Whiteboard: 1/60, f5.6, iso 1000, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT @ 95%** gain.

BBB 1/60, f5.6, iso 320, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT @ 95%** gain.

**BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!**

Title: Introductory Rotational Equilibrium Problem **[GoPro on ceiling & Sony close up]**

**Domino holding meterstick with 2 masses on it.**

**Film Geneve and Ryan on a seesaw.**

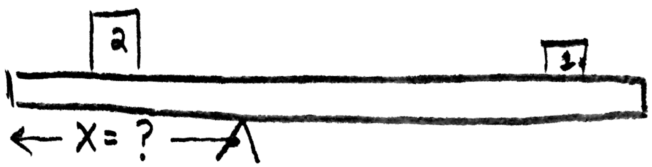
Mr.p: Good morning. A helpful problem from a textbook I use involves (two kids on a seesaw. The question asks "Where does the fulcrum need to be placed to balance the seesaw?" The fulcrum is the pivot point of a rotating structure and what always bothered me about this problem is that the fulcrum of a seesaw typically cannot be moved. So, we are going to model two kids on a seesaw with a movable fulcrum using a) meterstick, some masses, and a domino. {intro} Bo, please read the problem, and Billy, please translate.

- Bo: A 200.0 g mass is placed at the 20.0 cm mark on a uniform 93 g meterstick. A 100.0 g mass is placed at the 90.0 cm mark. Where on the meterstick should the fulcrum be placed to balance the system?
- Billy: Mass 2 equals 200.0 grams and is at the 20.0 cm mark. The mass of the uniform meterstick equals 93 grams. Mass 1 equals 100.0 grams and is at the 90.0 cm mark. … Because mass 2 is larger than the sum of mass 1 and the meterstick, I

think we can estimate the fulcrum needs to be somewhere between mass 2 and the center of mass of the meterstick. … We should draw a picture with the meterstick, the two masses, and the distance from the left end of the meterstick to the fulcrum as x, which equals question mark.

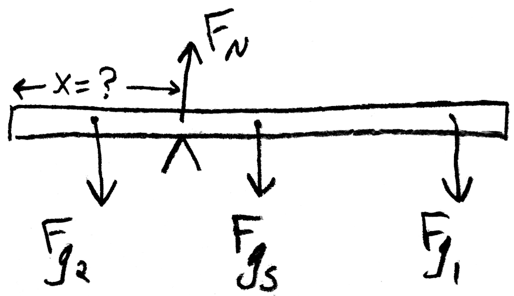
- Bo: Is the fulcrum going to be the domino?

Mr.p: [$m_2 = 200.0g\ @\ 20.0cm;\ m_s = 93g;\ m_1 = 100.0g\ @\ 90.0cm;\ x = ?$]




Yes, Bo, the fulcrum will be the domino. {demo} You can see right now that the meterstick by itself balances at the 50 cm mark because the center of mass of the uniform meterstick is in the middle of the meterstick. … {demo} After we solve this problem, we will place the masses on the meterstick and the meterstick on top of the domino at the location we determine the fulcrum to be, and the system should balance. … Bobby, please begin solving the problem.

Bobby: Sure. Let's start with a free body diagram. There are two forces of gravity acting down at each mass. Let's call them force of gravity 2 and force of gravity 1. There is also a force of gravity on the stick which acts down at the center of mass of the stick which is at 50 cm. When we place the meterstick on the domino, there will be a force normal acting upward at that point. … Now that we have the free body diagram, we can sum the forces on the meterstick in the y-direction. The net force equals force normal minus force of gravity 2 minus force of gravity of the stick, minus force of gravity 1. That all equals mass times acceleration in the y-direction. Because the meterstick is at rest, its acceleration is zero, so the net force is zero. … We can solve for the force normal and substitute mass times acceleration due to gravity for all three forces of gravity. … And we can factor out the acceleration due to gravity and substitute in numbers. … Force normal equals 9.81 times the quantity 200 … actually, we need the masses in kg so multiply all three masses times 1 kg over 1000 grams and then we get 9.81 times the quantity (0.2 plus 0.093 plus 0.1) which equals {calc} 3.85533 N.




Mr.p: [ $\sum F_y = F_N - F_{g_2} - F_{g_s} - F_{g_1} = ma_y = m(0) = 0$

$$\Rightarrow F_N = F_{g_2} + F_{g_s} + F_{g_1} = m_2 g + m_s g + m_1 g$$

$$\Rightarrow F_N = g\left(m_2 + m_s + m_1\right) \Rightarrow F_N = \left(9.81\right)\left(0.2 + 0.093 + 0.1\right) = 3.85533N$$

$$m_2 = 200.0g \times \frac{1kg}{1000g} = 0.2kg;\ m_s = 93g \times \frac{1kg}{1000g} = 0.093kg;\ m_1 = 100.0g \times \frac{1kg}{1000g} = 0.1kg$$ ]

And what should we do now?

- Bobby: … The meterstick is going to be at rest, so it will be in both translation and rotational equilibrium. Remember when it’s at rest like that it’s called static equilibrium and the net torque about any axis of rotation is zero.

- Bo: Right, so let’s sum the torques acting on the meterstick and pick the axis of rotation to be the location of the fulcrum or the force normal.
- Bobby: And pick counterclockwise or out of the page to be positive.
- Bo: Sure.
- Billy: The net torque then equals the torque caused by force of gravity 2 plus torque caused by the force normal, plus the torque caused by the

force of gravity of the meterstick plus the force of gravity of mass 1. … Actually, I forgot directions. Torque caused by force of gravity 2 is positive because it would cause the meterstick to rotate out of the board, or in the counterclockwise direction, which is positive. The torque caused by the force normal is actually zero because its r value in its torque equation equals zero. The torques caused by the force of gravity of the meterstick and the force of gravity of mass one are both negative because they would cause the meterstick to rotate into the board, or in the clockwise direction, which is negative. Net torque equals rotational inertia times angular acceleration which equals zero because the angular acceleration is zero. …

- Bo: Now we can substitute the r vector times force times the sine of theta for each torque. … And each angle is 90 degrees because all r values are horizontal and all forces are vertical and the sine of 90 degrees is one. …
- BBB: Everybody brought acceleration due to gravity to the party!

Mr.p: [

$$\sum \tau_{\substack{AoR\,@\\ fulcrum}} = \tau_2 + \tau_N - \tau_s - \tau_1 = I\alpha = I\left(0\right) = 0 \Rightarrow r_2 F_{g_2} \sin\theta_2 - r_s F_{g_s} \sin\theta_s - r_1 F_{g_1} \sin\theta_1 = 0$$

$$\tau_N = r_N F_N \sin\theta_N \Rightarrow r_N = 0 \Rightarrow \tau_N = 0 \quad \& \quad \theta_2 = \theta_s = \theta_1 = 90^\circ \ \& \sin\left(90^\circ\right) = 1$$

$\Rightarrow r_2 m_2 g - r_s m_s g - r_1 m_1 g = 0 \Rightarrow r_2 m_2 - r_s m_s - r_1 m_1 = 0$ **Add AoR!!**] This is great. But I thought we were solving for x. Where is x, the location of the fulcrum, in this equation?

- Bo: … We can define the r value for every torque equation in terms of x. For example x equals 20 plus r 2 so r 2 equals x minus 20.
- Bobby: And 50 equals x plus r for the stick so r for the stick equals 50 minus x.
- Bo: And 90 equals x plus r for force normal 1 so r for force normal 1 equals 90 minus x.
- Billy: And we can substitute all those back into our torque equation to get the quantity x minus 20 all times 200 minus the quantity 50 minus x all

times 93 minus the quantity 90 minus x all times 100 which all equals 0.

- Bobby: Which is 200x minus 4000 minus 4650 plus 93x minus 9000 plus 100x equals zero. We can rearrange that to make it easier to work with and then factor out x and combine all three constants to get 17,650. Solving for x gives us x equals 17,650 divided by 393 or 44.911 or 45 with two significant digits. … But what are the units?
- Billy: Oh, man. We forgot to use kilograms and meters.
- Bo: Actually, we did not have to. It ends up being 17,650 gram centimeters divided by 393 grams.

Grams cancel out, and our answer is 45 cm. The meterstick with the two masses on it will balance out when the fulcrum is placed at the 45 cm mark.

Mr.p: [ $x = 20 + r_2 \Rightarrow r_2 = x - 20 \; \& \; 50 = x + r_s \Rightarrow r_s = 50 - x \; \& \; 90 = x + r_1 \Rightarrow r_1 = 90 - x$

$$r_2 m_2 - r_s m_s - r_1 m_1 = 0 \Rightarrow (x-20)(200) - (50-x)(93) - (90-x)(100) = 0$$

$$\Rightarrow 200x - 4000 - 4650 + 93x - 9000 + 100x = 0 \Rightarrow 200x + 93x + 100x - 4000 - 4650 - 9000 = 0$$

$$\Rightarrow (200+93+100)x - (17650) = 0 \Rightarrow 393x = 17650 \Rightarrow x = \frac{17650 g \cdot cm}{393 g} = 44.911 cm \approx 45 cm$$

] Alright, let's see if the physics works. {hear Bo} Yes, Bo?

Bo: (Hold up mr.p.) Why did we sum the forces? We never used the fact that force normal equals 3.85533 newtons. So why did we sum the forces?

Mr.p: That is a good question. We summed the forces because that is what you are used to doing and what you often do right at the beginning of a problem that involves force, even if you do not have to. Summing the forces did not help us solve this particular problem. It was correct, however, not helpful for solving this problem.

Bo: Okay. Let’s not do that next time.

Mr.p: That’s fine with me. … Now, let’s check our physics. {demo} Let’s place the meterstick with the two masses on it on the domino, which becomes the fulcrum, with the domino at the 45 cm mark. … And will you look at that. The physics works, the physics works, uh huh uh huh, the physics works. … Thank you very much for learning with me today, I enjoyed learning with you.