Math 7

Gloucester County Public Schools
Curriculum Reference and Pacing Guide

Based on the 2016 Virginia Standards of Learning

GCPS Curriculum Guide Math 7

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June 2021

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Introduction

The Gloucester County Public Schools *Curriculum Reference and Pacing Guide* serves as a companion document to the 2016 *Mathematics Standards of Learning* (SOL) and the 2016 *Mathematics Standards of Learning Curriculum Framework*, and delineates in greater specificity the content that all teachers should teach and all students should learn. It serves as a guide for teachers when planning instruction and assessments.

The format of the *Curriculum Reference and Pacing Guide* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each objective. The Curriculum Guide is divided by unit and ordered to match the established GCPS pacing. Each unit is divided into three parts: Standards, Content, and Instruction.

The Standards Information section includes the SOL, SOL Strand, and Focus for the unit, as well as the anticipated pacing.

The Content section includes the Essential Knowledge and Skills along with strategies and resources to support them. It also includes the foundational objectives and/or future skills correlated to each SOL (Vertical Articulation), key vocabulary, essential questions, and key concepts that support successful instruction of the standard.

The Instruction section contains information to assist teachers with planning and implementing effective lesson plans. This section includes suggested assessment tools, links to VDOE's Mathematics Instructional Plans, common student misconceptions, and strategies for differentiating instruction. It also contains suggestions for incorporating the Mathematical Process Goals for Students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Math 7 Course Description

The seventh-grade standards continue to emphasize the foundations of algebra. The standards address the concept of and operations with rational numbers by continuing their study from grade six. Students will build on the concept of ratios to solve problems involving proportional reasoning. Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of solving linear equations and inequalities in one variable by applying the properties of real numbers. Students discern between proportional and non-proportional relationships and begin to develop a concept of slope as rate of change.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Source: Mathematics Standards of Learning for Virginia Public Schools – 2016 Grade Seven, Virginia Department of Education © September, 2016

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<u>GCPS Math 7 Course Overview</u>

Textbook: Glencoe Math Connects Course 2, Virginia Edition, 2012

Pacing Outline

First Quarter	Second Quarter	Third Quarter	Fourth Quarter
Unit 1: Square Roots & Scientific Notation (7.1abd) **Unit 2:** Rational Numbers (7.1ce) **Unit 3:** Operations with Rational Numbers (7.2)	**Unit 4:** Expressions & Equations (7.11/7.12) **Unit 5:** Inequalities (7.13) **Unit 6:** Proportions & Percent (7.3)	**Unit 7:** Transformations & Graphing Relationships (7.7/7.10) **Unit 8:** Quadrilaterals & Similar Figures (7.5/7.6) **Unit 9**: Volume & Surface Area (7.4)	**Unit 10:** Probability & Statistics (7.8/7.9) **SOL Review & Testing**

2016 SOL Test Blueprint

Reporting Category	**Strand**	**2016 SOL #**	**# of Items (CAT)**	**# of items (paper/pencil)**
RC01	Number & Number Sense	7.1	12	14
	Computation & Estimation	7.2, 7.3		
RC02	Measurement & Geometry	7.4, 7.5, 7.6, 7.7	10	12
RC03	Probability & Statistics	7.8, 7.9	20	24
	Patterns, Functions, & Algebra	7.10, 7.11, 7.12, 7.13		
Total Operational Items			42	50
Field-Test Items			8	0
Total Number of Items			50	50

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GCPS Math 7 Pacing Guide

First Quarter

Unit	# of days	2016 SOL #	SOL Content	Textbook Correlation
Unit 1: Square Roots & Scientific Notation	12	7.1a 7.1b 7.1d	Investigate and describe the concept of negative exponents for powers of ten Compare and order numbers greater than zero written in scientific notation (NO CALC) Determine square roots of perfect squares (NO CALC)	Additional Lessons 819-820 Ch. 3 Lesson 4C Ch. 1 Lessons 1A, 3A, 3B, 3C
Unit 2: Rational Numbers	12	7.1 c 7.1e	Compare and order no more than four positive or negative rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. (NO CALC) Identify and describe absolute value of rational numbers.	Ch. 3 Lessons 1A, 1B, & 1C Ch. 2 Lessons 1A & 1B
Unit 3: Operations with Rational Number	16	7.2	Solve practical problems involving addition, subtraction, multiplication, and division with positive and negative rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents.	Ch. 2 Lessons 2A through 3D Ch. 3 Lessons 2A through 3D
Extra Days			Introductions, Fall STAR, Review, Enrichment	

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Second Quarter

Unit	# of days	2016 SOL #	SOL Content	Textbook Correlation
Unit 4: Expressions & Equations	16	7.11 7.12	Evaluate algebraic expressions for given replacement values of the variables. Solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.	Ch. 1 Lessons 1B, 1C, 1D Ch. 4 Lessons 1A through 3B Additional Lesson 8
Unit 5: Inequalities	10	7.13	Solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.	Ch. 4 Lessons 4A, 4B, & 4C Additional Lesson 9
Unit 6: Proportions & Percent	11	7.3	Solve single-step and multistep practical problems, using proportional reasoning	Ch. 5 Lessons 1A through 2C Ch. 6 Lessons 1A through 2C, 3C, & 3D
Extra Days			Winter STAR, Review, Enrichment	

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Third Quarter

Unit	# of days	2016 SOL #	SOL Content	Textbook Correlation
Unit 7: Transformations & Graphing Relationships	14	7.7 7.10a 7.10b 7.10c 7.10d 7.10e	Apply translations and reflections of right triangles or rectangles in the coordinate plane. Determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form y = mx to represent the relationship. Graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in y = mx form where m represents the slope as rate of change. Determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form y = x + b to represent the relationship; Graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form y = x + b, where b represents the y-intercept; and Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.	Ch. 12 Lessons 2A, 2B, 3A, & 3B Ch. 7 Lessons 1B through 3C Additional Lesson 2
Unit 8: Quadrilaterals & Similar Figures	14	7.5 7.6a 7.6b	Solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles. Compare and contrast quadrilaterals based on their properties. Determine unknown side lengths or angle measures of quadrilaterals.	Ch. 5 Lesson 3A Ch. 12 Lesson 1D
Unit 9: Volume & Surface Area	12	7.4a 7.4b	Describe and determine the volume and surface area of rectangular prisms and cylinders; and Solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.	Ch. 10 Lessons 1A, 1B, 1C, 2A through 2D Additional Lesson 13
Extra Days			Review, Enrichment	

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Fourth Quarter

Unit	# of days	2016 SOL #	SOL Content	Textbook Correlation
Unit 10: Probability & Statistics	15	7.8a 7.8b 7.9a 7.9b 7.9c	Determine the theoretical and experimental probabilities of an event; and Investigate and describe the difference between the experimental probability and theoretical probability of an event. Collect, organize, and represent data in a histogram. Make observations and inferences about data represented in a histogram. Compare data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots.	Ch. 8 Lessons 1A, 3A, & 3B Additional Lesson 17 Ch. 9 Lessons 3A through 3E
Extra Days			Spring STAR, SOL Review & Testing	

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Unit 1 Square Roots and Scientific Notation	SOL 7.1abd Essential Knowledge and Skills and Key Instructional Information

Standards

SOL Strand:
Number & Number Sense

Focus:
Square Roots and Scientific Notation

VA SOL:
7.1 The student will
- a) investigate and describe the concept of negative exponents for powers of ten;
- b) compare and order numbers greater than zero written in scientific notation;
- d) determine square roots of perfect squares;

Anticipated Pacing:
12 days

The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:

Content

Content

<u>Recognize</u> powers of 10 with negative exponents by examining patterns.

<u>Represent</u> a power of 10 with a negative exponent in fraction and decimal form.

<u>Convert</u> between numbers greater than 0 written in scientific notation and decimals.

<u>Compare</u> and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order.

<u>Identify</u> the perfect squares from 0 to 400.

<u>Determine</u> the positive square root of a perfect square from 0 to 400.

Understanding The Standard :

- Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$).

- Negative exponents for powers of 10 can be investigated through patterns such as:

$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

- Scientific notation should be used whenever the situation calls for use of very large or very small numbers.
- A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310,000$ and $2.85 \times 10^{-4} = 0.000285$).
- Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line.
- Smaller numbers always lie to the left of larger numbers on the number line.
- A perfect square is a whole number whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \cdot 6 = 6^2$).
- A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since $11 \cdot 11 = 121$).
- The symbol $\sqrt{\ }$ may be used to represent a non-negative (principal) square root. Students in grade 8 mathematics will explore the negative square root of a number, denoted $-\sqrt{\ }$.
- The square root of a number can be represented geometrically as the length of a side of a square.
- Squaring a number and taking a square root are inverse operations.

Essential Questions:	**Vertical Articulation**	
	Previous Standards:	**Future Standards:**
• What is the difference between a negative exponent and a positive exponent? Explain. • What is the pattern for negative exponents raised to any power of 10?	6.2 The student will a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; and b) compare and order positive rational	8.1The student will compare and order real numbers. 8.2 The student will describe the relationships between the subsets

- How does a positive exponent affect the base? A negative exponent? - How to convert scientific notation numbers to standard form? - How to convert standard form to scientific notation? - How do you compare numbers written in scientific notation? - How to convert fraction, decimal percent, and scientific notation? - What is the difference between squaring a number and finding the square root of a number?	numbers. 6.3 The student will a) identify and represent integers; b) compare and order integers; and c) identify and describe absolute value of integers. 6.4 The student will recognize and represent patterns with whole number exponents and perfect squares. **Resources for Recovery:** 7.1abd Remediation	of the real number system. 8.3 The student will a) estimate and determine the two consecutive integers between which a square root lies; and b) determine both the positive and negative square roots of a given perfect square.

Vocabulary

- factors
- exponent
- base
- powers
- square roots
- Squared
- perfect squares
- standard form
- number
- exponential form
- scientific notation
- rational number
- terminating
- ascending order
- equivalent
- repeating number
- compare
- order
- integer
- numerator
- denominator
- percent
- whole number
- absolute value
- descending order
- ascending order
- decimals
- whole number
- opposites

Instruction

Process Goals

Problem-solving/ Communication:

- Many real life problems have fraction, decimal, percent aspects to them and can be applied to make the learning more realistic
- Emphasize that scientific notation is a method for communicating numbers that are inconvenient to write, and that instructions for translating it are loaded into the 10's exponent.

Reasoning/Connections:

- Have students make predictions about the relative size of the number before converting to decimals (the proportional relationship between the numerator and denominator, and how the overall value of the number changes as one increases or decreases)
- Science curriculum connections microscopic units and space travel, etc.

Representations:

- Using graphic organizers and manipulatives such as fraction tiles and fraction circles, base 10 blocks would be a way to assist students who struggle with fractional concepts.
- Build models of actual squares and using the side of the square to show the square root.
- Use bar modeling and SIngapore math to represent and solve algebraic relationships and problems.

Assessment Tools	Lesson Plans
Pre/Post Assessments: ● Prerequisite Quick Check ● Unit Test **Freckle** **Textbook** ● Ch. 1 Lessons 1A, 3A, 3B, 3C ● Ch. 3 Lesson 4C ● Additional Lessons: 819-820 **Tasks** ● Which Microscope?	**VDOE Lessons:** ● Powers of Ten ● Scientific Notation ● Square Roots

	Misconceptions/Mistakes
	• Students have confusion with negative exponents. • Scientific Notation has positive and negative exponents that can cause confusion with conversions. • Students may presume that a negative exponent produces a negative number (ex: 10^-3 = -1/1000). • Students can multiply the base by the exponent rather than actually squaring or cubing the base.

Unit 2 Rational Numbers	SOL 7.1ce Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use **problem solving**, **mathematical communication**, **mathematical reasoning**, **connections**, and **representations** to:
SOL Strand: Number & Number Sense **Focus:** Rational Numbers **VA SOL:** 7.1 The student will c) compare and order rational numbers; e) identify and describe absolute value of rational numbers. **Anticipated Pacing:** 12 days	**Content** **Content** **Compare** and **order** no more than four rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. **Demonstrate** absolute value using a number line. **Determine** the absolute value of a rational number. **Show** that the distance between two rational numbers on the number line is the absolute value of their difference, and **apply** this principle to solve practical problems. **Understanding The Standard :** ● Percent means “per 100” or how many “out of 100”; percent is another name for hundredths. ● A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). ● The set of integers includes the set of whole numbers and their opposites, {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. ● The opposite of a positive number is negative and the opposite of a negative number is positive. ● The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$

where a and b are integers and b does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$.

- Rational numbers may be expressed as positive and negative fractions or mixed numbers, positive and negative decimals, integers and percents.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can be positive or negative.
- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, colored counters, cubes, decimal squares, shaded figures, shaded grids, number lines and calculators). Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line. Smaller numbers always lie to the left of larger numbers on the number line.
- The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive (e.g., $\left|-\frac{1}{2}\right| = \frac{1}{2}$).
- The absolute value of zero is zero.

Essential Questions:	**Vertical Articulation**	
• How to convert fraction, decimal, & percent, ? • How do you use fractions and decimals to solve practical problems? • How do you compare rational numbers that are presented in different forms? • How does comparing quantities describe the relationship between them? • How can understanding	**Previous Standards:** 6.2 The student will a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; and b) compare and order positive rational numbers. 6.3 The student will a) identify and represent integers; b) compare and order integers; and c) identify and describe absolute value	**Future Standards:** 8.1 The student will compare and order real numbers.

benchmark numbers help when comparing rational numbers? • What is absolute value? • How is finding the absolute value of a number different from finding the opposite of a number?	of integers. **Resources for Recovery:** 7.1ce Remediation	

Vocabulary

- common denominator
- least common denominator
- proper fraction
- improper fraction
- rational numbers
- mixed numbers
- terminating decimals
- repeating decimals
- equivalent
- like fraction
- unlike fraction
- Numerator
- denominator
- percent
- standard form
- whole number
- fractions
- decimals
- descending order
- ascending order

Instruction

Process Goals

Problem-solving/ Communication:

- Many real life problems have fraction, decimal, percent aspects to them and can be applied to make the learning more realistic

Reasoning/Connections:

- Have students make predictions about the relative size of the number before converting to decimals (the proportional relationship between the numerator and

Representations:

- Using graphic organizers and manipulatives such as fraction tiles and fraction circles, base 10 blocks would be a way to assist students who struggle with fractional denominator, and how the overall value of the number changes as one increases or decreases)

concepts.

- Build models of actual squares and using the side of the square to show the square root.
- Use bar modeling and SIngapore math to represent and solve algebraic relationships and problems.

Assessment Tools	Differentiation
Pre/Post Assessments: • Prerequisite Quick Check • Unit Test **Freckle:** **Textbook** • Ch. 2 Lessons 1A, 1B • Ch. 3 Lessons 1A, 1B, 1C **Tasks**	• Graphic Organizers to help with memory, charts with strategies for students who cannot remember how to convert, manipulatives like fraction tiles, fraction, circles, and base ten blocks. • Use of flashcards to memorize benchmark fraction and decimal conversions. **Lesson Plans** **VDOE Lessons:** • Ordering Fractions, Decimals and Percents • Absolute Value

Misconceptions/Mistakes

- Students will have troubles with conversions of fractions to decimals and percents.
- Students are expected to have some details with basic fractions memorized and many students do not have strategies for memory work.
- Students not understanding that absolute value is always a positive number.

	• Students often think absolute value and opposite are the same thing.

Unit 3 Operations with Rational Numbers	SOL 7.2 Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:

SOL Strand:
Computation & Estimation

Focus:
Operations with Rational Numbers

VA SOL:
7.2 The student will solve practical problems involving operations with rational numbers.

Anticipated Pacing:
16 days

Content

Content

<u>Solve</u> practical problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.

Understanding The Standard :

- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). A fraction can have a positive or negative value.
- Solving problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. Practical problems involving rational numbers in grade seven provide students the opportunity to use problem solving to apply computation skills involving positive and negative rational numbers expressed as integers, fractions, and decimals, along with the use of percents within practical situations.

Essential Questions:	Vertical Articulation	
• How to convert scientific notation numbers to standard form? • How to convert standard form to scientific notation? • How to convert fraction, decimal percent, and scientific notation? • How do you use fractions and decimals to solve practical problems?	**Previous Standards:** 6.5 The student will b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. 6.6 The student will b) solve practical problems involving operations with integers; and **Resources for Recovery:** 7.2 Remediation	**Future Standards:** 8. 4 The student will solve practical problems involving consumer applications.

Vocabulary

- Numerator
- denominator
- proper fraction
- percent
- mixed numbers
- operations
- terminating decimals
- repeating decimals
- common denominator
- rational numbers
- least common denominator
- like fraction
- unlike fraction

Instruction

Process Goals

Problem-solving/ Communication:
- Have students write their own problems based off of real-life situations - shopping, recipes, measurements, etc.
- Have students orally or in written form to justify their reasoning behind their answers.

Reasoning/Connections:
- How is solving decimal, fraction and integers operations different? How are they the same?
- Have students use classroom money manipulatives to connect to real world situations.

Representations:
- Use Singapore Math strategies to teach word problems.

Assessment Tools

Pre/Post Assessments:
- Prerequisite Quick Check
- Unit Test

Freckle

Textbook
- Ch. 2 Lessons 2A-3D
- Ch. 3 Lessons 2A-3D

Tasks
- Bake Sale Fundraiser

Differentiation

- Singapore Math
- Number Line
- Drawing Diagrams

Lesson Plans

VDOE Lessons:

- Solve Problems Involving Operations with Rational Numbers

Misconceptions/Mistakes

- When word problems contain more than one form of number, students might forget to change the numbers into the same form before computing for the answer.
- Students might forget to complete all the step if the problem is a multi-step question.

Unit 4 Expressions and Equations	SOL 7.11/7.12
	Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:
SOL Strand: Patterns, Functions, & Algebra **Focus:** Expressions and Equations **VA SOL:** 7.11 The student will evaluate algebraic expressions for given replacement values of the variables. 7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable. **Anticipated Pacing:** 16 days	

Content
Content
<u>Represent</u> algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles.
<u>Use</u> the order of operations and **<u>apply</u>** the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression.
<u>Represent</u> and **<u>solve</u>** two-step linear equations in one variable using a variety of concrete materials and pictorial representations.
<u>Apply</u> properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational.
<u>Confirm</u> algebraic solutions to linear equations in one variable.
<u>Write</u> verbal expressions and sentences as algebraic expressions and equations.
<u>Write</u> algebraic expressions and equations as verbal expressions and sentences.
<u>Solve</u> practical problems that require the solution of a two-step linear equation.

Understanding The Standard :

- To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if a = 3 and b = -2 then 5a + b can be evaluated as: 5(3) + (-2) and simplified using the order of operations to equal 15 + (-2) which equals 13.
- Expressions are simplified by using the order of operations.
- The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. The order of operations is as follows:
  - First, complete all operations within grouping symbols[1]. If there are grouping symbols within other grouping symbols, do the innermost operations first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and /or divide in order from left to right.
  - Fourth, add and /or subtract in order from left to right.

  [1] Parentheses ( ), brackets [ ], and the division bar should be treated as grouping symbols.
- Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Associative property of addition: $(a + b) + c = a + (b + c)$.
  - Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
  - Subtraction and division are neither commutative nor associative.
  - Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
  - Identity property of addition (additive identity property): $a + 0 = a$ *and* $0 + a = a$.
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ *and* $1 \cdot a = a$.
  - Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$).
  - Inverse property of addition (additive inverse property): $a + (-a) = 0$ *and* $(-a) + a = 0$.
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ *and* $\frac{1}{a} \cdot a = 1$.
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0$ *and* $0 \cdot a = 0$.

- Division by zero is not a possible mathematical operation. It is undefined.
- Substitution property: If $a = b$, then b can be substituted for a in any expression, equation, or inequality.

- An equation is a mathematical sentence that states that two expressions are equal.
- The solution to an equation is the value(s) that make it a true statement. Many equations have one solution and can be represented as a point on a number line.
- A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.
- The inverse operation for addition is subtraction, and the inverse operation for multiplication is division.
- A two-step equation may include, but not be limited to equations such as the following: $2x + \frac{1}{2} =- 5;\ - 25 = 7.2x + 1;\ \frac{x-7}{-3} = 4;\ \frac{3}{4}x - 2 = 10.$
- An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, 5x, 140 - 38.2, 18 · 21, 5 + x).
- An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an “equal sign (=).” An expression cannot be solved.
- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression “a number multiplied by 5” could be represented by “n · 5” or “5n”.
- An algebraic expression is a variable expression that contains at least one variable (e.g., 2x – 3).
- A verbal sentence is a complete word statement (e.g., “The sum of twice a number and two is fifteen.” could be represented by “2n + 2 = 15”).
- An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., 2x – 8 = 7).
- Properties of real numbers and properties of equality can be applied when solving equations, and justifying solutions. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard):
    - Commutative property of addition: $a + b = b + a$.
    - Commutative property of multiplication: $a \cdot b = b \cdot a$.
    - Subtraction and division are not commutative.
    - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
    - Identity property of addition (additive identity property): $a + 0 = a$ *and* $0 + a = a$.
    - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ *and* $1 \cdot a = a$.
    - Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$).

- Inverse property of addition (additive inverse property): $a + (-a) = 0$ *and* $(-a) + a = 0$.
- Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ *and* $\frac{1}{a} \cdot a = 1$.
- Zero has no multiplicative inverse.
- Multiplicative property of zero: $a \cdot 0 = 0$ *and* $0 \cdot a = 0$.
- Division by zero is not a possible mathematical operation. It is undefined.
- Substitution property: If $a = b$, then b can be substituted for a in any expression, equation, or inequality.
- Addition property of equality: If $a = b$, then $a + c = b + c$.
- Subtraction property of equality: If $a = b$, then $a - c = b - c$.
- Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$.
- Division property of equality: If $a = b$ *and* $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Essential Questions:	Vertical Articulation	
• How can you use the square roots of non-perfect squares in problem-solving? • What is a mathematical expression, and how is it useful? • Why is it important to apply properties of operations when simplifying expressions? • How can algebraic expressions and equations be written? • What are possible solutions for the equation? • What is the solution to the equation using a manipulative to show the equation and the solution? • How can this problem be represented verbally? • When solving an equation, why is it important to perform identical operations on each side of the	**Previous Standards:** 6.6 The student will c) simplify numerical expressions involving integers. 6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable. **Resources for Recovery:** 7.11 Remediation 7.12 Remediation	**Future Standards:** 8.14 The student will a) evaluate an algebraic expression for given replacement values of the variables; b) simplify algebraic expressions in one variable. 8.17 The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.

equal sign?
- Why is it important to apply properties of operations when simplifying expressions & solving equations?

Vocabulary

- variable
- exponent
- powers
- terms
- substitution
- square roots
- parentheses
- evaluate
- Replacement
- coefficient
- define a variable
- replacement value
- brackets
- simplify
- Equation
- two-step equation
- formula
- Solution
- algebraic expression
- algebraic equations
- Evaluate
- standard form
- exponential form
- the multiplicative inverse
- reciprocal
- constant
- Coefficient
- properties
- the commutative and associative properties for addition and multiplication
- the distributive property
- the additive and multiplicative identity properties
- the additive and multiplicative inverse properties
- the multiplicative property of zero
- inverse
- inverse operations

Instruction

Process Goals

Problem-solving/ Communication:
- Identify patterns in

Reasoning/Connections:
- Teach properties with reasoning using cues within

Representations:
- Teach properties with developing physical

properties. ● Students can plug the answer into the equation to check their answers. This is a version of working backwards, a problem solving strategy. ● Students can write an equation word problem to represent the equation given.	the mathematical example. “Can two different expressions model the same situation? (ex: If my age is two years older than three times yours, is 3x + 2 as good as 2 + 3x?)” ● Have students evaluate already-completed problems, some deliberately solved incorrectly, to locate common errors and misconceptions. ● Hands On Equations help to build the reasoning component to equations by balancing equations visually to represent successful completion of the math problem. ● Have students build simple geometric formulas, such as the perimeter of a rectangle, by writing algebraic expressions. ● Students connect verbal equations to verbal expressions the same skills they had worked with before are connected to this segment of the curriculum	representations such as counters or chips. Use equation mats to visually demonstrate an equation

Assessment Tools	**Differentiation**

Pre/Post Assessments:
- Prerequisite Quick Check
- Unit Test

Freckle

Textbook
- Ch. 1 Lessons 1B, 1C, 1D
- Ch. 4 Lessons 1A-3B
- Additional Lesson 8

Tasks
- Expressions for Gardening

- Use a student-generated booklet to help remember properties.
- Use a mnemonic device to remember order of operations.
- Use graphic organizers for words that correspond with four functions (add, subtract, multiply, and divide).
- Use a highlighter for tracking steps with order of operations.
- Hands on Equations, using bar diagrams to illustrate equations, algebra tiles, cups and counters.
- Use of correct terms and application of properties can be reinforced when working through examples.
- Have students who are making repeated errors look at the same equation solved two ways side-by-side, one containing their type of error and one solved correctly, and have them identify the similarities and differences.

Offer students examples of equations that have fractional solutions (ex: $3x = 5$) and encourage them to leave their answers as fractions, even when improper ($x = 5/3$) instead of converting to decimals.

Lesson Plans

VDOE Lessons:
- Evaluating Algebraic Expressions
- Solving Two-Step Equations
- Translating Expressions and Equations

Misconceptions/Mistakes

- Students can multiply the base by the exponent rather than actually squaring or cubing the base.
- Students confuse coefficient of a number being part of a number instead of multiplying the coefficient by the replacement variable.
- Students replacing a variable without observing order of operations. Squaring a number and taking a square root are inverse operations.
- Students have difficulty when solving a two-step equation using the additive inverse so as to have the constants one side of the equation is the first step (ex: Given the equation 4 - 3x = 10, they may think the first step is to add 4 since the expression contains subtraction).
- Students may have difficulty managing the difference between equations like (2 + x)/4 = 10 and 2 + (x/4) = 10.
- Sometimes students have difficulty recognizing -x = -1x, so they get stumped when solving equations like 5 - x = 2 after they have eliminated the 5.

Unit 5 Inequalities	SOL 7.13 Essential Knowledge and Skills and Key Instructional Information
Standards **SOL Strand:** Patterns, Functions, & Algebra **Focus:** Inequalities **VA SOL:** 7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line. **Anticipated Pacing:** 10 days	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:** **Content** **Content** **Apply** properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational. **Represent** solutions to inequalities algebraically and graphically using a number line. **Write** verbal expressions and sentences as algebraic expressions and inequalities. **Write** algebraic expressions and inequalities as verbal expressions and sentences **Solve** practical problems that require the solution of a one- or two-step inequality. **Identify** a numerical value(s) that is part of the solution set of a given inequality. **Understanding The Standard :** • A one-step inequality may include, but not be limited to, inequalities such as the following: 2x > 5; $y - \frac{2}{3} \leq -6$; $\frac{1}{5}$ x < -3; $a - (-4) \geq \frac{11}{2}$.

- A two-step inequality may include, but not be limited to inequalities such as the following: $2x + 1 < -25$; $2x + \frac{1}{2} \geq -5$; $-25 > 7.2x + 1$; $\frac{x-7}{-3}$ 4; $\frac{3}{4}x - 2 \leq 10$.
- The solution set to an inequality is the set of all numbers that make the inequality true.
- The inverse operation for addition is subtraction, and the inverse operation for multiplication is division.
- The procedures for solving inequalities are the same as those to solve equations except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than.
- When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$).
- Solutions to inequalities can be represented using a number line.
- In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solution is $x > -7$. This means that x can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.)
- Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Subtraction and division are not commutative.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
  - Identity property of addition (additive identity property): $a + 0 = a$ *and* $0 + a = a$.
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ *and* $1 \cdot a = a$.
  - Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$).
  - Inverse property of addition (additive inverse property): $a + (-a) = 0$ *and* $(-a) + a = 0$.
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ *and* $\frac{1}{a} \cdot a = 1$.
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0$ *and* $0 \cdot a = 0$.
  - Division by zero is not a possible mathematical operation. It is undefined.
  - Substitution property: If $a = b$, then b can be substituted for a in any expression, equation, or inequality.
  - Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$.

- Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$..
- Multiplication property of inequality: If $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$; if $a > b$ and $c > 0$, then $a \cdot c > b \cdot c$.
- Multiplication property of inequality (multiplication by a negative number): If $a < b$ and $c < 0$, then $a \cdot c > b \cdot c$; if $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$.
- Division property of inequality: If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$; if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.
- Division property of inequality (division by a negative number): If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$; if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

Essential Questions:	Vertical Articulation	
• What numbers are included in the solution set? • How do you represent the solution using a numberline? • How can this problem be represented verbally? • How can algebraic inequalities be written? • When solving an inequality, why is it important to perform identical operations on each side of the equal sign? • How are the procedures for solving equations and inequalities the same? • How is the solution to an inequality different from that of a linear equation?	**Previous Standards:** 6.14 The student will a) represent a practical situation with a linear inequality in one variable; and b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line. **Resources for Recovery:** 7.13 Remediation	**Future Standards:** 8.18 The student will solve multi-step linear inequalities in one variable with the variable on one or both sides of the inequality symbol, including practical problems, and graph the solution on a number line.

Vocabulary

- Inequality
- solution
- properties
- solution set
- inverse operations
- graph
- less than
- greater than
- less than or equal to
- greater than or equal to
- additive inverse
- the multiplicative inverse
- reciprocal
- constant
- coefficient
- variable
- distributive property
- identity properties of addition and multiplication
- inverse properties of addition and multiplication
- commutative properties for addition and multiplication
- associative properties for addition and multiplication
- multiplicative property of zero

Instruction

Process Goals

Problem-solving/ Communication:

- Students can plug the answer into the equation to check their answers. This is a version of working backwards, a problem solving strategy.
- Teaching the students to solve inequalities and choose an answer to plug into the inequality that is in the solution set of the answer

Reasoning/Connections:

- Hands On Equations help to build the reasoning component to equations by balancing equations visually to represent successful completion of the math problem.
- Students connect verbal equations to verbal expressions the same skills they had worked with before are connected to this

Representations:

- Use equation mats to visually demonstrate an inequality or equation

also develops problem solving ability for inequalities • Students can write an equation/inequality word problem to represent the equation given.	segment of the curriculum

Assessment Tools	Differentiation
Pre/Post Assessments: • Prerequisite Quick Check • Unit Test **Freckle** **Textbook** • Ch. 4 Lessons 4A, 4B, & 4C • Additional Lesson 9 **Tasks**	• Have students verify the solutions by substituting the value into the inequality • help students understand the difference between the solution to an equation (one) and the solution to an inequality (infinite) which is why we demonstrate it by graphing it on a numberline. • Have students create or use number lines and use rays to demonstrate how to graph solutions. **Lesson Plans** **VDOE Lessons:** • Two-Step Inequality Practical Problems

Misconceptions/ Mistakes

- Students have misconceptions about solutions to inequalities, particularly that there are many solutions that will satisfy an inequality, and that the number that remains once the variable is isolated may not itself be a solution (ex: When given a choice of four solutions to an inequality problem students have difficulty selecting the number that is greater than or less than)

Unit 6 Proportions and Percent	SOL 7.3 Essential Knowledge and Skills and Key Instructional Information

Standards	**The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:**
SOL Strand: Computation & Estimation **Focus:** Proportions and Percent **VA SOL:** 7.3 The student will solve single-step and multistep practical problems, using proportional reasoning. **Anticipated Pacing:** 11 days	

Content
Content
<u>Given</u> a proportional relationship between two quantities, **<u>create</u>** and use a ratio table to determine missing values.
<u>Write</u> and **<u>solve</u>** a proportion that represents a proportional relationship between two quantities to find a missing value.
<u>Apply</u> proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor.
<u>Apply</u> proportional reasoning to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths.
<u>Using</u> 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number.
<u>Using</u> 10% as a benchmark, compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax, and discounts.
<u>Solve</u> problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.
Understanding The Standard :

- A proportion is a statement of equality between two ratios. A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, a:b = c:d, or a is to b as c is to d.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios.
- A proportion can be solved by determining the product of the means and the product of the extremes. For example, in the proportion a:b = c:d, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes ($5 \cdot 24$) is equal to the product of the means ($12 \cdot 10$).
- In a proportional relationship, two quantities increase multiplicatively. One quantity is a constant multiple of the other.
- A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions.
  - Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the unknown cups of flour needed, solve the proportion: $\frac{2}{3} = \frac{x}{9}$. To use a table of equivalent ratios to find the unknown amount, create the table:

flour (cups)	2	4	?
oatmeal (cups)	3	6	9

    To complete the table, we must create an equivalent ratio to 2:3; just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio.

- A proportion can be solved by determining equivalent ratios.
- A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute.
- Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions.
- A multistep problem is a problem that requires two or more steps to solve.

- Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example, if 1 inch is about 2.54 cm, how many inches are in 16 cm?

$$\frac{1\ inch}{2.54\ cm} = \frac{x\ inch}{16\ cm}$$
$$2.54x = 1 \cdot 16$$
$$2.54x = 16$$
$$x = \frac{16}{2.54}$$
$x = 6.299$ or about 6.3 inches

- Examples of conversions may include, but are not limited to:
  - Length: between feet and miles; miles and kilometers
  - Weight: between ounces and pounds; pounds and kilograms
  - Volume: between cups and fluid ounces; gallons and liters
- Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term weight (e.g., "How much does it weigh?" versus "What is its mass?").
- When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed.
- Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution.
- A percent is a ratio in which the denominator is 100.
- Proportions can be used to represent percent problems as follows: $\frac{percent}{100} = \frac{part}{whole}$

Essential Questions:	**Vertical Articulation**	
• What is a ratio, proportion? • What makes two quantities	**Previous Standards:** 6.1 The student will represent	**Future Standards:** 8.4 The student will solve practical

	proportional? ● How can I write proportions that represent equivalent relationships between two sets? ● How can I solve a proportion to find a missing term? ● What is unit rate? How can it help me? ● How can I apply proportions to convert units of measurement between the U.S. Customary System and the metric system? ● What is scale factor? ● What is a benchmark percent? ● What is tax? What is discount? What is tip? ● How to find percent of a whole number? ● What are common benchmarks? ● How can benchmarks help me solve situations like tips, tax and discounts? ● How can I solve problems involving tips, tax, and discounts? ● How do you mentally calculate 10% of any number? ● How can 10% be used as a benchmark to calculate other percentages?	relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*. **Resources for Recovery:** 7.3 Remediation	problems involving consumer applications.
	Vocabulary		
	● numerator ● denominator ● unit rate ● non-proportional ● proportion ● US customary ● equivalent ratio ● benchmark		

- ratio
- proportional relationship
- ratio table
- proportional
- conversion factor
- convert
- equivalent ratios cross products
- metric
- sale price
- total price
- percent
- Tax
- tip
- discount
- scale factor

Instruction

Process Goals

Problem-solving/ Communication:
- Proportions naturally lend themselves to a variety of rich practical problems**.**
- Have students who have used different proportions or strategies to solve a problem explain/defend/justify their approach to one another, and discuss whether one strategy was more efficient.

Reasoning/Connections:
- Discuss the flexibility of constructing a proportion to model or solve a given problem, and how multiple approaches can be used to determine a solution.
- Create ratio tables and relate back to the process of finding a common denominator when combining fractions, and how simplifying a fraction does not change its value.

Representations:
- Students can use diagrams,figures, and sketches to illustrate word problems.

Assessment Tools	Differentiation
Pre/Post Assessments: • Prerequisite Quick Check	• Present students with problems that move beyond solving a proportion and that require manipulation

	• Unit Test **Freckle** **Textbook** • Ch. 5 Lessons 1A-2C • Ch. 6 Lessons 1A-2C, 3C, & 3D **Tasks**	of the solution in order to answer the question (ex: one the amount of paint needed is found, determine the cost of the paint.) • Students can visualize patterns using ratio tables. **Lesson Plans** **VDOE Lessons:** • Sales Tax, Tip, and Discount • Conversions • Proportions
	Misconceptions/Mistakes	
	• In setting up proportions, students may not be consistent in which element they use for the numerator and which for the denominator.	

Unit 7 Transformations and Graphing Relationships	SOL 7.7/7.10 Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:
SOL Strand: Measurement & Geometry Patterns, Functions, & Algebra **Focus:** Transformations and Graphing Relationships **VA SOL:** 7.7 The student will apply translations and reflections of right triangles or rectangles in the coordinate plane. 7.10 The student will a) determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form y = mx to represent the relationship; b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in y = mx form where m represents the	(see Content table)

Content
Content
Given a preimage in the coordinate plane, **<u>identify</u>** the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation.
Given a preimage in the coordinate plane, **<u>identify</u>** the coordinates of the image of a right triangle or a rectangle that has been reflected over the x- or y-axis.
Given a preimage in the coordinate plane, **<u>identify</u>** the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the x- or y-axis or reflected over the x- or y-axis and then translated.
<u>Sketch</u> the image of a right triangle or rectangle that has been translated vertically, horizontally, or a combination of both.
<u>Sketch</u> the image of a right triangle or rectangle that has been reflected over the x- or y-axis.
<u>Sketch</u> the image of a right triangle or rectangle that has been translated and reflected over the x- or y-axis or reflected over the x- or y-axis and then translated.
<u>Determine</u> the slope, m, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and **<u>write</u>** an equation in the form y = mx to represent the relationship. Slope will be limited to positive values.
<u>Graph</u> a line representing a proportional relationship, between two quantities given an ordered pair on the line and the

slope as rate of change.
c) determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form y = x + b to represent the relationship;
d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form y = x + b, where b represents the y-intercept; and
e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Anticipated Pacing:
14 days

slope, m, as rate of change. Slope will be limited to positive values.

Graph a line representing a proportional relationship between two quantities given the equation of the line in the form y = mx, where m represents the slope as rate of change. Slope will be limited to positive values.

Determine the y-intercept, b, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation,and **write** an equation in the form y = x + b, b 0, to represent the relationship.

Graph a line representing an additive relationship *(y = x + b,* $b \neq 0$*)* between two quantities, given an ordered pair on the line and the y-intercept *(b)*. The y-intercept *(b)* is limited to integer values and slope is limited to 1.

Graph a line representing an additive relationship between two quantities, given the equation in the form *y = x + b,* $b \neq 0$. The y-intercept *(b)* is limited to integer values and slope is limited to 1.

Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding The Standard :

- A transformation of a figure called the preimage changes the size, shape, or position of the figure to a new figure called the image.
- Translations and reflections do not change the size or shape of a figure (e.g., the preimage and image are congruent figures). Translations and reflections change the position of a figure.
- A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction.
- A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection.
- The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation.
- A transformation of preimage point A can be denoted as the image A' (read as “A prime”).
- The preimage of a figure that has been translated and then reflected over the x- or y-axis may result in a different transformation than the preimage of a figure that has been reflected over the x- or y-axis and then translated.

- When two quantities, x and y, vary in such a way that one of them is a constant multiple of the other, the two quantities are "proportional". A model for that situation is y = mx where m is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of y to x.
- The slope of a proportional relationship can be determined by finding the unit rate.

  Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

x	*y*
4	2
6	3

The slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$

The equation of a line representing this proportional relationship of *y* to *x* is $y = \frac{1}{2}x$ or $y = 0.5x$.

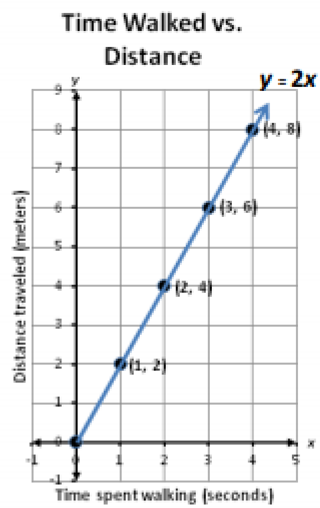
- The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

$$\text{slope} = \frac{\textit{change in } y}{\textit{change in } x} = \frac{\textit{vertical change}}{\textit{horizontal change}}$$

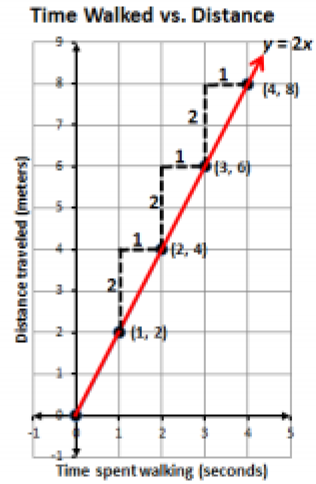
- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If x represents the number of seconds and y represents the number of meters he walks, this proportional relationship can be represented using a table of values:

x (seconds)	1	2	3	4
y (meters)	2	4	6	8

This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of y to x exists for every ordered pair. This proportional relationship could be represented by the following graph:



- A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given.
  - Example (using slope triangles): Cecil walks 2 meters every second. If x represents the number of seconds and y represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.



The rate of change from (1, 2) to (2, 4) is 2 units up (the change in y) and 1 unit to the right (the change in x), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.
    - Example:

Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

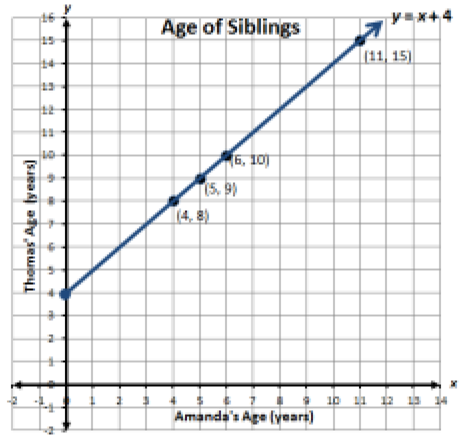
In the additive relationship, y is the result of adding 8 to x. In the multiplicative relationship, y is the result of multiplying 5 times x. The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Two quantities, x and y, have an additive relationship when a constant value, b, exists where y = x + b, where b≠ 0. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that b can be a positive value or a negative value. When b is negative, the right side of the equation could be written using a subtraction symbol (e.g., if b is −5, then the equation y = x – 5 could be used).
    - Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

Amanda's Age	4	5	6	11
Thomas' Age	8	9	10	15

+4 +4 +4 +4

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:



- Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.
    - Example: Graph the equation $y = x - 1$. In order to graph the equation, we can create a table of values by substituting arbitrary values for x to determine coordinating values for y:

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

These values can then be plotted as the points (-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

An equation written in $y = x + b$ form provides information about the graph. If the equation is $y = x - 1$, then the slope, m, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the y-axis can be located at (0, -1). We also know,

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$

So we can plot some other points on the graph using this relationship between y and x values.

A table of values can be used to determine the graph of a line. The y-intercept is located on the y-axis which is where the x-coordinate is 0. The change in each y-value compared to the corresponding x-value can be verified by the patterns in the table of values.

x	y
-1	-2
0	-1
1	0
2	1

Essential Questions:	Vertical Articulation	
• How does graphing on a coordinate plane help you understand the effect of reflections and translations? • What is a reflection? • How does a reflection affect a right triangle or rectangle in the coordinate plane? • What is a translation?	**Previous Standards:** 6.8 The student will a) identify the components of the coordinate plane; and b)identify the coordinates of a point and graph ordered pairs in a coordinate plane. 6.12 The student will	**Future Standards:** 8.7 The student will a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane; and b) identify practical applications of transformations.

	• How does a translation affect a right triangle or rectangle in the coordinate plane? • How can you use ordered pairs to identify translations? • How can you use ordered pairs to identify reflections? • What relationship exist between the slope of a line and a table of ordered pairs? • Where can we use slope in real-life? • How can a line represent a proportional relationship? • What effect does a multiplier have on a variable, and how does it change the graph of a function? • What effect does an addend have on a variable, and how does it change the graph of a function? • How do multipliers and addends show up in the various representations of a function?	a) represent a proportional relationship between two quantities, including those arising from practical situations; b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table; c) determine whether a proportional relationship exists between two quantities; and d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs. **Resources for Recovery:** 7.7 Remediation 7.10 Remediation	8.15 The student will a) determine whether a given relation is a function; and b) determine the domain and range of a function. 8.16 The student will a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero; b) identify the slope and y-intercept of a linear function, given a table of values, a graph, or an equation in y = mx + b form; c) determine the independent and dependent variable, given a practical situation modeled by a linear function; d) graph a linear function given the equation in y = mx + b form; and e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.
	Vocabulary		
	• ordered pair • transformation • quadrilateral • right triangle	• prime (?) • table of values • *y*-intercept slope triangle	• unit rate • constant • line • rate of change • change in y • slope form • equation • tables

- reflection
- translation
- pre-image
- image
- vertically
- horizontally
- Origin
- dependent variable
- independent variable
- addend
- coefficient multiplier
- Slope
- proportional relationship
- additive relationship
- change in x
- vertical change
- horizontal change
- ratio

Instruction

Process Goals

Problem-solving/ Communication:

- Have students determine a series of transformations that can move a point on the coordinate plane to a different point.
- Have students think about what sorts of situations show proportional and additive relationships: "Jenna has $10 in savings and earns $3 dollars each day for babysitting"; "Each cell in a petri dish divides once per day into two new cells"
- In writing and orally, have students explain their rationale for creating a

Reasoning/Connections:

- Have students come up with examples and nonexamples that help define categories of transformations.
- Explore how any representation can be used to create a different representation for a given relationship
- Explore the ideas of congruent polygons before and after transformations. Compare how slope and y-intercept manifest in each representation of a given relationship

Representations:

- Given a practical situation, represent the relationship in words, graph, a table of values, and with an equation representation of a proportional or additive relationship

Assessment Tools	Differentiation
Pre/Post Assessments: ● Prerequisite Quick Check ● Unit Test **Freckle** **Textbook** ● Ch. 12 Lessons 2A, 2B, 3A, & 3B ● Ch. 7 Lessons 1B-3C ● Additional Lesson 2 **Tasks** ● Summer Passes	● Use of graph paper, crayons, patty paper, mirrors, and brads to model transformations. ● Reinforce labeling of corresponding vertices to help distinguish between transformations that are ambiguous. ● Modeling with graph paper and very simple situations to illustrate how a graph is an illustration for a practical situation, and how an equation is just a boiled down version. ● Present two or three situations that yield the same equation and the same graph so that students can find the commonalities and see how they relate (ex: Sarah earns $4 per day; the plant grows 4 mm per week; the temperature increases 4 degrees every hour.) **Lesson Plans** **VDOE Lessons:** ● Translation and Reflection ● Discover Slope (m) ● Discover y-intercept (b) ● Making Connections

Misconceptions/Mistakes

- Students may confuse the words “transformation” and “translation.”
- Students may confuse vertical and horizontal movement, or may not understand negative translations to be movements left or down.
- Students may perform reflections over the wrong axis, especially when the axis intersects the preimage.
- Students may confuse the preimage and image.
- Students may plot the y-intercept on the x-axis, or if they know where the y-intercept is in the equation, assume that the slope is the x-intercept.
- Because points are plotted (*x,y*) with horizontal movement followed by vertical movement, they may get confused when slope asks them to consider “rise over run.

GCPS Curriculum Guide Math 7

Unit 8 Quadrilaterals and Similar Figures	SOL 7.5/7.6
	Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>**problem solving**</u>, <u>**mathematical communication**</u>, <u>**mathematical reasoning**</u>, <u>**connections**</u>, and <u>**representations**</u> to:
SOL Strand: Measurement & Geometry **Focus:** Quadrilaterals and Similar Figures **VA SOL:** 7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles. 7.6 The student will a) compare and contrast quadrilaterals based on their properties; and b) determine unknown side lengths or angle measures of quadrilaterals. **Anticipated Pacing:** 14 days	

Content
Content
<u>**Identify**</u> corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles.
Given two similar quadrilaterals or triangles, <u>**write**</u> similarity statements using symbols.
<u>**Write**</u> proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles.
<u>**Solve**</u> a proportion to determine a missing side length of similar quadrilaterals or triangles.
Given angle measures in a quadrilateral or triangle, <u>**determine**</u> unknown angle measures in a similar quadrilateral or triangle.
<u>**Compare**</u> and <u>**contrast**</u> properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid.
<u>**Sort**</u> and <u>**classify**</u> quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties.
Given a diagram, <u>**determine**</u> an unknown angle measure in a quadrilateral, using properties of quadrilaterals.

Given a diagram **determine** an unknown side length in a quadrilateral using properties of quadrilaterals.

Understanding The Standard :

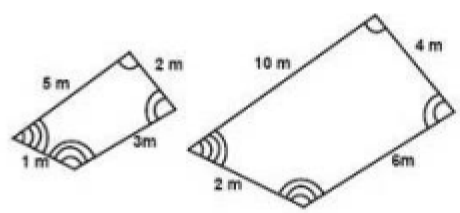
- Similar polygons have corresponding sides that are proportional and corresponding interior angles that are congruent.
- Similarity has practical applications in a variety of areas, including art, architecture, and the sciences.
- Similarity does not depend on the position or orientation of the figures.
- Congruent polygons have the same size and shape. Corresponding angles and sides are congruent.
- Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent.
- The symbol ~ is used to represent similarity. For example, ∆ABC ~ ∆DEF.
- The symbol ≅ is used to represent congruence. For example, A ≅ B
- Similarity statements can be used to determine corresponding parts of similar figures such as:

Given: $\Delta ABC \sim \Delta DEF$

$\angle A$ corresponds to $\angle D$

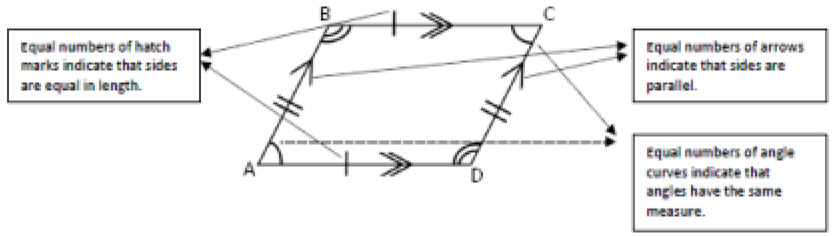
$\overline{AB}$ corresponds to $\overline{DE}$

- A proportion representing corresponding sides of similar figures can be written as $a\ b = c\ d$.
    - Example: Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.



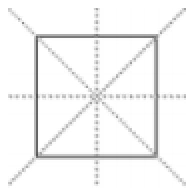
$\frac{5}{10} = \frac{2}{4}$ or $\frac{5}{10} = \frac{3}{6}$ or $\frac{1}{2} = \frac{2}{4}$ are some of the ways to express the proportional relationships that exist.

- The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if angle A is congruent to angle C, then both angles will be marked with the same number of curved lines.

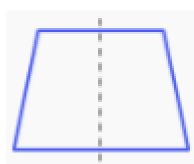


- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.
- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A quadrilateral is a polygon with four sides.
- Properties of quadrilaterals include: number of parallel sides, angle measures, number of congruent sides, lines of symmetry, and the relationship between the diagonals.
- A diagonal is a segment in a polygon that connects two vertices but is not a side.
- To bisect means to divide into two equal parts.
- A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.
- A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following:
  - opposite sides are parallel and congruent;
  - opposite angles are congruent; and
  - diagonals bisect each other and one diagonal divides the figure into two congruent triangles.
- Parallelograms, with the exception of rectangles and rhombi, have no lines of symmetry. A rectangle and a rhombus have two lines of symmetry, with the exception of a square which has four lines of symmetry.
- A rectangle is a quadrilateral with four right angles. Properties of a rectangle include the following:
  - opposite sides are parallel and congruent;
  - all four angles are congruent and each angle measures 90; and
  - diagonals are congruent and bisect each other.
- A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of

a square include the following:
  - opposite sides are congruent and parallel;
  - all four angles are congruent and each angle measures 90; and
  - diagonals are congruent and bisect each other at right angles.
- A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following:
  - all sides are congruent;
  - opposite sides are parallel;
  - opposite angles are congruent; and
  - diagonals bisect each other at right angles.
- A square has four lines of symmetry. The diagonals of a square coincide with two of the lines of symmetry that can be drawn. Example: Square with lines of symmetry shown:



- A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs.
- An isosceles trapezoid has legs of equal length and congruent base angles. An isosceles trapezoid has one line of symmetry. Example: Isosceles trapezoid with line of symmetry shown:



- A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to properties such as sides and/or angles.
- Quadrilaterals can be classified by the number of parallel sides: parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has one pair of parallel sides; other quadrilaterals have no parallel sides.
- Quadrilaterals can be classified by the measures of their angles: a rectangle and a square have four 90° angles; a

trapezoid may have zero or two 90° angles.

- Quadrilaterals can be classified by the number of congruent sides: a rhombus and a square have four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides, and an isosceles trapezoid has one pair of congruent sides.
- A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals.
- Any figure that has the properties of more than one subset of quadrilaterals can belong to more than one subset.
- The sum of the measures of the interior angles of a quadrilateral is 360°. Properties of quadrilaterals can be used to find unknown angle measures in a quadrilateral.

<table>
<tr><th>Essential Questions:</th><th colspan="2">Vertical Articulation</th></tr>
<tr>
<td>Why can some quadrilaterals be classified in more than one category?
What are the critical attributes of various quadrilaterals?
How can you use attributes to compare and contrast quadrilaterals?
How can properties of quadrilaterals be used to determine a missing angle measure in a quadrilateral?
What are the relationships between corresponding angles and sides in similar figures?
How can I write proportions that represent equivalent relationships between two sets?
What is scale factor?</td>
<td>Previous Standards:
5.12 The student will classify and measure right, acute, obtuse, and straight angles.

5. 13 The student will
a) classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles; and
b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

6.9 The student will determine congruence of segments, angles, and polygons.

Resources for Recovery:
7.5 Remediation</td>
<td>Future Standards:
G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.</td>
</tr>
</table>

• What are corresponding parts, sides and angles? • How can I write proportions to express the relationships between the lengths of corresponding sides of similar figures? • How can I determine if quadrilaterals or triangles are similar? • How can I write a similarity statement? • How do you find corresponding parts and determine if a polygon is similar?	7.6 Remediation	

Vocabulary

- square
- set
- subset
- diagonal
- vertex
- perpendicular
- parallel
- interior angle
- polygon
- quadrilateral
- trapezoid
- isosceles trapezoid
- parallelogram
- rhombus
- rectangle
- Bases
- right angle
- bisect
- opposite
- adjacent
- congruent
- Venn Diagram
- intersect
- line of symmetry
- ratio
- triangle
- quadrilateral
- corresponding parts
- similar
- Nonproportional
- congruent
- proportional
- scale factor
- similarity symbol (~)
- proportion
- angles

Instruction

Process Goals

Problem-solving/ Communication:

- Have students explain and justify their rationales orally and in writing for categorizing shapes.
- Have students who have used different proportions or strategies to solve a problem explain/defend/justify their approach to one another, and discuss whether one strategy was more efficient.

Reasoning/Connections:

- Have students come up with examples and nonexamples that help define categories of quadrilaterals
- Point out that the scale factor from one figure to a similar one is the reciprocal of the scale factor in the reverse direction. Use opportunities to preview dilations.

Representations:

- Use a variety of organizational tools and strategies for categorizing quadrilaterals.
- Have students create models to solve practical problems that are analogous to similar figures.
- Students can use diagrams,figures, and sketches to illustrate word problems.

Assessment Tools	Differentiation
Pre/Post Assessments: ● Prerequisite Quick Check ● Unit Test **Freckle** **Textbook** ● Ch. 12 Lesson 1D ● Ch. 5 Lesson 3A **Tasks** ● A Walk in the Park	● Use of graphic organizers (Venn Diagrams, trees) and foldables to reinforce quadrilateral vocabulary, critical attributes, and other attributes. ● If students grasp setting up proportions for similar figures, but struggle with practical problems, have them create pictures or models to visualize a situation before setting up a proportion. ● Students can use colored pencils to highlight corresponding sides. Students can visualize patterns using ratio tables.

	Lesson Plans
	VDOE Lessons: • Similar Figures • Missing Measurements • Classifying Quadrilaterals • Quadrilaterals - Measures of Sides and Angles

Misconceptions/Mistakes

- Students ascribe attributes that are not necessarily true (rectangles must be longer than wide, rhombi must "lean," trapezoids are always isosceles), so fail to identify a square is a rectangle and a rhombus.
- Students may try to assume an additive relationship instead of a multiplicative one (ex: "they are similar because we added 2 to all of the sides.")
- In setting up proportions, students may not be consistent in which element they use for the numerator and which for the denominator.
- Similar shapes given with different alignments may present problems when determining which elements correspond.
- Students may not recognize congruent figures as also being similar.

Unit 9 Volume and Surface Area	SOL 7.4 Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:
SOL Strand: Measurement & Geometry **Focus:** Volume and Surface Area **VA SOL:** 7.4 The student will a) describe and determine the volume and surface area of rectangular prisms and cylinders; and b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders. **Anticipated Pacing:** 12 days	

Content

Content
<u>Determine</u> the surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas.
<u>Determine</u> the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas.
<u>Determine</u> if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area.
<u>Solve</u> practical problems that require determining the surface area of rectangular prisms and cylinders.
<u>Solve</u> practical problems that require determining the volume of rectangular prisms and cylinders.

Understanding The Standard :

- A polyhedron is a solid figure whose faces are all polygons.
- A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges.
- A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders.
- A face is any flat surface of a solid figure.

- The surface area of a prism is the sum of the areas of all 6 faces and is measured in square units.
- The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units.
- Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure.
- A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$).
- A cylinder can be represented on a flat surface as a net that contains two circles (the bases of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ($SA = 2\pi r^2 + 2\pi rh$).
- The volume of a rectangular prism is computed by multiplying the area of the base, B, (length times width) by the height of the prism (V = lwh = Bh).
- The volume of a cylinder is computed by multiplying the area of the base, B, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$).
- The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator.

Essential Questions:	**Vertical Articulation**	
	Previous Standards:	**Future Standards:**
• How are volume and surface area related? • How can you use a net, diagram, or concrete object to understand surface area or volume? • How can you use a net to develop the formula for surface area or volume?	5.8 The student will a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.	8.6 The student will a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

	6.7 The student will a) derive π (pi); b) solve problems, including practical problems, involving circumference and area of a circle; and c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles. **Resources for Recovery:** 7.4 Remediation	

Vocabulary

- area
- surface area
- volume
- net
- face
- lateral face
- base
- prism
- cylinder
- attribute
- Two-dimensional
- Three-dimensional
- height
- Pi
- polyhedron

Instruction

Process Goals

Problem-solving/ Communication:

- Have students solve a variety of real-world problems involving volume and surface area

Reasoning/Connections:

- Explore the link between units and how they are affected algebraically when included with their numbers in the formulas for volume

Representations:

-

- Have students describe their steps for computing volume and surface area orally and in writing and surface area (ex: $V = lwh$, $V = (2\ in)(3\ in)(4\ in)$, $V = 2 \cdot 3 \cdot 4 \cdot in \cdot in \cdot in$, $V = 24\ in^3$)

Assessment Tools	Differentiation
Pre/Post Assessments: ● Prerequisite Quick Check ● Unit Test **Freckle** **Textbook** ● Ch. 10 Lessons 1A, 1B, 1C, 2A-2D ● Additional Lesson 13 **Tasks** ● Shipping Box	● Have students assemble rectangular prisms from nets made on graph paper to see the relationship between the areas of the faces and the surface area. ● Use manipulatives to compare volume and surface area. **Lesson Plans** **VDOE Lessons:** ● Volume and SA of Rectangular Prisms and Cylinders

Misconceptions/Mistakes

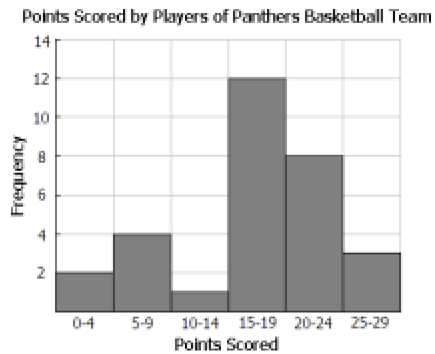
- Some students may not read carefully and choose the wrong measure to solve a practical problem.
- For cylinders given “on their sides,” diameter may be confused for height and height may be interpreted as length.
- For cylinders, the student must use the radius in the formula. If student is given the diameter, they may use the diameter in the formula instead of the radius.

Unit 10 Probability and Statistics	SOL 7.8/7.9 Essential Knowledge and Skills and Key Instructional Information

Standards	The student will use <u>problem solving</u>, <u>mathematical communication</u>, <u>mathematical reasoning</u>, <u>connections</u>, and <u>representations</u> to:
SOL Strand: Probability & Statistics **Focus:** Probability and Statistics **VA SOL:** 7.8 The student will a) determine the theoretical and experimental probabilities of an event; and b) investigate and describe the difference between the experimental probability and theoretical probability of an event. 7.9 The student, given data in a practical situation, will a) represent data in a histogram; b) make observations and inferences about data represented in a histogram; and c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.	**Content**

Content
<u>Determine</u> the theoretical probability of an event.
<u>Determine</u> the experimental probability of an event.
<u>Describe</u> changes in the experimental probability as the number of trials increases.
<u>Investigate</u> and **<u>describe</u>** the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.
<u>Collect</u>, **<u>organize</u>**, and **<u>represent</u>** data in a histogram.
<u>Make observations and inferences</u> about data represented in a histogram.
<u>Compare</u> data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots.
Understanding The Standard :

Anticipated Pacing: 12 days	● In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space. ● The probability of an event occurring can be represented as a ratio or equivalent fraction, decimal, or percent. ● The probability of an event occurring is a ratio between 0 and 1. - A probability of 0 means the event will never occur. - A probability of 1 means the event will always occur. ● The theoretical probability of an event is the expected probability and can be determined with a ratio. ● If all outcomes of an event are equally likely, the theoretical probability of an event = $\frac{\textit{number of possible favorable outcomes}}{\textit{total number of possible outcomes}}$ ● The experimental probability of an event is determined by carrying out a simulation or an experiment. ● The experimental probability of an event = $\frac{\textit{number of times desired outcomes occur}}{\textit{number of trials in the experiment}}$ ● In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers). ● A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class or bin is depicted by the use of a bar. Every element of the data set is not preserved when representing data in a histogram. ● All graphs must include a title and labels that describe the data. ● Numerical data that can be characterized using consecutive intervals are best displayed in a histogram. ● Teachers should be reasonable about the selection of data values. Students should have experiences constructing histograms, but a focus should be placed on the analysis of histograms. ● A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.



- A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.

Number of Cappuccinos Made per Hour at the Cafe

Number of Cups of Coffee	Tally	Frequency
0 - 3	||	2
4 – 7	|||	3
8 – 11	~~||||~~ |||	8
12 – 15	|||	3
16 – 19	||	2

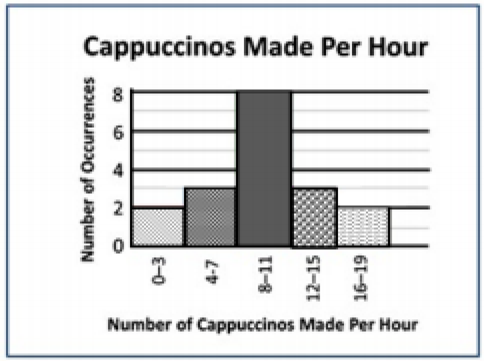
To construct a histogram:

- Organize collected data into a table. Create one column for data range categories (bins), divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
    - Bins should be all the same size.
    - Bins should include all of the data.
    - Boundaries for bins should reflect the data values being represented.
    - Determine the number of bins based upon the data.

- If possible, the number of bins created should be a factor the number of data values (e.g., a histogram representing 20 data values might have 4 or 5 bins).

- Create a graph. Mark the data range intervals on the x-axis (horizontal axis) with no space between the categories. Mark frequency on the y-axis (vertical axis), also in equal intervals.

- Plot the data. For each data range category (bin), draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category (bin).



- Note: histograms may be drawn so that the bars are horizontal. To do this, interchange the x- and y-axis. Mark the data range intervals (bins) on the y-axis and the frequency on the x-axis. Draw the bars horizontally.

- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. Data analysis helps describe data, recognize patterns or trends, and make predictions.
- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate.
- A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number

line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.
- A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. Every element of the data set is not preserved when representing data in a circle graph.
- A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. A stem and leaf plot displays the entire data set and provides a picture of the distribution of data.
- Different situations or contexts warrant different types of graphs, and it helps to have a good knowledge of what graphs are available. Students can determine which graph makes the most sense to use based on the type of data provided and which graph can help them answer questions most easily.
- Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides.
- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or “what could happen if” (inference).

<table>
<tr><th>Essential Questions:</th><th colspan="2">Vertical Articulation</th></tr>
<tr>
<td>What is theoretical probability?
What is experimental probability?
What is the difference between the theoretical and experimental probability of an event?
How does increasing the number of trials affect the experimental probability?
What is the relationship between the theoretical probability and experimental probability of the same event?
What types of data are most appropriate to display in a histogram?
What information does or</td>
<td>Previous Standards:
5.15 The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

5.16 The student, given a practical problem, will
a) represent data in line plots and stem-and-leaf plots;
b) interpret data represented in line plots and stem-and-leaf plots; and
c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.</td>
<td>Future Standards:
8.11 The student will
a) compare and contrast the probability of independent and dependent events; and
b) determine probabilities for independent and dependent events.

8.12 The student will
a) represent numerical data in boxplots;
b) make observations and inferences about data represented in boxplots; and</td>
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doesn't a histogram provide? • Why can't specific numbers be determined from data presented in a histogram? • How can a histogram help you infer or draw conclusions about a given set of data? • How can you interpret and compare data sets using data displays?	6.10 The student, given a practical situation, will a) represent data in a circle graph; b) make observations and inferences about data represented in a circle graph; and c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots. **Resources for Recovery:** 7.8 Remediation 7.9 Remediation	c) compare and analyze two data sets using boxplots. 8.13 The student will a) represent data in scatterplots; b) make observations about data represented in scatterplots; and c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

Vocabulary

- theoretical probability
- experimental probability
- outcomes
- trials
- Law of Large Numbers
- event
- ratio
- equally likely
- fraction
- decimals
- percentages
- histogram
- interval
- frequency
- circle graph
- line plot
- stem and leaf plots
- categorical data
- numerical data
- consecutive
- frequency distribution
- inference
- data
- title
- labels
- clusters

Instruction

Process Goals

Problem-solving/ Communication:

- Using strategies solve and make prediction in a variety of real-world problems and situations (games of chance, etc.).
- Given a real-world problem, have students make decisions about a policy or to solve a community problem based on data presented in a variety of graphs.
- Describing steps taken to arrive at a solution to a problem orally and in writing.
- Have students explain their inferences made from a histogram orally and in writing.

Reasoning/Connections:

- Given multiple situations, determining which one is more likely to have a successful outcome based on calculations, and comparing that to predictions; reconciling differences between theoretical and experimental probabilities. “Given a histogram, can one identify the mean? median? mode?”
- Creating two different real world situations that are expressed by the same given arbitrary probability.
- Have students make comparisons between the same information presented in different kinds of graphs to determine each type’s strengths and weaknesses.

Representations:

- Expressing probability in different forms (ratio, percent, decimal), understanding its place on the number line between 0 and 1 (0% and 100%);
- Graphing experimental vs theoretical on a line graph over time to show how experimental probability stabilizes and gravitates toward the theoretical as the number of trials increases.
- Have students develop their own research questions, create frequency distributions, and histograms to represent the data.

Assessment Tools	Differentiation
Pre/Post Assessments: ● Prerequisite Quick Check ● Unit Test **Freckle** **Textbook**	● Use of manipulatives to demonstrate experimental probability; instructional sequence that builds on a problem to increase its complexity (ex: On a number cube, what is P(4)? P(4,4)?, P(4,4,4)?) ● Graphing probabilities on a number line for students who have not developed intuition about the relative size of fractions (ex: 17/30 is slightly

- Ch. 8 Lessons 1A, 3A, & 3B
- Additional Lesson 17, 21
- Ch. 9 Lessons 3A-3E

Tasks

- Spin, Spin, Spin

more than one half).

- Use dice, cards, spinners, objects to choose out of a bag/basket, online dice, spinners, etc.
- Use games such as Skunk, Obstacle Course and Motorway, horse race to engage students.
- Have students use the same set of data to create three different histograms that use different intervals for the bars.
- Create two sets of data that would create the same histogram based on the intervals used in order to reinforce that specific numbers are disguised in a histogram.

Lesson Plans

VDOE Lessons:

- What are the Chances?
- Numbers in a Name
- All Graphs Are Not The Same

Misconceptions/Mistakes

- The probability of rolling a 4 is 4/6 not 1/6
- Students may believe they can pull specific numbers from a data set presented in a histogram, or try to create a histogram by drawing a bar graph without spaces between bars, even when the numbers are not grouped into intervals.