

Term Project | Term 211 | Group 25

Solar Stoves Distribution

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For

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Optimization Methods

ISE 321

Abstract

This report presents the corresponding planning phase in which the NPO decides on the best distributing quantity of solar cooking stove while minimizing the overall costs for the following year, and then generalize the model to handle P port countries and D destination countries.

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I. Introduction Questions

(a) Identify potential global standards that should be considered in this project. For example, standards related to solar stoves, standards related to components, standards related to shipping container, standards related to shipping, etc

A **standard container** is the most common type of container in the market. It's usually made of steel and sometimes aluminum. Standard ISO shipping containers are 8ft (2.43m) wide, 8.5ft (2.59m) high and come in two lengths; 20ft (6.06m) and 40ft (12.2m). Extra tall shipping containers called high-cube containers are available at 9.5ft (2.89m) high.

The dimensions of a shipping container are determined by industry regulations to ensure that there are no problems during transport. The International Organization for Standardization (ISO) establishes the size of shipping containers.

The typical solar stove has three reflectors and a vapor wiper mechanism made from locally accessible materials. The external box dimensions of the box cooker are 550 mm x 550 mm x 230 mm, and the pyramidal interior box measurements are 460 mm x 460 mm upper face and 300 mm x 300 mm lower face with a depth of 135 mm. outer Box: The outer box of a solar cooker is generally made of G.I. or aluminum sheet or fibre reinforced plastic. Inner Cooking Box (Tray): This is made from aluminum sheet. Double Glass Lid: A double glass lid covers the inner box or tray.

standards related to shipping

Cost (Retail pricing) for First Class Package International is 26.36\$

It takes 2-10 days for delivery.

(b) Identify how different cultures across the globe have different requirements for stoves in general, and solar stoves in particular.

It depends about the method of using the stove and the number of family members which related to the size of the stove. In the Africa continent, which is the subject of focus for this project, the size of the stove used is large because some villages use one stove with large cooking bowl to cook food for all families in the village.

(c) Discuss how the reduced taxes for components will affect the indigenous society in terms of local manufacturing and future industrial development.

In general, and in all domains, the first priority in a society is to pay less. Reducing taxes of the components will cause an impact on GDP growth rates especially for extremely poor countries such as Somalia, Ghana and other African countries.

Reducing taxes can be considered as a golden opportunity for them to manufacture and develop their industry.

(d) Discuss how the environment in general gets benefit from using such stoves.

Regarding developing countries, due to their significant fuel use, the three-stone fires, lighting fire using wood and charcoal etc., used by three billion people around the world present dangers to the local environments where they are used and contribute to global climate change. Therefore, one solution is to use solar stoves because they are clean, renewable, and readily available solar energy as a fuel, conserving natural resources by not requiring the use of wood or other biomass fuels for cooking and not producing dangerous emissions that pollute local environments and contribute to climate change.

On the other hand, in the developed countries such as the European Union (EU), households in consume 25% (402 Mtoe) of the final energy, cause 20% (846 Mt) of annual greenhouse gas emissions (GHG) emissions and produce 8% (209 Mt) of total waste in the EU (Eurostat, 2016). The share of these, including >11% of household electricity consumption, is due to the use of electrical appliances. Consequently, using home-made solar cookers instead of microwaves for example where possible could lead to significant environmental benefits.

II. Assumption

- * We allow for mix delivery of fully and assembly stoves containers to the port country, so we can have more options which will minimize the total costs.
- * We assume that the capacity of the 40ft high cube containers 76 m 3 , for this the maximum number of units it can hold of fully stoves is = 76 / (0.30x0.30x0.35) = 2412 boxes and for assembly = 12,666 boxes.
- * We allow for mix delivery of fully and assembly stoves unis to the destination country from any port that does not have a plant, so we can have more options which will minimize the total costs and do not conflict with the type 2 flow.

III. Decision Variables Definitions

• Integer Variables ≥ 0:

 x_0 = The number of fully stoves shipped from Brazil to the port country **p**

 y_p = The number of solar assemblies shipped from Brazil to the port country ${f p}$

 $\mathbf{w}_{p,d}$ = The number of fully stoves shipped from the port country \mathbf{p} to destination country \mathbf{d}

 $z_{p,d}$ The number of solar assemblies shipped from the port country **p** to destination country **d**

 cx_p = The number of fully stoves containers shipped from Brazil to the port country **p**

 cy_p = The number of fully stoves containers shipped from Brazil to the port country ${f p}$

• Binary Variables:

 $p_d = \{1 \text{ if there is a plant in the country } d = 0$ otherwise

IV. Parameters Definitions

 cc_p = Fixed cost per container shipped to port country **p**

cu_p = Shipping cost per solar stove from Brazil to port country **p**

 $cf_{p,d}$ = Shipping costs per solar stove from port country **p** to the final destination country **d**.

t_d = Setup costs per assembly plant funded by the organization in the country d

 a_d = Assembly costs per unit funded by the organization in the country **d**

 r_d = Price of reflector in the country **d**

 D_d = The total demand for every country **d**

V. Constraints Formulation

* Logic constraint to ensure that the port country does not ship more than what it has.

$$x_p \ge \sum_{d=1}^{12} w_{p,d} \ \forall \ p = 1 \dots 6$$

$$y_p \ge \sum_{d=1}^{12} z_{p,d} \ \forall \ p = 1 \dots 6$$

* Maximum capacity for every container

$$x_p \ge 2412 \text{ cx}_p \ \forall \ p = 1 \dots 6$$

$$y_p \ge 12666 \text{ cy}_p \ \forall \ p = 1 \dots 6$$

* Constraint to link the decision variable $\boldsymbol{z}_{p,d}$ with \boldsymbol{p}_d

$$\sum_{p=1}^{6} z_{p,d} \ge 1 - M(1 - p_d) \quad \forall d = 1 \dots 12$$

$$\sum_{p=1}^{6} z_{p,d} \le M(p_d) \quad \forall d = 1 \dots 12$$

* Demand Constraint for every country

$$\sum_{p=1}^{6} w_{p,d} + \sum_{p=1}^{6} z_{p,d} = D_{d} \ \forall \ d = 1 \dots 12$$

* Type 2 flow constraints

$$\sum_{d=1, d \neq i}^{12} z_{p,d} \le M(1 - p_i) \ \forall (p, i) \in S$$

Where S =

$$\{(1,3),(2,4),(3,5),(4,6),(5,9),(6,10)\}$$

* Logic constraint to not allow port country to ship units of fully stoves more than what it has is if there is a plant in this port country.

$$\sum_{d=1}^{12} w_{p,d} \le x_p + y_p + M(1 - p_i) \quad \forall p = 1 \dots 6 \quad \forall i = 3, 4, 5, 6, 9, 10$$

Where M is a big number

VI. The Objective Function

Minimize:

$$\begin{split} &\sum\limits_{p=1}^{6} cc_{p}(cx_{p} + cy_{p}) + \sum\limits_{p=1}^{6} cu_{p}x_{p} + \frac{1}{2}\sum\limits_{p=1}^{6} cu_{p}y_{p} + \sum\limits_{p=1}^{6}\sum\limits_{d=1}^{12} cf_{p,d}w_{p,d} + \\ &\frac{1}{2}\sum\limits_{p=1}^{6}\sum\limits_{d=1}^{12} cf_{p,d}z_{p,d} + 40\sum\limits_{p=1}^{6}\sum\limits_{d=1}^{12} w_{p,d} + \sum\limits_{d=1}^{12} t_{d}p_{d} + 30\sum\limits_{p=1}^{5}\sum\limits_{d=1}^{5}z_{p,d} + \sum\limits_{p=1}^{6}\sum\limits_{d=1}^{12} r_{d}z_{p,d} + \\ &\sum\limits_{p=1}^{6}\sum\limits_{d=1}^{12} a_{d}z_{p,d} \end{split}$$

VII. The Problem size

The model is Pure Integer Linear Program which has an overall:

- 180 Integer Variable
 - 12 of them are binary variables
 - ❖ 168 of them are general integer variables.
- 72 Linear constraints

VIII. The Optimal Solution

We have used GAMS optimization software to solve the model (the modeling codes are shown in (**Appendix**), and the global optimal solution cost was equal to = 5,274,890 \$. we assume **M** = 1000,000

The nonprofit organization (NPO) should follow the following distribution plan to minimize the total cost:

- Flow Type 1

It should send fully stoves units from Brazil to three port countries which are:

- 1- Cameroon = 26532 units in 11 containers, which will use from it 8068, and send 2442 units to Rwanda, 9317 units to and 6705 units to .
- 2- Ghana = 9648 units in 4 containers, which will use from it 5441 units, and send 1491 units to Kenya, and 2716 to Zimbabwe.
- 3- Senegal = 38592 units in 16 containers, which will use from it 6719 units, and send to Chad 3021 units, D. R. Congo 13374 units, Ghana 1307 units, Mali 8346 units, Senegal 6719 and Uganda 5825 units.

- Flow Type 3

Ghana will receive 12454 units in 1 containers of assembly stoves and send to Burkina Faso 5640 units and to Kenya 6814 units where both have plants.

Receive	Keep	Send
26522	9069	1846
26532	8068	4

IX. The Greedy Heuristic Algorithm

a- Finding Random Initial Solution

We Start with random solution created by excel, using some penalties to not exceed the constraints. We start with x_p , y_p to generate the initial solution:

	A	В	С		
1	Intermediate country	×	У		
2	Cameroon	11339	34018		
3	D. R. Congo	0	26168		
4	Chana	9595	28785		
5	Kenya	5234	0		
6	Senegal	9595	0		
7	Tanzania	6106	18317		

And from this we find the other variables $w_{\text{p,d}}$, $z_{\text{p,d}}$ and p_{d} :

8	Burkina Faso	Chad	Cameroon	D. R. Congo	Chana	Kenya	Mali	Rwanda	Senegal	Tanzania	Uganda	Zimbabwe	Burkina Faso	Chad	Carneroo	D. R. Cong	Chana	Kenya	Mali	Rwanda	Senegal	Tanzania	Uganda	Zimbabwe
9 Cameroon	0	0	8068	0	0	0	0	0	0	0	7257	0	0	0	0	0	0	0	0	0	0	0	0	0
10 D. R. Congo	0	0	0	13374	0	0	0	0	0	0	0	3076	0	0	0	0	0	0	0	0	0	0	0	0
11 Chana	1391	2070	2469	0	6748	1920	8904	3060	0	4344	0	7189	0	0	0	0	0	0	0	0	0	0	0	0
12 Kenya	0	0	0	0	0	5234	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13 Senegal	1556	1916	4	0	0	0		0	6719	0	0		0	0	0	0	0	0	0	0	0	0	0	0
14 Tanzania	3090	0	0	0	485	0	4108	0	0	6106	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15 Sum =	6037.27	3985.53	10540.35	13374.00	7232.64	7153.67	13012.13	3060.00	6719.00	10449.80	7257.20	10265.20	0.00	0.00	3324.10	2698.55	570.01	2489.12	1711.43	0.00	257.20	513.31	1533.24	556.91
16 D =	5640.00	3021.00	8068.00	13374.00	6748.00	8305.00	8346.00	2442.00	6719.00	9317.00	5825.00	9421.00	5640.00	3021.00	8068.00	13374.00	6748.00	8305.00	8346.00	2442.00	6719.00	9317.00	5825.00	9421.00
17						₩												Z						
18 W+Z	6037.27	3985.53	10540.35	6563.31	7232.64	7153.67	13012.13	3060.00	6719.00	10449.80	7257.20	10265.20	0	0					0	0			0	0
19																		р						

The clarifications of the constraint penalty are as follow:

For x_p we use this formula :

$$= |F(RAND()<0.6,0.13*SUM(B16:M16),0)|$$

To generate a random solution that penalize by the reciprocal of the shipping cost for every port country and multiped by the total demand, below is the penalties for every port country:

0.13	
0.10	
0.11	
0.06	
0.11	
0.07	

For y_p it is the same as x_p but the reciprocal is multiplied by 3, because you can ship more with less cost.

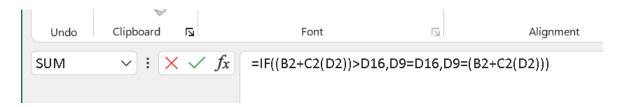
Now, because the function of a random variables is a random variable, we use x_p and y_p values to find $w_{p,d}$, $z_{p,d}$ and p_d with some penalties to control the output. For $w_{p,d}$ we use this formula

- ·				_ ·		J	
: X ✓ j	fx =IF(R	AND()<0.	4,0.64* <mark>(</mark> \$	B\$2+\$C\$2*	(\$D\$2) <mark>)-\$</mark> I	O\$15, 0)	
В	С	D	Е	F	G	Н	
ж	У	р					Г
11339	34018	1.00					
O	26168	1.00					
9595	28785	1.00					
5234	0	0.00					
9595	0	0.00					
6106	18317	1.00					L
Burkina Faso	Chad	Cameroon	D. R. Congo	Chana	Kenya	Mali	R
\$D\$2))-\$D\$15,0)	0	8068	0	o	0	0	С
0	0	0	13374	0	0	0	C
1391	2070	2469	o	6748	1920	8904	3
0	0	0	0	0	5234	0	С
1556	1916	4	0	0	0		이이에이이이
3090	0	О	О	485	0	4108	С
6037.27	3985.53	10540.35	13374.00	7232.64	7153.67	13012.13	
5640.00	3021.00	8068.00	13374.00	6748.00	8305.00	8346.00	

which generate a random solution with the penalty of shipping cost from this port country to the final destination country multiplied by the summation of both fully and assembly units coming from Brazil subtracted from it the demand of the port country. The table below show the whole penalties

\$0.64 \$0.79 \$1.00 \$0.67 \$0.93 \$0.64 \$0.73 \$0.79 \$0.76 \$0.82 \$0.73 \$0.79 \$0.77 \$0.88 \$0.92 \$1.00 \$0.64 \$0.61 \$0.96 \$0.71 \$0.64 \$0.77 \$0.58 \$0.84 \$0.96 \$0.96 \$0.88 \$0.92 \$1.00 \$0.64 \$0.61 \$0.96 \$0.71 \$0.64 \$0.77 \$0.58 \$0.84 \$0.96 \$0.96 \$0.96 \$0.88 \$0.96 \$0.88 \$0.64 \$0.67 \$0.88 \$0.71 \$0.64 \$0.67 \$0.71 \$0.74 \$0.84 \$0.79 \$0.79 \$0.79 \$0.70 \$0.79 \$0.75 \$1.00 \$0.89 \$0.70 \$0.62 \$0.70 \$0.66 \$0.89 \$0.72 \$0.95 \$0.90 \$0.85 \$0.90 \$0.57 \$0.81 \$0.57 \$1.00 \$0.89 \$0.70 \$0.62 \$0.70 \$0.60 \$0.90 \$0.50 \$0.66 \$0.89 \$0.66 \$0.89 \$0.89 \$0.85 \$0.90 \$0.57 \$0.81 \$0.57 \$0.66 \$0.89 \$0.70 \$0.60 \$0.72 \$0.90 \$0.50 \$0.66 \$0.89 \$0.89 \$0.80 \$0.89 \$0.80 \$0.80 \$0.80 \$0.80 \$0.80 \$0.60 \$0.80 \$0.60 \$0.80 \$0.70 \$0.60 \$0.70 \$0.70 \$0.60 \$0.70													
\$0.96 \$0.88 \$0.64 \$0.64 \$1.00 \$0.88 \$0.71 \$0.64 \$0.67 \$0.71 \$0.74 \$0.84 \$0.79 \$0.79 \$0.79 \$0.70 \$0.75 \$1.00 \$0.89 \$0.70 \$0.65 \$0.90 \$0.85 \$0.90 \$0.57 \$0.81 \$0.57 \$1.00 \$0.72 \$0.90 \$0.50	\$0.64	\$0.79	\$1.00	\$0.67	\$0.93	\$0.64	\$0.73	\$0.79	\$0.76	\$0.82	\$0.73	\$0.79	
\$0.79 \$0.79 \$0.70 \$0.75 \$1.00 \$0.89 \$0.70 \$0.62 \$0.70 \$0.66 \$0.89 \$0.72 \$0.95 \$0.90 \$0.85 \$0.90 \$0.57 \$0.81 \$0.57 \$1.00 \$0.72 \$0.90 \$0.50	\$0.77	\$0.88	\$0.92	\$1.00	\$0.64	\$0.61	\$0.96	\$0.71	\$0.64	\$0.77	\$0.58	\$0.84	
\$0.72 \$0.95 \$0.90 \$0.85 \$0.90 \$0.57 \$0.81 \$0.57 \$1.00 \$0.72 \$0.90 \$0.50	\$0.96	\$0.88	\$0.64	\$0.64	\$1.00	\$0.88	\$0.71	\$0.64	\$0.67	\$0.71	\$0.74	\$0.84	
	\$0.79	\$0.79	\$0.70	\$0.79	\$0.75	\$1.00	\$0.89	\$0.70	\$0.62	\$0.70	\$0.66	\$0.89	
\$0.66 \$0.89 \$0.82 \$0.69 \$0.93 \$0.75 \$0.89 \$0.66 \$0.69 \$1.00 \$0.61 \$0.72	\$0.72	\$0.95	\$0.90	\$0.85	\$0.90	\$0.57	\$0.81	\$0.57	\$1.00	\$0.72	\$0.90	\$0.50	
	\$0.66	\$0.89	\$0.82	\$0.69	\$0.93	\$0.75	\$0.89	\$0.66	\$0.69	\$1.00	\$0.61	\$0.72	

In case of the port country, we use this formula to get as many as the demand from the coming shipment.



For $z_{p,d}$ is almost the same as $w_{p,d}$ expect for little penalty to control the constraint of flow type 2:

$$\times f_x$$
 =IF(RAND()<0.6,0.64*N\$16,0 *(1-\$D2)

In case of p_d there are two situations, first when the plant is on port country, then it is just a condition :

Second, when it is a destination country, then will check if the port country has a plant or not, where N9:N14 are the summation of assembly units coming from all port countries.

$$f_x$$
 =IF(SUM(N9:N14)>1,1,0)

Also, we established a row to monitor the demand, in case is something get wrong:

D =	5640.00	3021.00	8068.00	13374.00	6748.00	8305.00	8346.00	2442.00	6719.00	9317.00	5825.00	9421.00
		•	•	•		w		•			•	
w+z	6037.27	3985.53	10540.35	6563.31	7232.64	7153.67	13012.13	3060.00	6719.00	10449.80	7257.20	10265.20

And from what shown, every country is satisfied with demand.

B- Greedy Heuristic Algorithm

We consult we one of the experts ,Dr Syed Mujahid, and we find the best way is to use x_p from the initial solution to add and subtract a container for every neighbor where the number of units for fully stoves container is 2412 units, and use the excel to find the objective function values and the pervious mentioned penalties to control the constraints:

We get:

	X_1	X_2	X_3	X ₄	X_5	X_6	Obj
X _o	11339	0	9595	5234	9595	6106	\$15,358,613
N_1	13751	0	9595	5234	9595	6106	\$15,467,000
N_2	8927	0	9595	5234	9595	6106	\$15,562,631
N_3	11339	2412	9595	5234	9595	6106	\$17,653,613
N_4	11339	0	12007	5234	9595	6106	\$15,557,436
N_5	11339	0	7183	5234	9595	6106	\$15,400,842
N_6	11339	0	9595	7646	9595	6106	\$16,360,725
N ₇	11339	0	9595	2822	9595	6106	\$15,641,756
N ₈	11339	0	9595	5234	12007	6106	\$15,543,124

N ₁₀	11339 11339	0	9595 9595	5234 5234	9595 9595	8518 3694	\$15,641,756 \$15,217,436
N ₉	11339	0	9595	5234	7183	6106	\$15,562,631

 N_{11} have the best (Minimum) value, and it can be used to second iteration. $X^{(1)} = N_1$

X. The General MIP case

It will be much different from what we build, except for small different :

A.Decision Variables

• Integer Variables ≥ 0:

 x_p = The number of fully stoves shipped from Brazil to the port country \mathbf{p} y_p = The number of solar assemblies shipped from Brazil to the port country \mathbf{p} $\mathbf{w}_{p,d}$. The number of fully stoves shipped from the port country \mathbf{p} to destination country \mathbf{d} $z_{p,d}$. The number of solar assemblies shipped from the port country \mathbf{p} to destination country \mathbf{d} cx_p = The number of fully stoves containers shipped from Brazil to the port country \mathbf{p} cy_p = The number of fully stoves containers shipped from Brazil to the port country \mathbf{p}

• Binary Variables:

 $p_d = \{1 \text{ if there is a plant in the country } d = 0$ otherwise

where p = 1...P port countries, d = 1....D destination countries.

B. Parameters Definitions

 cc_p = Fixed cost per container shipped to port country **p**

cu_p = Shipping cost per solar stove from Brazil to port country **p**

 $cf_{p,d}$ = Shipping costs per solar stove from port country ${m p}$ to the final destination country ${m d}$.

t_d = Setup costs per assembly plant funded by the organization in the country d

 a_d = Assembly costs per unit funded by the organization in the country **d**

 r_d = Price of reflector in the country **d**

 D_d = The total demand for every country **d**

C. Constraints Formulation

* Logic constraint to ensure that the port country does not ship more than what it has.

$$\mathbf{x}_{\mathbf{p}} \ge \sum_{d=1}^{D} w_{p,d} \ \forall \ \mathbf{p} = 1 \ \mathbf{P}$$

$$y_p \ge \sum_{d=1}^{D} z_{p,d} \ \forall \ p = 1 \dots P$$

* Maximum capacity for every container

$$x_p \ge 2412 \ cx_p \ \forall \ p = 1 \P$$

$$y_p \ge 12666 \ cy_p \ \forall \ p = 1 \P$$

* Constraint to link the decision variable $z_{\text{p,d}}$ with p_{d}

$$\sum_{p=1}^{P} z_{p,d} \ge 1 - M(1 - p_d) \quad \forall d = 1 \dots D$$

$$\sum_{p=1}^{P} z_{p,d} \le M(p_d) \quad \forall d = 1D$$

* Demand Constraint for every country

$$\sum_{p=1}^{P} w_{p,d} + \sum_{p=1}^{P} z_{p,d} = D_{d} \ \forall \ d = 1 \dots D$$

* Type 2 flow constraints

$$\sum_{d=1, d \neq i}^{D} z_{p,d} \le M(1 - p_i) \ \forall (p, i) \in S$$

Where S = is a set that link the index of the port country from the short list of port counties $(1 \dots P)$ with the long list of destination counties $(1 \dots D)$

*Logic constraint to not allow to ship units of fully more than what you have in case of there is a plant in port country

 $\sum_{d=1,}^{D} w_{p,d} \le x_p + y_p + M(1 - p_i) \quad \forall \ p = 1 \dots P \quad \forall \ i = set \ of \ the \ index \ of \ the \ port \ country \ in \ he long list of destination counties <math>(1, \dots D)$

Where M is a big number

D. The Objective Function

Minimize:

$$\begin{split} &\sum_{p=1}^{P} cc_{p}(cx_{p} + cy_{p}) + \sum_{p=1}^{P} cu_{p}x_{p} + \frac{1}{2}\sum_{p=1}^{P} cu_{p}y_{p} + \sum_{p=1}^{P}\sum_{d=1}^{D} cf_{p,d}w_{p,d} + \\ &\frac{1}{2}\sum_{p=1}^{P}\sum_{d=1}^{D} cf_{p,d}z_{p,d} + 40\sum_{p=1}^{P}\sum_{d=1}^{D}w_{p,d} + \sum_{d=1}^{D}t_{d}p_{d} + 30\sum_{p=1}^{P}\sum_{d=1}^{D}z_{p,d} + \sum_{p=1}^{P}\sum_{d=1}^{D}r_{d}z_{p,d} + \\ &\sum_{p=1}^{P}\sum_{d=1}^{D}a_{d}z_{p,d} \\ &\sum_{p=1}^{P}\sum_{d=1}^{D}a_{d}z_{p,d} \end{split}$$

APPENDIX

