Week 2: Jan 20	Multisensory mathematics	Reflections: Jan 24, 8M Responses: Jan 26, midnight

EDCP 553 Week 2, Jan 20-26

Week 2: Multisensory mathematics

Schooling generally emphasizes only the senses of sight and hearing -- and these in very specific, mostly linguistic ways (reading and writing words and symbols, and listening to talk). But mathematics can be sensed in other modalities: through tangible objects, movement, orientation, balance, hearing musical and other sounds, seeing pattern and colour, even through taste and smell. Historically, mathematicians learned and observed mathematical patterns using all their senses. What would mathematics lessons be like if we took multisensory learning seriously?

Introduction:

When learners are approaching a new mathematical topic, would it be helpful for them to *hear* a mathematical relationship, to *touch* it, or to know it through *movement*? Can even smell and taste be enlisted as multisensory modalities for understanding mathematical patterns and relationships? Can students learn the equivalence of a variety of representations that include, but go beyond, those usually invoked in mathematics classes?

Typically, students in secondary school math courses are now taught that a *word problem*, *table of values*, *equation* and *graph* may be considered equivalent representations of a single mathematical function. Would it be beneficial for them to learn that same function in terms of mathematically-equivalent patterns of musical sounds, colours, tactile objects and/or kinesthetic movements? What new insights or ways of grasping a concept might emerge?

Multisensory, bodily experiences of mathematical patterns and relationships have potential to serve as meaningful symbolic or representational resources as students develop the ability to work with generalization and abstraction in mathematics. Our puzzle as teachers and pedagogical designers is to find the resonances that can be brought to students' awareness when focused multimodal experiences and abstract mathematical symbolism are *both* present in learners' attention in the math classroom.

Another pedagogical question is: When should multisensory experiences be included as part of mathematics learning? Are these experiences only worthwhile at the beginning of learning something, or for the very young, or could they be helpful at other stages of life and learning too?

In more traditional, linear and hierarchical approaches, the senses are considered coarse and inferior ways of knowing -- a harkening back to Christian and other religious or philosophical traditions that considered "the things of this world" to be unreliable and transient, and only the ethereal worlds of abstraction and the afterlife to be eternal and trustworthy. In this world view (a very Cartesian and Platonic one, as we've discussed) our sense perceptions were to be doubted in all cases because sometimes our perceptions can be fooled. So in this way of thinking about things, sensory perceptions and experiences should be used as little as possible and jettisoned in favour of abstractions, logic, proofs and abstract symbolism. Working in an abstract way is treated as far superior to working in a multisensory way.

But that is not the only way to think about embodied, multisensory math learning. A recursive model is an interesting way to think about using multisensory math learning experiences as resources to think with. This is especially true for embodied experiences that are shared by a whole class. A really engaging and well-designed, shared multisensory activity can help introduce a new mathematical pattern, and can be returned to again and again as learners explore the details, elaborations and extensions of a pattern or concept. A movement vocabulary, collection of physical objects and manipulatives, sound patterns, the process of making something, and other experiences can be a touchstone and a source of embodied metaphors that help students take the new work further. They can use the sensory activities to experiment with imagery and metaphor that lets them see, hear, move, balance, touch, taste, design, etc. with the new mathematical pattern or idea. These activities can become learning resources that help learners synthesize multiple sense images to learn, remember and reconstruct these patterns with a deeper understanding.

This visceral, whole-bodied way of engaging with mathematics has been documented as the way that research mathematicians explore and communicate mathematical ideas: using metaphor and narrative, tactile and visuospatial models, and through gesture, movement and voice (Sinclair & Gol Tabaghi 2010; Burton 1999; Sfard 1994). Until recently, however, most learners have been expected to learn mathematics without access to these experiential and communicative modalities; in the familiar, stereotypical school math classroom, students sit silently in rows of desks, quietly taking notes while the teacher lectures at a blackboard or overhead projector. (See Boaler 2014 -- starting at about 21:00 -- for an elaboration of this typical scene). I contend, with Boaler and others, that a pedagogy that offers nothing but this kind of passive reception of teacher lectures results in an impoverished mathematical education for the majority of students, who do not thrive in such circumstances.

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One way to think about multisensory mathematics learning is to **consider students with sensory impairments**: for example, those who ‘don’t see with their eyes’ or ‘don’t hear with their ears’. How can we design multisensory mathematical experiences that work well for people with visual, auditory and other sensory impairments/ different ways of perceiving?

A fundamental conceptual premise I've been working with is that innovations supporting learning for students with sensory impairments will support learning for all. These innovations, emerging from close work with learners, have the potential to revolutionize mathematics pedagogy for sighted and hearing students as well, by offering engaged, movement-oriented ways of representing and exploring mathematical relationships.

This involves rethinking ideas about disabilities. Rather than thinking of people as disabled, we can consider whether society disables people in a number of ways, including physical barriers, non-inclusive attitudes and lack of infrastructure. A person might have particular *impairment*, but the *disability* results from a restriction of activity that disadvantages and oppresses the person, rather than from the person's own way of being.

A simple example would consider a person who had an injury or impairment (say, a sprained ankle) that prevented them from climbing stairs. If ramps, elevators and other alternatives were available everywhere, there would be no disability, as the person would be able to move freely as they needed to, without restriction; otherwise, the person would be disabled by social and physical barriers. In order to minimize disability, the society would need to stop disabling people!

This social model contrasts with an older individualized, medicalized model, which saw the disability residing in the impaired individual, who needed special treatment and equipment, and who might have to live with reduced expectations of participation in the 'normal' life of the typically-abled majority.

For the most part, students' sensory and other impairments are diagnosed individually within a psychological/ medical paradigm, and those students are often treated as having a deficiency or lack carried within themselves from class to class and year to year. Special education treatments may be offered, including individual pull-out instruction, separate classes or schools for special needs students, or modifications to curriculum and assessment. But these methods have been critiqued for being stigmatizing and sometimes leading to drastically-reduced expectations for full participation in schooling and later life. Even where students with impairments are integrated into mainstream classes (as they are in Canadian schools), there are still many difficulties. Even school systems with an inclusive approach often label individuals as *disabled*, rather than thinking of the social and built environment as *disabling*.

Some researchers and activists point out that there is no clear line between the 'normal' and 'disabled'; everyone experiences impairments to some degree at different points in their life. By acknowledging that we must all deal with frailty, limitations and vulnerabilities, we can remove a false dividing line between 'able-bodied' and 'disabled' persons, and "expose the essential connection between impairment and embodiment" (Shakespeare & Watson 2001, 25; Davis 1995).

By learning from the lived experiences of mathematics learning for students with sensory impairments, we may start to create new pedagogical and curricular approaches that will benefit all. These new approaches, materials, technologies and pedagogies have the potential to make mathematics learning a sense-making undertaking for all students.

A similar approach to technological and business applications via impairment and disability was reported on an item on the CBC National News a few years ago (see Roumeliotis 2014). Roumeliotis states in this report that "after decades as a 'niche market', the disabled have become a driving force in technology and a giant inspiration for innovation". Roumeliotis documents business' discovery that what's good for the disabled tends to be good for everyone else too. She offers the examples from Google of a self-driving car being tested by a blind driver, Google Glass used by a person in a wheelchair, and closed captioning used by hearing second language viewers and people in noisy situations, as well as by the deaf. Eve Anderson, a developer at Google, is quoted as saying, “It’s not just about taking existing products and making them accessible, but rather creating completely new things to make the world around us accessible […] We all have some kind of impairment at some time […] Making things that work without relying on all of our senses and all of our capabilities at all times is really helpful for the population at large. (Roumeliotis 2014)”.

These insights from the world of business, marketing and technological innovation mirror the findings of research work in innovative embodied mathematics pedagogies over the past decade. If we reframe ‘disability’ as a driver for innovation and creativity, rather than as a deficiency, there is the potential to benefit the population at large, in education as in business. Offering more learners access to a deep understanding of mathematics in new engaging and multifaceted ways has the potential to benefit our societies through a much wider understanding of patterns and relationships in science, technology, engineering and the arts, and within mathematics itself.

Viewing:

This week's viewing is of several short films about tactile and even edible mathematics, by the wonderful daughter and father mathematicians, Vi Hart and George Hart. While watching these, consider the ways that multisensory, embodied mathematical experiences might affect learner's understandings of the mathematical patterns these films explore.

** (Note that Vi Hart’s videos are very fast-talking. You might want to try slowing down the speed on YouTube by clicking on the ‘gear’ icon, selecting ‘playback speed’, and choosing 0.75 or even 0.5. Similarly, if you are watching a video that goes very slowly, you might want to choose a playback speed of 1.5 or even 2!)

1. Vi Hart: Math improv Smarties (note that what she calls 'Smarties' is what we in Canada might call Rocket candies) and Supersmarties

2. Vi Hart 4 short Hexaflexagon films: Hexaflexagons part 1 -- Hexaflexagons part 2 -- How to make a Hexaflexagon -- Hexaflexagon Safety Guide --- and the ultimate, edible hexaflexagon, Flex Mex!!- (For some additional optional Vi Hart edible math viewing, you can also check out the Green Bean Matherole & Borromean Onion Rings)

3. George Hart: Mathematically correct breakfast (instructions) with video link at the bottom

Activities:

For those reading (a) (Kepler reading) -- mathematical fruit, constructing 3-D Platonic solids and other polyhedra

For those reading (b) and (c): Doing data analysis and geometry with candy. Making hexaflexagons, and hexa-mexa-flexagons, linked bagels (and perhaps a bagel Möbius strip?)

Here's the link to the activities sheet for this week.

Readings:

a) Excerpts from Johannes Kepler (1611/ 2010) On the Six-pointed Snowflake: A New Year's Gift. Please read pp. 31 (starting with last paragraph) -- 65 (second paragraph), and note that these are short pages, only the odd-numbered pages, and there are illustrations.

This small and rather delightful book was a little "nothing" that Kepler gave as a gift to his wealthy patron on New Year's, 1611, just 413 years ago. It was originally written in Latin, and published in English translation in the 1960s and again in 2010.

I am interested in this book because small, everyday observations of the physical world (for example, looking at a snowflake that falls on his wool coat) inspires Kepler's imagination -- and those imaginings ended up as the foundations of whole new fields of mathematics, including close-packing problems and crystallography. Kepler playfully observes snowflakes, beehives, pomegranates, apples, frost on the window of a steam bath, and various polyhedra (cubes, triangular pyramids -- tetrahedra, and all the other shapes that Dungeons and Dragons dice come in, plus a few more!) It will be helpful for those reading this piece to actually look at some snowflakes, honeycomb, apples, pomegranates, etc. if they are available, and to construct models of the polyhedra (see Activities sheet).

b) Lulu Healy & Solange Fernandes (2013), Multimodality and mathematical meaning-making: Blind students' interactions with symmetry.

c) Angelika Stylianodou & Elena Nardi (2019), Tactile construction of mathematical meaning: Benefits for visually impaired and sighted pupils