CHAPTER 1 – RELATIONS AND FUNCTIONS

DAY 1 – RELATIONS – REFLEXIVE, SYMMETRIC, TRANSITIVE, EQUIVALENCE

- A relation R on a set A is called an ***empty relation*** if no element of A is related to any element of A.
- A relation R on a set A is called a ***universal relation*** if each element of A is related to every element of A.
- A relation R on a set A is called
  - ***Reflexive***, if $(a,\ a) \in R\ \forall\ a \in A$.
  - ***Symmetric***, if $(a,\ b) \in R\ implies\ (b,\ a) \in R$ for $a,\ b \in A$.
  - ***Transitive***, if $(a,\ b),\ (b,\ c) \in R\ implies\ (a,\ c) \in R$ for $a,\ b,\ c \in A$.
- A relation R on a set A is ***equivalence relation*** if R is reflexive, symmetric, transitive.

Questions

1. Let R be the relation on $A = \{1,\ 2,\ ....\ 14\}$ defined as $R = \{(x, y) :\ 3x - y = 0\}$
   a. Write R in roster form
   b. Is R reflexive, symmetric, transitive?

2. Let R be the relation on $A = \{1,\ 2,\ ..\ 6\}$ defined as $R = \{(x, y) :\ y\ is\ divisible\ by\ x = 0\}$
   a. Write R in roster form
   b. Is R reflexive, symmetric, transitive?

3. Let R be the relation on $A = \{1,\ 2,\ ....\ 6\}$ defined as $R = \{(a, b) :\ b = a + 1\}$
   a. Write R in roster form
   b. Is R reflexive, symmetric, transitive?

4. Let $A = \{ 1,\ 2,\ 3\}$. Give an example of a relation on A which is
   a. Symmetric but neither reflexive nor transitive
   b. Transitive but neither reflexive nor symmetric.

Show that the relation R on the set R of real numbers defined by

5. $R = \{ (a,\ b) :\ a \leq b^2 \}$ is neither reflexive, nor symmetric nor transitive.
6. $R = \{ (a,\ b) :\ a \leq\ b \}$ is reflexive, symmetric but not transitive.

Show that the relation R on Z of integers defined by

7. $R = \{ (a,\ b): |a - b|\ is\ even \}$ is an equivalence relation.
8. $R = \{ (a,\ b): 2\ divides\ a - b \}$ is an equivalence relation.

Note: A relation R on a set A means, relation $R : A \to A$.

Do more problems of your own!