Time Value of Money

1. Why money has time value

The reason why money has time value can be explained through two dimensions: consumers' consumption preference and the opportunity for money to earn a return over time.

First, people generally prefer to consume goods and services today rather than in the future. For example, human needs and desires are immediate. People require food, shelter, clothing, and other essentials to maintain their well-being. They also have immediate desires for leisure and enjoyment. Due to this present consumption preference, individuals are willing to pay a premium for having access to money today. This premium is the time value of money, reflecting the extra value placed on money received in the present compared to the same amount of money received in the future.

Second, Money has the potential to earn a return when invested or used for productive purposes. This return can come in various forms, such as interest, dividends, capital gains, or profits from investments. The potential to earn a return on money provides financial motivation for individuals and organizations to postpone immediate consumption in favor of saving or investing. Money that is invested wisely has the capacity to grow over time, making it more valuable in the future.

2. Time value of money calculation

2.1. Some basic concepts related to the Time Value of Money

- **Compounding:** Compounding is the process of earning interest on both the initial principal and any previously earned interest. It leads to exponential growth in the value of money over time.
- **Discounting:** Discounting is the opposite of compounding. It involves reducing the future value of money to its present value by applying a discount rate. It reflects the idea that the value of money diminishes over time.

- **Future Value (FV):** This represents the value of a sum of money at a specific point in the future, taking into account the interest or return it can earn. The formula to calculate future value is:
- **Present Value (PV):** Present value is the current worth of a sum of money to be received or paid in the future, discounted at a specific interest rate. The formula to calculate present value is:
- **Interest Rate (r):** The interest rate used in time value of money calculations is crucial. It represents the rate at which money grows or declines in value over time. It can be an annual, monthly, or any other relevant rate, depending on the context.
- **Number of Periods (n):** This represents the length of time over which the money is invested or borrowed. It could be expressed in years, months, quarters, or any other time unit consistent with the interest rate.
- **Perpetuity:** A perpetuity is an infinite series of cash flows that continue indefinitely. The formula for calculating the present value of a perpetuity is PV = PMT / r, where PMT represents the constant payment received or made each period.

2.2. Basic formula to calculate the Time Value of Money

$$FV = PV * (1 + r)^n$$

$$PV = \frac{FV}{(1+r)^n}$$

Where:

- FV = Future Value
- PV = Present Value (initial amount of money)
- r = Interest rate per period
- n = Number of periods

2.3. Timeline for a single CF

PV_0 **FV_T**

|---------|---------|---------|----- … -----|-------------------> time

0 1 2 3 T

If you have a CF of $100 at t = 0

$$PV_0 = \$100$$

then calculate $FV_T = PV_0 * (1 + r)^T$

If you have a CF of $100 at t = T

$$FV_T = \$100$$

then calculate $PV_0 = \frac{FV_T}{(1+r)^T}$

2.4. Value Additivity Principle for CFs:

Add/Subtract CFs on the same date.

If CFs are on different dates, discount/compound the CFs to the same date and add/subtract the values.

The present value of $100 in one year, $100 in two years, and $100 in three years is:

PV_0 (CF_1, CF_2, and CF_3)

= PV_0 (CF_1) + PV_0 (CF_2) + PV_0 (CF_3)

= $100/(1+r)^1 + 100/(1+r)^2 + 100/(1+r)^3$.

The future value of the above cash flows at year 3 is:

FV_3 (CF_1, CF_2, and CF_3)

$= FV_3 (CF_1) + FV_3 (CF_2) + FV_3 (CF_3)$

$= 100*(1+r)^2 + 100/(1+r)^1 + 100.$

2.5.Timeline for multiple CFs over multiple periods

CF_1 CF_2 CF_3 … CF_t … CF_T

|---------|---------|---------|-------- … --------|-------- … --------|------> time

0 1 2 3 t T

The general PV equation:

$$PV_0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_t}{(1+r)^t} + \cdots + \frac{CF_T}{(1+r)^T}$$

2.6.Perpetuity

A cash stream that generates the same cash flow in each period and continues forever.

CF CF … CF …

|----------|----------|-------- … -------|------- … --------> time

0 1 2 t

$$PV = \frac{CF_1}{r}$$

The interest rate is 5%. An asset that pays you $1,000 for all the future years will be priced at 1000/0.05 = 20,000 today.

2.7.Growing Perpetuity

A cash stream that generates the cash flow growing at a constant rate g in each period and continues forever.

CF CF*(1+g) … CF*(1+g)^t …

|----------|----------|-------- … -------|------- … --------> time

0 1 2 t

$$PV = \frac{CF_1}{r-g}$$

The interest rate is 5%. An asset that pays you $1,000 at t=1 and grows at 1% every year for all the future years will be priced at 1000 / (0.05 – 0.01) = 25,000 today.

3. Terminal value

3.1.Definition and significance

Terminal value refers to the estimated future value of an investment or project at the end of a defined forecast period. It represents the value that will be realized beyond the explicit projection period, often accounting for the long-term cash flows and growth potential of the investment.

Terminal value is crucial in investment analysis as it captures the majority of the value of a project, especially in cases where cash flows extend significantly beyond the forecast period. It helps provide a more comprehensive picture of the investment's worth and aids in decision-making and valuation.

3.2.Estimating Terminal Value

The Perpetuity Growth Method (Gordon Growth Model) assumes that the cash flows of the investment will continue growing at a constant rate indefinitely after the forecast period. It involves estimating the cash flow at the end of the forecast period and dividing it by the discount rate minus the growth rate.

3.3.Formula

$$TV_{t-1} = \frac{CF_t}{r}$$

where

- TV represents the terminal value,
- CF_t represents the cash flow in the next period
- r is the discount rate
- g is the growth rate.

Terminal value will always be **one period before** the **first cash flow** of the perpetuity / growing perpetuity you used in the discount formula above.