

Standard of Learning (SOL) 4.2a Compare and order fractions and mixed numbers.



Student Strengths	Bridging Concepts	Standard of Learning
Students can name, write, represent and compare fractions and mixed numbers represented by a model	Students can make generalizations about fractional relationships across representations (i.e., when the numerator is half of the denominator, the fraction is always equal to $\frac{1}{2}$).	Students can compare and order fractions and mixed numbers with and without models

Understanding the Learning Trajectory

Big Ideas:

- Initial fraction ideas begin with creating experiences with a variety of visual representations (area models, number lines, set models, etc.) which can be utilized to compare and order fractions. Over time, reasoning strategies and generalizations are developed through modeling and mental imagery. There may be a transition from creating models to applying those generalizations and reasoning strategies to solve problems.
- A variety of reasoning strategies can be applied to compare fractions. Potential strategies include but are not limited to:
 - reasoning about relative magnitude ;
 - unit fraction reasoning;
 - benchmark reasoning (i.e. I know $\frac{4}{8}$ is $\frac{1}{2}$ so $\frac{5}{8}$ is a little more);
 - distance to 1 whole (ie. I know $\frac{9}{10}$ is $\frac{1}{10}$ from 1 whole, and $\frac{8}{9}$ is $\frac{1}{9}$ from 1 whole. $\frac{1}{10}$ is smaller than $\frac{1}{9}$, so $\frac{9}{10}$ is closer to 1 whole than $\frac{8}{9}$.); and
 - equivalence.
- Behr and Post (1992) indicate that “a child’s understanding of the ordering of two fractions needs to be based on an understanding of the ordering of unit fractions” (1992, p.21).
- The reasoning strategy applied is often impacted by number choice. For example, I might use the benchmark of $\frac{1}{2}$ if I am comparing $\frac{3}{10}$ (a little less than $\frac{1}{2}$) and $\frac{5}{8}$ (a little more than $\frac{1}{2}$).
- Generalizations about fractional relationships can be extended to understanding improper fractions and mixed numbers.

Research Referenced:

- Bray, W. S. & Abreu-Sanchez, L. (2010). *Using Fraction Sense to Compare Fractions*, NCTM. *Teaching Children*

Mathematics, 17(2), 90-97

- Behr M. J., Harel G., Post T. and Lesh R: 1992, 'Rational number, ratio and proportion', in D. A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, New York, pp. 296–333.
- Cramer, K., Behr, M., Post T., Lesh, R., (2009) Rational Number Project: Initial Fraction Ideas.
- [Rational Number Project](#) Lesson and research on learning trajectory of initial fraction ideas
- [Fraction Circles Colored](#)
- [OGAP Fraction Framework](#)-OGAP is a systematic, intentional, and iterative formative assessment system grounded in the research on how students learn mathematics, Learn more <http://www.ogapmath.com/about>

Formative Assessment:

- VDOE [Just in Time Mathematics Quick Check 4.2a \(PDF\)](#)
- VDOE [Just in Time Mathematics Quick Check 4.2a \(Desmos Version\)](#)

Important Assessment Look Fors:

- Students recognize and utilize benchmark fractions.
- Students use their understanding of the ordering of unit fractions to ordering of two fractions.
- Students recognize and apply their understanding of equivalent fractions.
- Students recognize fractions equivalent to $\frac{1}{2}$.
- Students represent and work with fractions greater than 1.
- Students organize their thinking in a way that helps them make sense of the problem. Student representations support their thinking/reasoning.
- Student strategies make sense for the given set of numbers.
- It is evident in the strategy used that the student understands the relationship between the numerator and the denominator.

Purposeful Questions:

- Can you share where you started and why?
- How might you use your understanding of unit fractions to compare and order these fractions?
- How can benchmark be used to help you understand the value of these fractions?
- What representation might help you visualize these fractions?
- Is that fraction greater than 1? Less than 1? How do you know?
- What do you know about the value of these numbers?
- What relationships do you see?
- I'm looking at your number line and I'm noticing _____. Tell me more about that...

Bridging Activity to Support Standard	Instructional Tips
Routine: Same but Different, samebutdifferent.com Would You Rather?, adapted from wouldyourathermath.com	The routine serves as a bridge to help students access the task and apply their understandings of $\frac{1}{2}$ to compare and order fractions. Select fractions to compare that lead to generalizations, such as the purple and gray squares comparing $\frac{2}{4}$ and $\frac{4}{8}$. Consider selecting a variety of different fractions to compare over several days, including fractions greater than 1. Engage students in conversation during this routine. This number choice was designed to support students thinking about fractions equivalent and close to $\frac{1}{2}$. As students build their understanding of this benchmark, they can apply it as a strategy to compare and order numbers. Consider providing models to support student thinking.
Rich Tasks	You may want to adapt the task directions to “using no digit more than twice”. This will

Open Middle: Using $\frac{1}{2}$ as a Benchmark , Open Middle	provide more opportunities to create fractions with a denominator less than 12. Encourage students to explain their thinking. Can they use reasoning? Do they use models to support their understanding? Provide concrete and virtual models for students that opt to use it.
Games/Tech Smaller to Larger, Theresa Wills Directions Desmos 4.2a Ordering Fractions on a Number Line 4.2a Ordering on a Number Line ↗	Scroll down to Fractions Games to find this game. Consider providing concrete and virtual fraction manipulatives as students play the game. Reflect with students to help them notice patterns and generalizations • How did you decide where to place the fraction? • What do you notice? • Why did you put that card in the trash? In this Desmos activity, students will compare and order fractions, improper fractions and mixed numbers on a number line.
Other Resources: • Doubling Numerators and Denominators ○ This task does not explicitly compare fractions but it can serve as a bridge to help students notice patterns and relationships with the numerator and denominator. Building on this understanding will strengthen fraction equivalence and comparisons. • Fraction Comparison (Scroll down and search under Fraction Games) ○ Fraction Comparison might be supportive for students that may need models to build their reasoning. • Fractions Greater Than 1 (from FCPS) ○ This resource can be used to support students developing understanding of working with fractions greater than 1. • VDOE Mathematics Instructional Plans (MIPS) ○ 4.2ab - Fraction Strips: Comparing and Ordering Fractions (Word) / PDF Version ○ 4.2a - Circle Fractions: Comparing and Ordering with Benchmarks (Word) / PDF Version ○ 4.2a - Comparing Fractions: Developing Strategies (Word) / PDF Version • VDOE Algebra Readiness Remediation Plans ○ Compare Fraction Strategies (Word) / PDF • VDOE Word Wall Cards: Grade 4 (Word) (PDF) • VDOE Instructional Videos for Teachers ○ Models for Teaching Fractions • Desmos Activities: ○ Ordering Fractions on a Number Line ○ Ordering on the Number Line ○ Equivalent Fractions ○ Fraction Card Sort Learning Trajectory Resources:	

- Bray, W. S. & Abreu-Sanchez, L. (2010). *Using Fraction Sense to Compare Fractions*, NCTM. *Teaching Children Mathematics*, 17(2), 90-97.
- Behr M. J., Harel G., Post T. and Lesh R: 1992, 'Rational number, ratio and proportion', in D. A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, New York, pp. 296–333.
- Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership*, 7(3), NCSM.
- Cramer, K., Behr, M., Post T., Lesh, R., (2009) Rational Number Project: Initial Fraction Ideas. [Rational Number Project](#)
- Common Core Standards Writing Team. (2019). [Progressions for the Common Core State Standards for Mathematics](#). Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Van De Walle, J., Karp, K. S., & Bay-Williams, J. M. (2018). *Elementary and Middle School Mathematics: Teaching Developmentally*. (10th edition) New York: Pearson (2019:9780134802084)
- [Blackline masters from older addition available on web](#)
- VDOE Curriculum Framework for All Grades - [Standard of Learning Curriculum Framework \(SOL\)](#)