HAI PHONG DEPARTMENT OF EDUCATION AND TRAINING

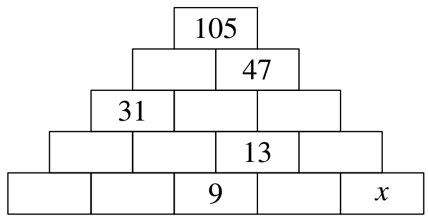
MATHEMATICS AND SCIENCE COMPETITION IN ENGLISH, SHOOL YEAR: 2022-2023

TEST: MATHEMATICS – SECONDARY LEVEL

Time allowance: 120 minutes

PART I. MULTIPLE CHOICE QUESTIONS (100 points)

Question 1: In this partly completed pyramid, each rectangle is to be filled with the sum of the two numbers in the two rectangles immediately below it. What number should replace x?



A. 3.
B. 4.
C. 5.
D. 7.
E. 6.

Question 2: A workshop has two dedicated machines M1 and M2 that produce two types of products denoted by I and II. One ton of type I products generates a profit of two million VN dong and that of type II products generates a profit of 1.6 million VN dong. It takes machine M1 three hours and machine M2 one hour to produce one ton of type I products. It takes machine M1 and machine M2 one hour each to produce one ton of type II products. It is impossible for one machine to produce two types of products at the same time. Machine M1 can work no more than 6 hours per day and machine M2 no more than 4 hours per day. Make a production plan to obtain the largest total of profits.

A. 1 ton of type I products and 3 tons of type II products.
B. 2 tons of type I products and 2 tons of type II products.
C. 3 tons of type I products and 1 ton of type II products.
D. 1,5 tons of type I products and 2,5 tons of type II products.
E. None of the above.

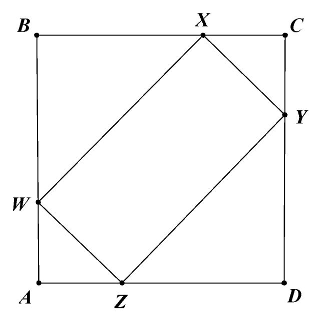
Question 3: What is the value of the sum $S=\frac{3}{4}+\frac{8}{9}+\frac{15}{16}+...+\frac{2023^2-1}{2023^2}$?

A. None of the above.
B. 2020.
C. 2021.

D. 2022.
E. 2023.

Question 4: Given that the roots of the equation $x^2 - mx + 2m = 0$ (m is a parameter) are integers. What is the sum of the possible values of m?
A. 16.
B. 0.
C. 18.
D. 8.
E. 17.

Question 5: In the figure below, $ABCD$ is a square whose side equals 3. Knowing that $AW = AZ = CX = CY = 1$, what is the perimeter of rectangle $WXYZ$?



A. $6\sqrt{2}$.
B. $3\sqrt{2}$.
C. $4\sqrt{2}$.
D. 8.
E. $8\sqrt{2}$.

Question 6: Let ABC be an equilateral triangle, with each side of a. A set of points M which is satisfy the equality $4MA^2 + MB^2 + MC^2 = \frac{5a^2}{2}$ lie on a circle $(O;R)$, calculate the radius R in terms of a.

A. $R = \frac{a\sqrt{6}}{6}$.

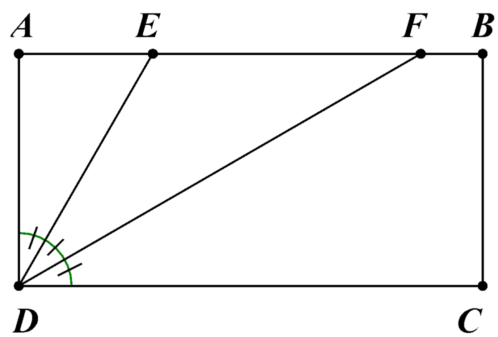
B. $R = \frac{a\sqrt{3}}{3}$.

C. $R=\frac{a}{4}$.

D. $R=\frac{a\sqrt{3}}{2}$.

E. None of the above.

Question 7: Given rectangle $ABCD$, in which $DC=2CB$ and points E and F lie on side AB such that ED and FD trisect angle ADC as shown in the figure below. What is the ratio of the area of ΔDEF to the area of rectangle $ABCD$?



A. $\frac{\sqrt{3}}{6}$.

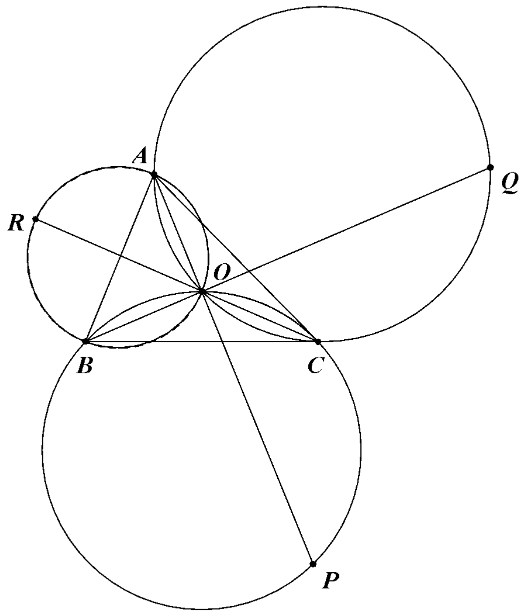
B. $\frac{\sqrt{6}}{8}$.

C. $\frac{2\sqrt{3}}{3}$.

D. $\frac{3\sqrt{3}}{16}$.

E. $\frac{\sqrt{2}}{4}$.

Question 8: Given triangle ABC inscribed in a circle $(O;R_1)$, in which lines AO, BO and CO intersect the circumcircles of triangles OBC, OAC and OAB at P, Q and R, respectively and the smallest value of product $OP.\,OQ.\,OR$ can be written in the form of $k.R_1^3$, where k is the natural number, find the value of k?



A. 8.
B. 6.
C. 7.
D. 9.
E. None of the above.

Question 9: How many positive integer triples (x, y, z) are there satisfying the equation $x^2 + 8z - 2y^2 = 3$?

A. 0.
B. 3.
C. 2.
D. 1.
E. None of the above.

Question 10: Given that three circles of the same diameter d cover an equilateral triangle with side of $3,$ which of the following statements is TRUE?

A. $d \geq \sqrt{3}.$
B. $d < \sqrt{3}.$
C. $d < \frac{\sqrt{3}}{2}.$
D. $d \geq \frac{3\sqrt{3}}{2}.$
E. $d \geq 3.$

PART II: COMPOSE (200 points)

Problem 1: Given that polynomial $f(x) = 3x^3 - kx^2 + mx + 2,$ where k and m are real constants and that $f(x)$ is divisible by $x-2$ and $f(1)=1,$ find the values of k and m?

Problem 2: Given equation $x^2 - 2(m+1)x + 6m - 2 = 0$ (m is a parameter), determine all the values of parameter m such that the above equation has two distinct roots x_1, x_2 satisfying $x_1^2 + x_2^2 = 8.$

Problem 3: Let ABC be an equilateral triangle inscribed in circle $(O;R).$ Prove that:

$$M \in (O;R) \Leftrightarrow MA^2 + MB^2 + MC^2 = 2AB^2.$$

Problem 4: Let ABC be a triangle such that $\dfrac{\sin B + 2023\sin C}{2023\cos B + \cos C} = \sin A$ and the lengths of its sides are natural numbers and let M be the midpoint of BC and G be a centroid of triangle $ABC.$ Prove that the value of the area of triangle MBG is a natural number.

Problem 5: Let d be the distance between the circumcenter and the centroid of a triangle and let R be its circumradius and r be the radius of its inscribed circle. Prove that $d^2 \le R(R-2r).$

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