

Solving Quadratic Programming

$$T1 = 1$$

$$T2 = -1$$

$$T3 = -1$$

$$T4 = 1$$

$$X1=[0,0], X2=[0,1], X3=[1,0], X4=[1,1]$$

Using kernel function as $K(x,y)=(1+x.y)^2$

With $X=[x_1, x_2]$

Inner product kernel in terms of monomials of different orders as follows

$$K(X_i, X_j) = 1 + X_{i1}^2 \cdot X_{j1}^2 + 2X_{i1} \cdot X_{i2} \cdot X_{j1} \cdot X_{j2} + X_{i2}^2 \cdot X_{j2}^2 + 2X_{i1} \cdot X_{j1} + 2X_{i2} \cdot X_{j2}$$

$$\phi(X) = [1, x_1^2, \sqrt{2x_1 \cdot x_2}, x_2^2, \sqrt{2x_1}, \sqrt{2x_2}]^T$$

$$\max_{\alpha} \left\{ \sum_{j=1}^4 \alpha_j - \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 T_i \cdot T_j \cdot \alpha_i \cdot \alpha_j (1 + x_i \cdot x_j)^2 \right\}$$

Such that

$$\alpha_j \geq 0 \quad j = 1, 2, 3, 4$$

$$\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 0$$

Make above maximization problem into minimization problem to use quadprog tool in matlab to solve

$$\min_{\alpha} \left\{ \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 T_i \cdot T_j \cdot \alpha_i \cdot \alpha_j (1 + x_i \cdot x_j)^2 - \sum_{j=1}^4 \alpha_j \right\}$$

Hessian Matrix for above problem

Therefore we can transform this problem into this form

$$\frac{1}{2} X^T \cdot H \cdot X + f \cdot X$$

$$H = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 4 & 1 & -4 \\ -1 & 1 & 4 & -4 \\ 1 & -4 & -4 & 9 \end{bmatrix}$$

$$F = [-1 \ -1 \ -1 \ -1]$$