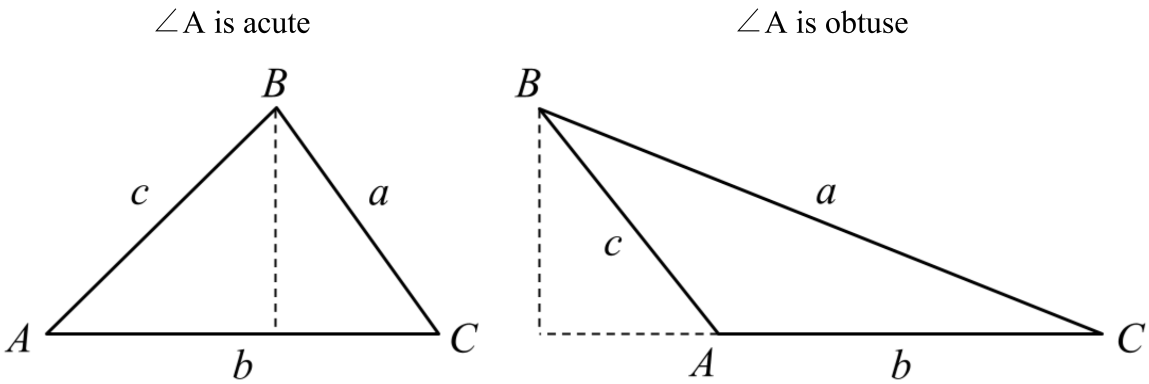
The Law of Sines



For both triangles, using the altitude shown (the dotted line), we can claim:

$$\sin A = \frac{altitude}{c} \qquad \sin C = \frac{altitude}{a}$$

$$\downarrow \qquad \downarrow$$

$$\text{altitude} = c \sin A \qquad \text{altitude} = a \sin C$$

Therefore $c \sin A = a \sin C$. Rearranging, we get:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Similar proofs can show that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \text{ and } \frac{\sin B}{b} = \frac{\sin C}{c}$$

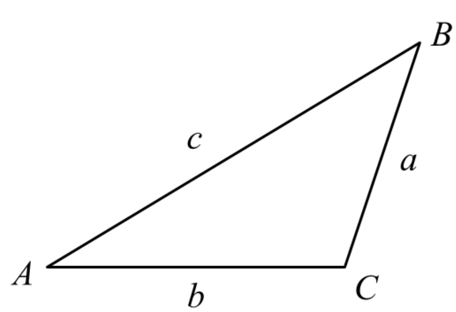
The Law of Sines

Let ΔABC be any triangle (oblique or right) with angle measures *A*, *B*, and *C*, and sides of length *a*, *b*, and *c*. The law of sines can be used to solve any triangle when given **ASA** or **AAS** (and sometimes **SSA**).

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

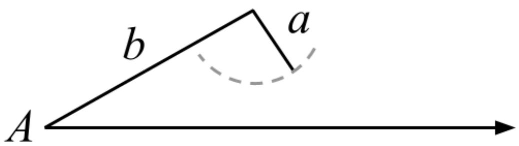
or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

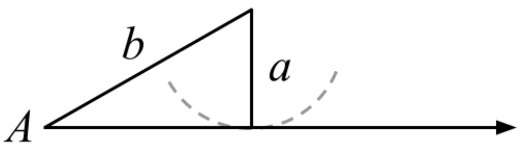


The Ambiguous Case (SSA)

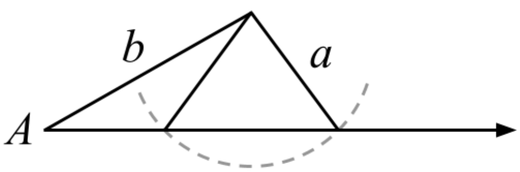
Case 1: The Known Angle is Acute



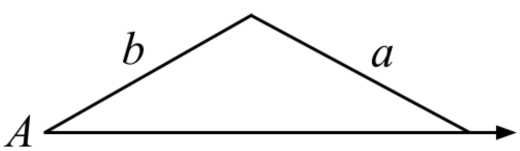
No Triangle — $a <$ altitude



One Right Triangle — $a =$ altitude



Two Triangles — altitude $< a < b$

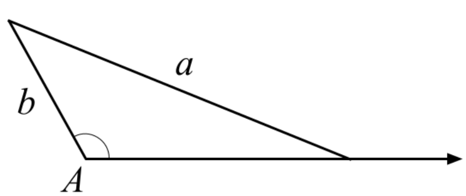


One Triangle — $a \geq b$

Case 2: The Known Angle is Obtuse

No Triangle — $a < b$

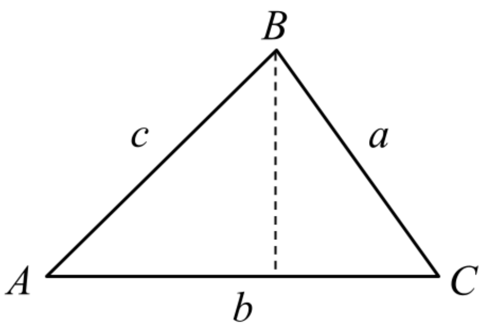
No Triangle — $a = b$



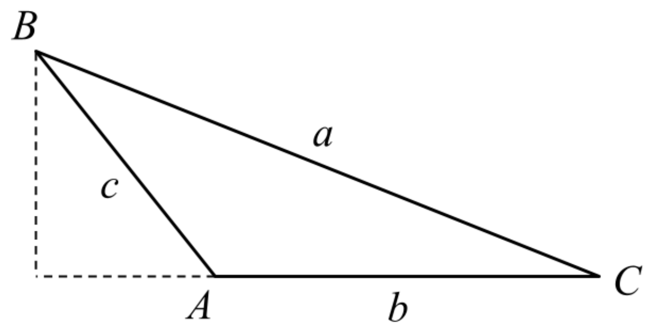
One Triangle — $a > b$

The Law of Cosines

$\angle$A is acute



$\angle$A is obtuse



Applying the Pythagorean Theorem, we get:

$$a^2 = (b - c\cos A)^2 + (c\sin A)^2$$
$$a^2 = b^2 - 2bc\cos A + c^2\cos^2 A + c^2\sin^2 A$$
$$a^2 = b^2 - 2bc\cos A + c^2(\cos^2 A + \sin^2 A)$$
$$a^2 = b^2 - 2bc\cos A + c^2$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

The Law of Cosines

Let ΔABC be any triangle (oblique or right) with angle measures *A*, *B*, and *C*, and sides of length *a*, *b*, and *c*. The law of cosines can be used to solve any triangle when given **SAS** or **SSS**.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$