(2 points)

$x+x^2$ $\quad$ $x(1+x)$

x^2 $\quad$ $x+x^2$

$$E(x) = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} x(1-x)\,dx = \int x(1+x)\,dx + \int x(1-x)\,dx$$

$$= \frac{x^2}{2} + \frac{x^3}{3}\Big|_{-1}^{0} + \frac{x^2}{2} - \frac{x^3}{3}\Big|_{0}^{1}$$

$$= \left(\frac{(-1)^2}{2} + \frac{(-1)^3}{3} - \frac{0}{2} + \frac{0}{3}\right) + \left(\frac{0}{2} - \frac{0}{3} - \frac{1}{2} - \frac{1}{3}\right)$$

$$= \left(\frac{1}{2} + \frac{1}{3}\right) + \left(-\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} = 0$$

C. V(X) $\quad E(x) = 0$

(2 points)

$V(x) =$

3. Assume that the number of misprints per page in a certain type of book has a Poisson distribution with a mean of one misprint per one page.

a. What percentage of pages contains no misprints? **(3 points)**

$P(x=x) \quad \frac{x^x \cdot e^{-x}}{x!}$ $\quad$ 0.5

19.5/50

Bahir Dar Institute of Technology

Probability and Statistics Final Exam

Date: July 5, 2021 **Time allotted: 2:45 hrs** **Exam value: 50%**

Name [redacted] Department WRIE IDNo. 120[redacted]

PART I: write True if the statement is correct or false if it is not (1 point each)

1. True The length of time between telephone calls is continuous random variable.
2. True If events X and Y are independent, then they are also mutually exclusive.
3. False If $P(A) = 0.3$ and $P(B) = 0.2$ and events A and B are independent, then $P(A \text{ and } B) = 0.06$.
4. False A type II error is the error that is committed if the null hypothesis is rejected when in fact it is true.
5. False A two-tailed hypothesis test is always more powerful than a one tailed test.
6. True The probability of committing a type I error is called level of significance (a) of the test.

4

Part II: Fill blank space. (1 point each)

1. If the mean and variance of a binomial distribution are 4 & 2 respectively, then the number of trials (n) and probability of success (p) for this distribution are n= 3.1 & p= 4/12 respectively.
2. Two events A and B are such that P(A)=0.2, P(B)=0.3, and P(A∪B) =0.4. Find the following:
   a. $P(A \cap B)$ = 0.1
   b. $P(\bar{A} \cup \bar{B})$ = 0.9
   c. $P(\bar{A} \cap \bar{B})$ = 0.6
   d. $P(\bar{A}|B)$ = 0.66
3. A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random,
   a. The probability that all nationalities are represented 1/136
   b. The probability that all nationalities except Italian are represented 3/495

2

4

$P(A)=0.2$
$P(B)=0.3$
$P(A\cup B)=0.4$

$P(A\cap B)=?$ $P(\bar{A}\cup\bar{B})=?$ $P(\bar{A}\cap\bar{B})=?$ $P(\bar{A}/B)$

$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{2}=\frac{15+10+6+15}{30}=\frac{46}{30}$

can jap ita ger

$\frac{2}{12}+\frac{3}{12}+\frac{5}{12}+\frac{2}{12}=\frac{12}{12}=1$ (a) (c) = 0.3 (D) 1

can jap ger

$\frac{2}{12}+\frac{3}{12}+\frac{2}{12}=\frac{7}{12}$ (b)

$0.4-0.1=0.3$

$\frac{11}{4}$ $\frac{12}{4}$ $\frac{2\cdot3}{12}$ 10

$\frac{\frac{1}{2}+\frac{1}{2}}{1}=\frac{\frac{1+1}{2}}{1}=\frac{\frac{2}{2}}{1}=\frac{1}{1}=1$

$A\cap B$ $A\cup B = A|B + B|A + A\cap B$

A journalist claims that all adults in her city spend an average of 30 hours per month on general reading, such as newspapers, magazines, novels, and so forth. A recent sample of 25 adults from this city showed that they spend an average of 27 hours per month on general reading. The population of such times is known to be normally distributed with the population standard deviation of 7 hours.

a. Determine the point estimate for the mean time spent per month such reading by all adults in this city. **(1 point)**

$\mu_x = \bar{x}$, $\bar{x}_x = 27$ hours per month

b. Construct the 95% confidence interval for the mean time spent per month on such reading by all adults in this city? **(3 points)**

c. Using the 5% significance level, would you conclude that the mean time spent per month on such reading by all adults in this city is less than 30 hours? **(5 points)**

$$\begin{array}{r} 14.4 \\ -\ 15.9 \\ \hline -2.5 \end{array}$$

200 employees are selected from this company, approximately how many of them receive wages higher than \$14.4? **(2 points)**

$P(x > 14.4) = P\left(Z > \frac{14.4 - 15.9}{1.5}\right)$

$P(x > 14.4) = P(Z > 1)$

c. The highest 2.5% of the employee hourly wages is greater than what amount? **(2 points)**

d. What is the probability that the mean hourly wage of randomly selected 16 employees will lie between \$15.15 and \$16.15? **(2 points)**

$n = 16$

$P(15.5 < x < 16.15) < Z < \frac{16.15 - 15.5}{1.5}$

How many pages would you expect to have no misprints in a 1000-page book? **(3 points)**

no misprints in a 1 page book $\frac{1^0 e^{-1}}{0!} = e^{-1} = \frac{1}{e}$

if $n = 1000$, no of misprints is $n\left(\frac{1}{e}\right)$

$\frac{n}{e} = \frac{1000}{e}$ thus, for 1000 page book, no misprints are

$\boxed{\frac{1000}{e}} = 437$ pages

3

c. What is the probability that there are at least two no misprints in a 1000-page book? **(3 points)**

$1 - P(x=0) - P(x=1)$

$1 - \frac{1^0 e^{-1}}{0!} - \frac{1^1 e^{-1}}{1!}$

$1 - \frac{1}{e} - \frac{1}{e}$

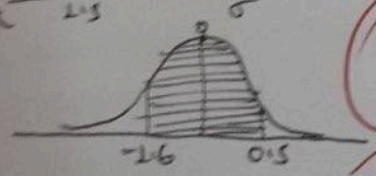
$\frac{e-1-1}{e}$

$= \boxed{\frac{e-2}{e}}$

4. A company pays its employees an average wage of \$15.90 an hour with a standard deviation of \$1.50. If the wages are approximately normally distributed and paid to the nearest cent,

a. What percentage of the workers receives wages between 13.5 and \$16.65 an hour inclusive? **(2 points)**

$P(13.5 < x < 16.5) = P\left(\frac{13.5 - 15.9}{1.5} < \frac{\bar{x} - \delta}{\sigma} < \frac{16.65 - 15.9}{1.5}\right)$



1

$P\left(\frac{-3.6}{1.5} < \frac{\bar{x} - \delta}{\sigma} < \frac{0.85}{1.5}\right)$

$P(-2.4 < \frac{\bar{x} - \delta}{\sigma} <$

ST III: Show the necessary steps.

1. For three persons A, B, C the chance of being the selected as a manager of the firm is in the ratio of 4:1:2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.4, 0.8 and 0.6.

   A. Find the probability that the change does takes place. **(2 points)**

   P(a) =

   P(A) = 0.57
   P(B) = 0.14
   P(C) = 0.286

   P(change/a) = 0.4 P(change/b) = 0.8 P(change/c) = 0.6

   P(change) = P(change/a) · P(a) + P(change/b) · P(b) + P(change/c) · P(c)

   P(change) = (0.4 * 0.57) + (0.8 * 0.14) + (0.6 * 0.286)

   P(change) = 0.22854 + 0.1143 + 0.1714

   P(change) = 0.51

   2

   B. If the change does take place, find the probability that it is due to the appointment of person A? **(2 points)**

   P(due to appt of per A) = $\frac{P(A)}{P(change)}$

   $= \frac{0.57}{0.51}$

   =

2. Let X be a continuous random variable with probability density function

$$f(X)=\begin{cases} 1+X, -1 \le X < 0 \\ 1-X, 0 \le X \le 1 \\ 0, otherwise \end{cases}$$

Find

A. P(-1/2≤ X ≤ 1/2) **(2 points)**

$P(-1/2 \le X \le 1/2)$ $(-1/2 \le Z \le 1/2) \Rightarrow \frac{\left(\frac{1}{2}+\frac{1}{2}\right)}{1}$

$\rightarrow \frac{1}{1} = 1$