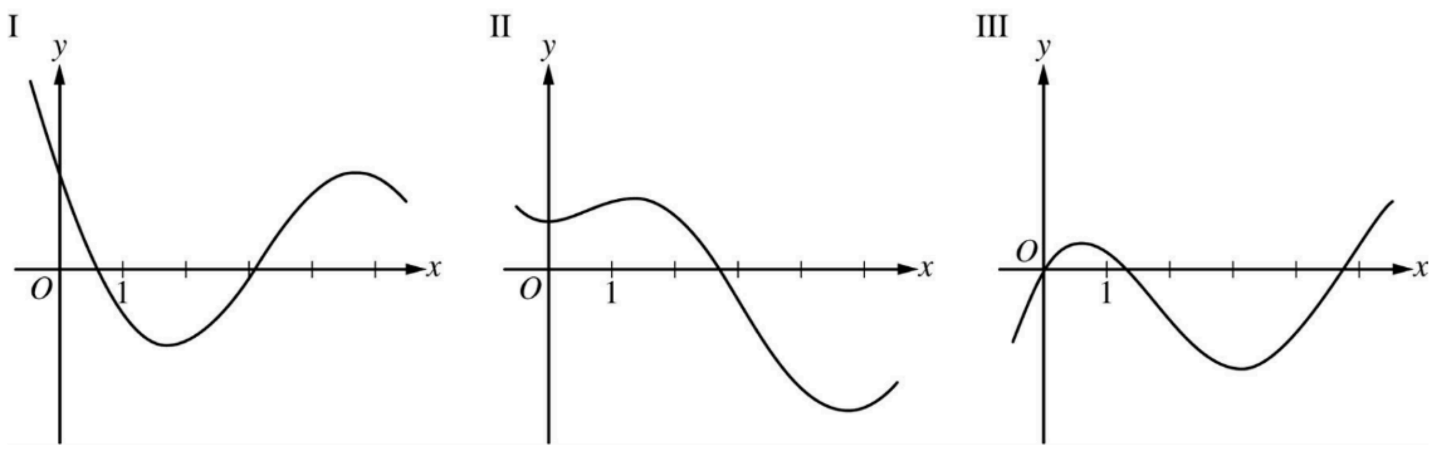
   b. On which interval(s) is *g* increasing?

7. Let *f* be the function defined by $f(x) = x^3 + 3x^2 - 9x + 7$. On which of the following intervals is the graph of *f* both increasing and concave down?

# Unit 4 Lesson 7: Compare Graphs of f, f', and f"

Lesson Objectives

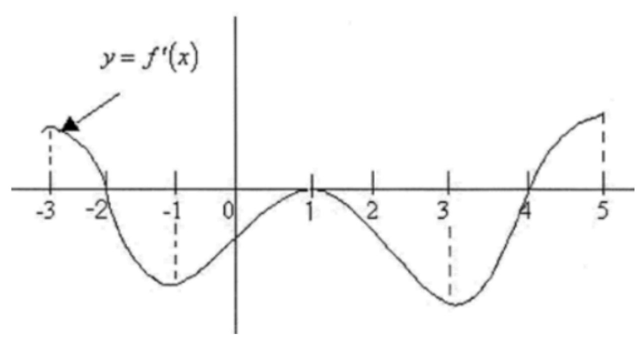
- Answer questions comparing graphs of f, f', f"



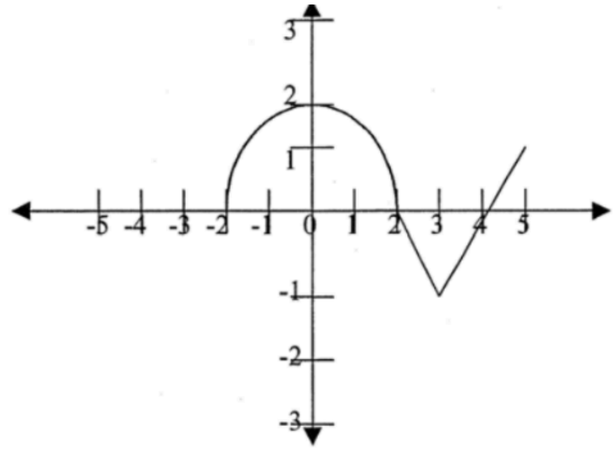

Three graphs labeled I, II, and III are shown above. One is the graph of $f$, one is the graph of $f'$, and one is the graph of $f''$. Which of the following correctly identifies each of the three graphs?

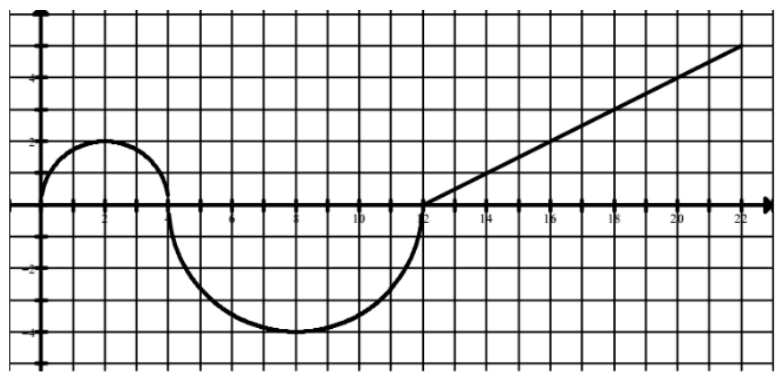
| | $f$ | $f'$ | $f''$ |
|---|---|---|---|
| (A) | I | II | III |
| (B) | II | I | III |
| (C) | II | III | I |
| (D) | III | I | II |

For what value(s) of x does $f$ have a relative maximum? Why?
For what value(s) of x does $f$ have a relative minimum? Why?
On what intervals is the graph of $f$ concave up? Why?
On what intervals is $f$ increasing? Why?
For what value(s) of x does $f$ have an inflection point? Why?



Find g(3).
For what value(s) of x does *g* have a relative maximum? Why?
For what value(s) of x does *g* have a relative minimum? Why?
For what value(s) of x does g have an inflection point? Why?
Write an equation for the line tangent to the graph of *g* at *x*=3



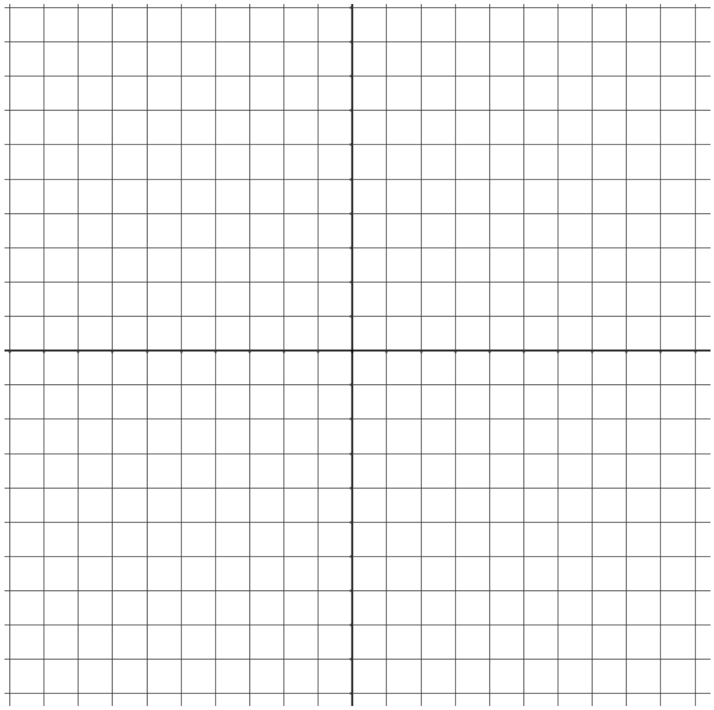



1. On what intervals is f increasing? Justify your answer.
2. For what values of x does f have a relative minimum? Justify.
3. On what intervals is f concave up? Justify.
4. For what values of x is f″ undefined?
5. Identify the x−coordinates for all points of inflection of f.
6. For what value of x does f reach its maximum value? Justify.
7. If f (4) = 5, find f(12).

# Unit 4 Lesson 8: Prepare for Test On Curve Sketching

1. For the equation $y = 10x/e^{-x}$, determine the following features and use them to sketch the graph.

   a. roots   b. asymptotes   c. critical points   d. points of inflection



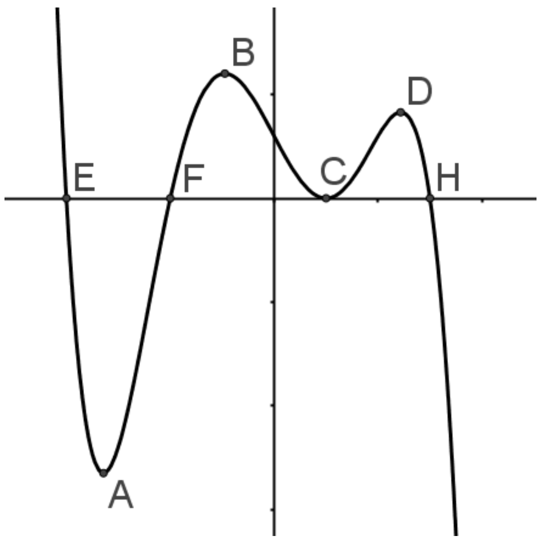
2. The graph below shows f'. State the points which have x-values that correspond to the following.
    a. Critical points:

    b. Relative maxima:

    c. Relative minima:

    d. Inflection points:

    e. Roots:




3. For the function $f(x) = -\frac{x^3}{3} + \frac{3}{2}x^2 + 4x + 1$
    a. On what intervals is the graph both increasing and concave down?
    b. What is the absolute max and absolute min on the interval [-4, 6]?