Explorations of the Alternating Harmonic Series

This notebook provides explorations of the rearrangements of the alternating harmonic series. Click on the formula given below and then press the ENTER key. When you are asked if you want to evaluate all the initialization cells, answer YES. When Mathematica warns that there might be a problem, proceed and answer EVALUATE.

The command AHS[r,s,m] calculates the sum of the first (r+s)m terms from the rearranged alternating harmonic series in which we rearrange the series so that the first r positive terms are followed by the first s negative terms, then the next r positive terms followed by the next s negative terms, and so on. Check the value of the original alternating harmonic series and the series with r=1, s=2, both taken out to a total of six million terms.

```
> AHS := (r, s, m) -> evalf(Sum(1/(2*j-1),j = 1 .. r*m))-evalf(Sum(1/2/j,j = 1 .. s*m)) ;
```

```
> AHS(2,3,100000);
```

```
> AHS(1,2,2000000);
```

The command AHSGraph[r,s,m] finds the sum of the first (r+s)m terms of the alternating harmonic series as well 20 partials sums. It lists the values of these partial sums and plots their values.

```
> AHSList := (r,s,m) -> [seq([(r+s)*floor(k*m/20)+r*(k-2*floor(1/2*k)),
  AHS(r,s,floor(k*m)/20)],k = 1 .. 20)];
```

```
> AHSList(2,3,1000);
```

```
> plots[listplot](AHSList(2,3,1000));
```

Challenge problem

We know that lim_{m -> ∞ }AHS(1, 1, m) = Log[2] and lim_{m -> ∞ }AHS(1, 2, m) = log(2)\/2. There is a general formula for lim_{m -> ∞ } AHS(r, s, m). Try to guess it . You may want to use the Inverse Symbolic Calculator at http://oldweb.cecm.sfu.ca/projects/ISC/ISCmain.html. The ISC takes a decimal and returns all exact quantities (such as log 2) whose decimal digits agree with those that have been entered.