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B.Sc. (Hons.) Math (Semester – 2nd)

ALGEBRA-II

Subject Code: BMATS1222

Paper ID: [22131206]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define: Ring, A commutative ring with unity, A non- commutative ring with unity, Ring without zero divisors.

b) Let R be an integral domain with unity such that R has finite number of ideals. Show that R is a field.

c) Let $f : R \rightarrow R'$ be an onto homomorphism, where R is a ring with unity. Show that f (1) is unity of R′.

d) Every integral domain is not a field since ring of integers is an integral domain but it is not a field.

e) If A is a left and B is a right ideal of a ring R then show that AB is a two sided ideal of R whereas BA need not be even a one-sided ideal of R.

f) Show that $\{0, 3, 6, 9\}$is a subring of the ring $(Z_{12}, +_{12}, \times_{12})$.

g) Every ideal in a Euclidean domain is a principal ideal.

h) Give an example of a ring which is not an integral domain.

i) Let V be an inner product space. Show that

(a) $(0, v) = 0$ for all $v \in V$ (b) $(u, v) = 0$ for all $v \in V \Rightarrow u = 0$

j) Obtain an orthonormal basis, with respect to the standard inner product for the subspace of R^3 generated by (1, 0, 3) and (2, 1, 1).

Section – B **(5 marks each)**

Q2. Let R be a ring having more than one element such that aR = R, for all $0 \neq a \in R$. Show that R is a division ring.

Q3. If N be an ideal of a ring R then there exists a one-one onto mapping between the set of all ideals of R, containing N and the set of ideals of R/N.

Q4.(a) Prove that $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$ is a field.

(b) If A and B are two ideals of R then A + B is an ideal of R, containing both A and B.

Q5. Let R[x] be the ring of polynomials over a ring R then

(i) R is commutative iff R[x] is commutative.

(ii) R has unity iff R[x] has unity.

Q6. Let S be an orthogonal set of non-zero vectors in an inner product space V. Then S is a linearly independent set.

Section – C **(10 marks each)**

Q7.(i) Let D be an integral domain. Let F be a field such that F ⊆ D. Suppose unity *1* of F is also unity of D. Then D can be regarded as a vector space over F. Show that D is a field if [D : F] = finite.

(ii) Let V be any inner product space. Given a linearly independent subset $\{u_1, u_2 \ldots, u_n\}$ there exists an orthonormal set of vectors $\{v_1, \ldots, v_n\}$ such that $L(\{u_1, \ldots, u_k\}) = L(\{v_1, \ldots, v_k\})$ for all $1 \leq k \leq n$. In particular, if V is finite dimensional, then it has an orthonormal basis.

Q8.(i) Every homomorphic image of a group G is isomorphic to some quotient group of G.

(ii) Let B ⊆ A be two ideals of a ring R. Then $\frac{R}{A} \cong \frac{R/B}{A/B}$.

Q9.(i) Find the field of quotients of the integral domain $Z(\sqrt{2}) = \{a + b\sqrt{2} \quad a, b \in Z\}$

(ii) Let W_1 , W_2 be two subspaces of a vector space V. If W_1 , W_2 are inner product spaces, show that $W_1 + W_2$ is also an inner product space.