FORM FOUR PRE- EXAMINATION

MVOMERO DISTRIC COUNCIL

FORM FOUR MOCK EXAMINATION 2023

CODE: 041 **BASIC MATHEMATICS**

(For Both School and Private Candidates)

MARKING SCHEME

---

1. (a) Simplify the sum of 85% of 9861 and $\frac{3}{5}$ of 12458. Write your answer correct to two significant figures.

$\frac{85}{100}\times 9861 + \frac{3}{5}\times 12458$ **1 mark**

$8381.85 + 7474.8 = 15856.65$ **1 mark**

$15856.65 \approx 16000$ *in two significant figures* **1 mark**

(b) (i) Mr. Tumai Distributed Tshs 960,000/= awards to students who passed well in their examinations and their respective teachers as follow: 22% to all students who passed Arts Subjects. 17% to students who passed Mathematics and 26% to students who passed science subjects. The remained amount was distributed to teachers. Required to Find the amount that were awarded to teachers.

22% + 17% + 26% + Teachers = 100%

65% + Teachers = 100% **1 mark**

Teachers = 100% - 65% **1 mark**

Teachers = 35% **1 mark**

= $\frac{35}{100}\times 960,000 = 336,000$; Teachers were awarded **Tshs. 336,000/=**

(ii) Given time intervals: 8, 10 and 12 seconds

To find the time that 3 bells range together, we find the L.C.M of 8, 10 and 12 seconds; By prime factorization:

8 = $2\times 2\times 2$

10 = 2×5

12 = $2\times 2\times 3$ **1 mark**

L.C.M = $2\times 2\times 2\times 3\times 5$

= 120 seconds or 2 minutes **1 mark**

Time they will toll together is (50+2) minutes = 52 minutes or (120+3000) seconds = **3120 seconds.** **1 mark**

2. (a) If $a = \sqrt{5},\ b = \sqrt{3}\ ,\ and\ c = \sqrt{2}$. Simplify $\frac{a-b}{b+c}$

$\frac{a-b}{b+c} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ By rationalizing the denominator

$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{\sqrt{15}-\sqrt{10}-3+\sqrt{6}}{3-\sqrt{6}+\sqrt{6}-2}$

$= \sqrt{15} - \sqrt{10} + \sqrt{6} - 3$

(b) Describe the applications of Logarithm in real life situations **(1mark @, Total = 3marks)**

- In chemistry:
  - ✔ To determine acidity and alkalinity of the pH ($p(t) =- t$)
  - ✔ In radioactive decay by geologists (finding Half-life)
  - ✔ In carbon dating – Radiometric dating techniques
- Measure of population growth
  - ✔ Determine increment of certain measurement (Natural and Exponential functions)
  - ✔ Predict death rates, population increments (birth rates)
- Measure of Earthquakes
  - ✔ Measure of magnitude of an Earthquakes
  - ✔ Predict the coming of an Earthquake

3. In a certain class MSUVA displayed counting numbers less than 30. From the displayed numbers, OKWI mentioned all numbers completely divisible by 2 and FEITOTO mentioned all numbers that were the multiples of 3.

(a) (i) Outline all numbers that were mentioned by OKWI and FEITOTO in Common

OKWI = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28} **0.5 marks**

FEITOTO = {3, 6, 9, 12, 15, 18, 21, 24, 27} **0.5 marks**

Numbers mentioned in common = {6, 12, 18, 24} **0.5 marks**

(ii) How many numbers were mentioned BY Either OKWI or FEITOTO

n(OKWI U FEITOTO) = n(OKWI) + n(FEITOTO) – n(Common numbers) **0.5 marks**

n(OKWI U FEITOTO) = 14 + 9 – 4 =19 **1 mark**

(b) A number was selected in random from the number displayed by MSUVA, Req. to find the probability that a selected number was a multiple of 3.

$P(\, multiple\ of\ 3) = \frac{n(\, multiple\ of\ 3)}{n(Counting\ number\ less\ than\ 30)}$ **1.5 marks**

$$P(\, multiple\ of\ 3) = \frac{9}{29}$$ **1.5 marks**

$$p(\, multiple\ of\ 3) = \frac{9}{29}$$

4. (a) Given that $\underline{a}$ = 4i + 3j and $\underline{b}$ = 6i – 3j, Required to find the value of "h" and "k" if h$\underline{a}$ + k$\underline{b}$ = 10i + j

given that $\underline{a}$ = 4i + 3j and $\underline{b}$ = 6i – 3j

h$\underline{a}$ + kb = h(4i + 3j) + k(6i – 3j)

= 4hi + 3hj + 6ki - 3kj **(1mark)**

Comparing h$\underline{a}$ + k$\underline{b}$ = 10i + j

4hi + 3hj + 6ki - 3kj = 10i + j

4hi + 6ki = 10i

4h+ 6k = 10 …………………… (i) **(0.5 marks)**

3hj -3kj = j

3h-3k=1 ………………………….(ii) **(0.5 marks)**

$Solving\ equation\ (i) and\ (ii) Simultaneously,\ h\ =\ 1.2\ and\ k\ =\ 0.87$ **(1 mark)**

(b) A perpendicular line from the point *P(2,-4*) to the line meets the line at point *Z(-1,3).* Required to find:

i. Distance $\overline{PZ}$

$Distance\ \overline{PZ} = \sqrt{(2\ +\ 1)^2 + (-\ 4\ -\ 3)^2} = \sqrt{58} =\ 7.615\ unit\ of\ lengt$ **(1.5 marks)**

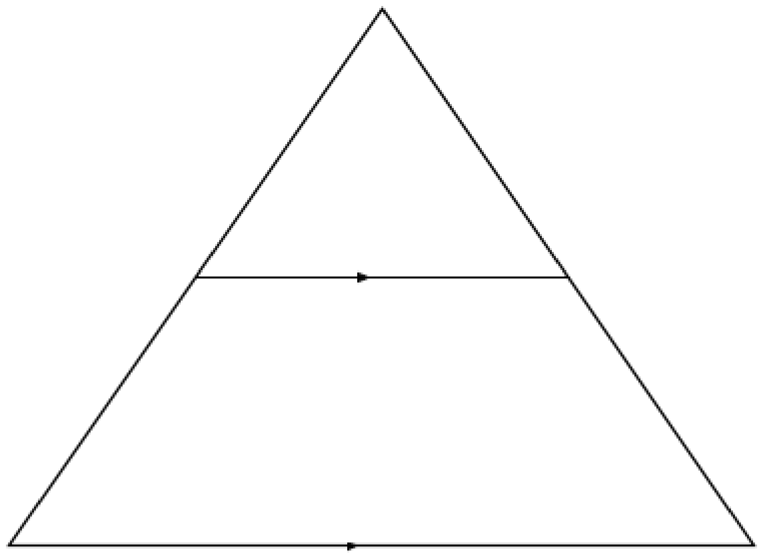
ii. If the point *Z(-1,3)* is a mid-point of the line $\overline{PQ}$, find the coordinates of point Q

$Midpoint\ = \left(\frac{x+2}{2}, \frac{y-4}{2}\right)$

$(-\ 1, 3) = \left(\frac{x+2}{2}, \frac{y-4}{2}\right)$ by comparing, we get; $(x, y) = (-\ 4, 10)$

$Hence\ Q(x, y) =\ (-4{,}10)$ **(1.5marks)**

5. (a) In the following figure; $\overline{ED} // \overline{BC}$ and $\overline{AB} = \overline{AC}$; If $\overline{AB}\ =\ 13cm$ and $\overline{BC}\ =\ 10cm$, find the Area of BCDE



E

A

D 0.5 mark

B 0.5 marks

C

6.5cm

6.5cm 0.5 marks

10cm

$B\hat{A}C = E\hat{A}D$ (common Angle)

$A\hat{B}C = A\hat{E}D$ (alternate angle $\overline{ED} /\!/ \overline{BC}$)

$B\hat{C}A = E\hat{D}A$ (alternate angle $\overline{ED} /\!/ \overline{BC}$)

$ABC \sim AED$ (AAA)

$Hence\ \frac{13}{6.5} = \frac{6.5}{ED}$ and $\overline{ED} = 5cm$

Consider the height from ΔABC

From Pythagoras theorem; $a^2 + b^2 = c^2$

$h^2 = 13^2 - 5^2$

$h = 12$

Area of $\Delta ABC = \frac{1}{2} \times h \times BC$

$Area\ of\ \Delta ABC = \frac{1}{2} \times 10 \times 12$

Area of $\Delta ABC = 60cm^2$ **0.5 marks**

From area of similar figures

Ratio of areas $k^2 = \frac{A_1}{A_2}$ since $A_1 = 60cm^2$

$$\left(\frac{10}{5}\right)^2 = \frac{60}{A_2}$$

$100A_2 = 60 \times 25$

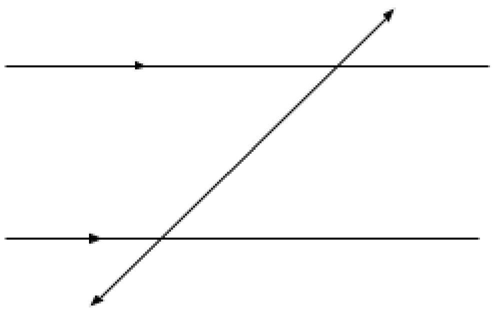
$A_{2} = \frac{1500}{100} = 15cm^2$ **0.5 marks**

Therefore $Area\ of\ BCDE = Area\ of\ \Delta ABC - \Delta AED$

Hence $Area\ of\ BCDE = 60cm^2 - 15cm^2$

Area of BCDE = 45cm² **0.5 marks**

(b) By considering the Alternating opposite angles theorem, required to draw the diagram hence identify the corresponding angles



1 mark

1 mark

a

b

x

y

1 mark

b = x

a = y

6. (a) The time "t" taken to buy fuel at LAKE OIL PETROL STATION in Morogoro varies directly as the number of vehicle "V" in the queue and inversely as the number of pumps "P" available in the station. In petrol station with 10 pumps it takes 20 minutes to fuel 40 vehicles.

i. Required to write vehicle "v" in terms of pumps "P" and time "t"

$t \propto \frac{v}{p}$ which implies that $t = k\frac{v}{p}$ **(0.5 marks)**

$v = \frac{pt}{k}$ **(0.5 marks)**

ii. Req. to find the time it will take to fuel 60 vehicles in a station with 4 pumps.

$From,\ t = \frac{kv}{p}$ this implies $k = \frac{pt}{v}$ **(1 mark)**

(1mark)

$when\ p = 10,\ t = 20\ and\ v = 40\ therefore\ k = \frac{10\times20}{40} = 5$ **(1mark)**

$when\ v = 60,\ p = 4\ and\ k = 5,\ therefore\ t = \frac{5\times60}{4} = 75\ minutes$

(b) Mayele bought 3 bottles of juice of capacity 350 ml and Dialo bought 1 bottle of juice of capacity 1 litre.

i. required to find Who had more juice to drink.

$Mayele\ juice = 3\times350ml = 1050ml = 1.05\ litres$

$Dialo\ juice = 1\ litre$ **(1 mark)**

$Mayele\ had\ more\ Juice\ to\ drink$

ii. required to find How much more

$amount = 1.05\ litres - 1\ litre = 0.05 litre = 50\ ml$ **(1 mark)**

7. (a) Require to give the meaning of the following terms as used in Accounts
   (i) C**ash book**: Is the book/account which records cash transactions only.
   (ii) **Assets**: Are properties or possessions of the business; For example buildings, motor vans, etc **(1mark @)**
   (iii) **Credit transaction:** Are transactions where by payments are done later.

(b) A company bought two cars for Tshs 25,000,000/= each. If one car was sold at a profit of 18% and another was sold at a loss of 6%. In the whole transactions there were no loss. What was the profit made by a company?

$Car\ 1: Profit = \frac{18}{100}\times25,000,000 = 4,500,000$ **1 mark**

$Car\ 2: Loss = \frac{6}{100}\times25,000,000 = 1,500,000$ **1 mark**

$Profit\ made = Profit\ of\ Car\ 1 - Loss\ of\ car\ 2$ **1 mark**

$= 4,500,000 - 1,500,000 = 3,000,000$

8. (a) A BODA BODA driver rates after each kilometre. The fare is Tshs. 1000/= for the first kilometre and raise by Tshs. 500/= for each additional kilometre. If BALEKE want to travel 10 kilometres by BODA BODA. What will he be charged by BODA BODA driver?

$First\ kilometre\ (A_1) = 1,000$

$Amount\ added\ to\ preceeding\ (d) = 500$

Required to find charge* (A_{50}) *after travelling 50km (n = 10)

$A_n = A_1 + (n - 1)d$ **1 mark**

$A_{10} = 1,000 + (10 - 1)500$ **1 mark**

1 mark

$A_{10} = 5,500$ Tshs.

(b) The number 19683 is in which term in the following Geometric sequence 3, 9, 27, …?

$G_n = 19683, G_1 = 3, r = 3$ required to find number of terms "n"

0.5 mark

$G_n = G_1 r^{n-1}$

0.5 mark

$19683 = 3 \times 3^{n-1}$

1 mark

$6561 = 3^{n-1}$

1 mark

$3^8 = 3^{n-1}$

$n - 1 = 8, \ n = 9$

32768 is the Nineth term of the sequence

9. (a) If the square of the hypotenuse of an isosceles right-angled triangle is 128 cm^2 find the length of each other sides

let a side be x

1 mark

$x^2 + x^2 = 128$

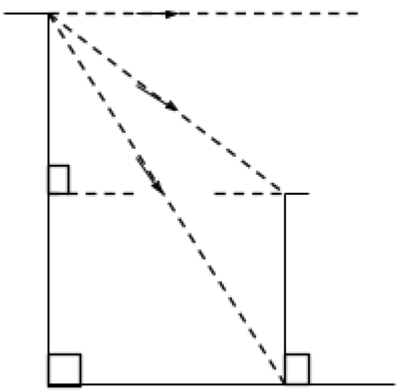
1 mark

$2x^2 = 128$

1 mark

$x^2 = 64 \quad , \quad x = 8cm$

(b) From the top of a tower of height 60m the angles of depression of the top and the bottom of a building are observed to be 30^0 and 60^0 respectively. Find the height of the building.



1 mark

0.5 marks

0.5 marks

0.5 marks

0.5 marks

30^0

30^0

60^0

30^0

A

B

C

D

E

$$\overline{AD} = 60m$$

$$\tan tan\ 60^0 = \frac{\overline{AD}}{\overline{DC}} = \frac{60}{\overline{DC}}$$

$$\overline{DC} = \frac{60}{\sqrt{3}}$$

Also, $\tan tan\ 30^{\ 0} = \frac{\overline{AE}}{\overline{BE}} = \frac{60-\overline{DE}}{\overline{BE}}$

$$\overline{BE} = \sqrt{3}(60 - \overline{DE})$$

But $\overline{DC} = \overline{BE}$

$$\frac{60}{\sqrt{3}} = \sqrt{3}(60 - \overline{DE})$$

$\overline{DE} = \overline{BC} = 40cm$ **height**

9. (a) If the length of each side of a square is increased by 6cm, the area become increased by $144cm^2$ of the area of the small square. Required to find the length of one side of the original square.

$$Area\ of\ small\ square = Length \times Lenght = (Length)^2$$

$$Area\ of\ Large\ square = (6 + Lenght)(6 + length)$$ **0.5 marks**

But, $Area\ of\ large\ square = 144 + Area\ of\ small\ square$ **0.5 marks**

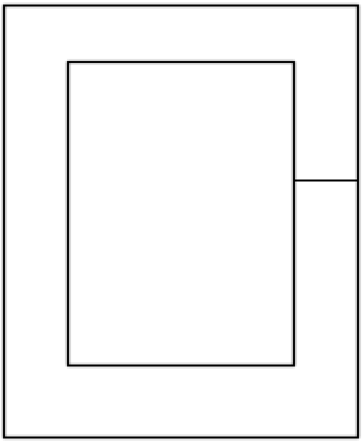
$$36 + 12Lenght + (Length)^2 = 144 + (Length)^2$$ **0.5 marks**

$$36 + 12Length = 144$$ **0.5 marks**

$$12Lenght = 108$$ **1 mark**

$$Length = 9cm$$

(b) A large rectangular garden in a park is 120m wide and 150m long. A contractor is called in to add a brick walkway to surround this garden by the same width. If the area of the walkway is $2800m^2$, how wide is the walkway?



0.5 marks

0.5 marks

0.5 marks

1 mark

0.5 marks

120m

150m

x

$Area\ of\ small\ rectangle = 120 \times 150 = 18000$ m²

$Area\ of\ large\ rectangle = (120 + 2x)(150 + 2x)$

But, $Area\ of\ large = Area\ of\ small + Area\ walkway$

$$(120 + 2x)(150 + 2x) = 18000 + 2800$$

$$18000 + 540x + 4x^2 = 18000 + 2800$$

$$4x^2 + 540x - 2800 = 0$$

$$x^2 + 135x - 700 = 0$$

$$x = 5\ or\ x = -140$$

$$Since,\ Length \neq negative\ value,\ x = 5m$$

$$The\ walkway\ is\ 5m\ wide$$

10. (a) In certain research the data were summarized as shown in a table below:

Class Mark	10	15	20	25	30	35
Frequency	3	2	10	5	4	1

Required to reconstruct a frequency distribution table including class interval and frequency.

Let $a - b,\ be\ the\ first\ class\ interval.\ Then\ \frac{a+b}{2} = 10$

$hence\ a + b = 20$........................ (i) **(0.5mark)**

$$class\ Size = 15 - 10 = 5\ (Consecutive\ Classmarks)$$

$$But\ Also,\ Class\ Size = Upper\ boundary - Lower\ boundary$$

$$Class\ Size = b + 0.5 - a + 0.5$$

$$5 = b - a + 1$$

$b - a = 4$…………………(ii) **(1mark)**

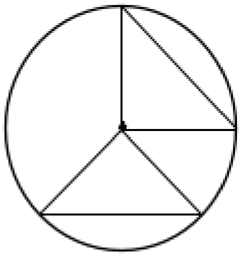
$Solving\ (i) and\ (ii)\ simultaneously\ we\ get\ a = 8,\ b = 12$ **(1.5 mark)**

$$The\ first\ interval\ is\ 8 - 12$$

interval	8 – 12	13 – 17	18 – 22	23 – 27	28 – 32	33 – 37
Frequency	3	2	10	5	4	1

(2 marks)

(b) Required to prove that equal chords of a circle subtend equal angles at a centre.



A

B

C

D

O

$chord\ \overline{AB} = chord\ \overline{DC}\ Given$ **(1 mark)**

$\overline{AO} = \overline{DO} = \overline{BO} = \overline{CO}\ radii$ **(1 mark)**

$\Delta AOB \equiv \Delta COD \quad SSS$ **(1 mark)**

$Hence\ A\hat{O}B = C\hat{O}D$ **(1 mark)**

(1 mark)

11. (a) Two towns P and Q on the Latitude 48^0 are 370 Nautical miles apart. Find the difference in their longitude.

$Nm\ Distance = (60\theta cos\alpha)Nm$ **(1 mark)**

$$where\ \theta = Difference\ in\ longtude\ (to\ be\ found)$$

$$and\ \alpha = Latitude = 48°N\ (Given)$$

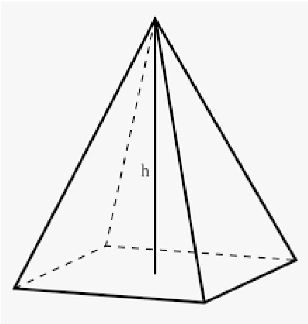
$$Distance = 370Nm\ (Given)$$

$Therefore,\ 370Nm = (60\theta \cos cos\ 48°\ )Nm$ **(1 mark)**

$\theta = 9.2°$ **(1mark)**

(b) A pyramid with vertex V and edges VA, VB, VC, VD each 15cm long has a rectangular base ABCD where AB = CD = 10cm and AD = BC = 8cm.

i. Sketch the pyramid using the above information.



V

B

C

A

D

O

15cm

8cm

10 cm

1.5 marks

ii. Calculate the height "VO" of the pyramid where "O" is the centre of the rectangle.

Consider ΔAOV,

$$\overline{VO}^2 + \overline{AO}^2 = \overline{AV}^2 \text{ but } \overline{AO} = \frac{1}{2}\overline{AC}$$

$\overline{VO}^2 = 15^2 - \frac{1}{4}\overline{AC}^2$ ………………………. (i) **(0.5 marks)**

consider ΔABC,

From Pythagoras theorem

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$$

$\overline{AC}^2 = 10^2 + 8^2 \text{ hence } \overline{AC}^2 = 164$……………. (ii) **(0.5 marks)**

By Subst equation (ii) *into* (i)

$$\overline{VO} = h = 13.6cm$$ **(1mark)**

iii. Calculate the angle between the base and edge

From ΔAOV, *let* θ *represent the angle between* $\overline{VA}$ *and* $\overline{AO}$

$$Then,\ Cos\,\theta = \frac{\overline{AO}}{\overline{VA}} = \frac{\frac{\sqrt{164}}{2}}{15}$$

(1mark)

$$Hence\ \theta = 64.7°$$ **(0.5 marks)**

12. (a). (i). Find the possible values of x if the matrix $(x + 1\ 9\ 3\ x + 5)$ has no inverse.

(ii). By using matrix method, solve the equations

$\{4x - 9 =- 3y\ 5x - 17 - 2y = 0$

(i). Let A = $(x + 1\quad 9\ 3\quad x - 5)$

Since matrix A has no inverse

Then

|A| = 0 **(0.5 marks)**

So

$(x + 1)(x - 5) - (3x\ 9) = 0$ expanding $x^2 - 5x + x - 5 - 27=0$ hence $x^2 - 4x - 32=0$

Solving quadratically we get **(1mark)**

<u>X = 8 or -4</u>

(ii). Given that $\{4x - 9 =- 3y\ 5x - 17 - 2y = 0$

Rearranging $\{4x + 3y = 9\ 5x - 2y = 17$

Let B = $(4\quad 3\ 5\quad - 2)(x\ y) = (9\ 17)$

|B| = -8 – 15 hence |B|= - 23 **(0.5 marks)**

$B^{-1} = \frac{1}{-23}(- 2\ \ - 3\ \ - 5\ \ 4)$

$B^{-1} = \left(\frac{2}{23}\quad \frac{3}{23}\ \frac{5}{23}\quad \frac{-4}{23}\right)$

(1mark)

Then

$B^{-1}B = B^{-1}\ (9\ 17)$

$(1\quad 0\ 0\quad 1)(x\ y) = \left(\frac{2}{23}\ \frac{5}{23}\quad \frac{3}{23}\ \frac{-4}{23}\right)(9\ 17)$

(1mark)

$(x\ y) = \left(\frac{18}{23}\quad \frac{51}{23}\ \frac{45}{23}\quad \frac{-68}{23}\right)$

$(x\ y) = \left(\frac{69}{23}\ \frac{-23}{23}\right)$

$(x\ y) = (3\ \ - 1)$ hence <u>x = 3 and y = -1</u> **(1mark)** (b). A transformation is given $x' = 2x + 5y$ and $y' = -x$, Required to find the image of the equation 2x – 3y = 6 under the transformation matrix which performs the above transformations.

Given

$$x' = 2x + 5y$$
$$y' = -x$$

Then

The transaction matrix = $(2 \quad 5 \ -1 \quad 0\,)$

(1mark)

Required to find the image of 2x – 3y =.

Its x and y intercept points are:

A(0, -2) and B(3, 0).

Their images are obtain as

A' = $(2 \quad 5 \ -1 \quad 0\,)\,(0 \ -2\,)$

(1mark)

A' = (-10, 0)

Also

B' = $(2 \quad 5 \ -1 \quad 0\,) \quad (3\,0\,)$

B' = $(6, -\ 3)$ **(1mark)**

M = $\frac{y_2 - y_1}{x_2 - x_1}$ M = $\frac{-3-0}{6-(-10)}$ M= $\frac{-3}{16}$

Also to find equation

M= $\frac{y - y_0}{x - x_0}$ M= $\frac{y - 0}{x - (-10)}$ $such\ that\ \frac{-3}{16} = \frac{y}{x+10}$ **(1mark)**

-3x – 30 =16y rearranging, -3x -16y = 30

Its image is <u>3x + 16y = -30</u> **(1mark)**

13. (a) A function f is defined as;

$f(x) = \{x - 4,\ if\ x < 2\ 15,\ if\ -2 < x < 2\ x^2, \qquad if\ x > 2\}$

Req. to Find

(i) f(1.95) =15 **1 mark**

(ii) $f(15) = 15^2 = 225$ **1 mark**

(iii) Domain of f = $\{x \epsilon R,\ x \neq -2\ and\ x \neq 2\}$ **2 marks**

(iv) The Type of a function f is ***STEP FUNCTION*** **1 mark**

(b) A school is preparing a trip for 400 students. The company who is proving the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only 9 drivers available. The rental cost for a large bus is 800,000/= and 600,000/= for the small bus. How many buses of each type should be used for the trip for the least possible cost?

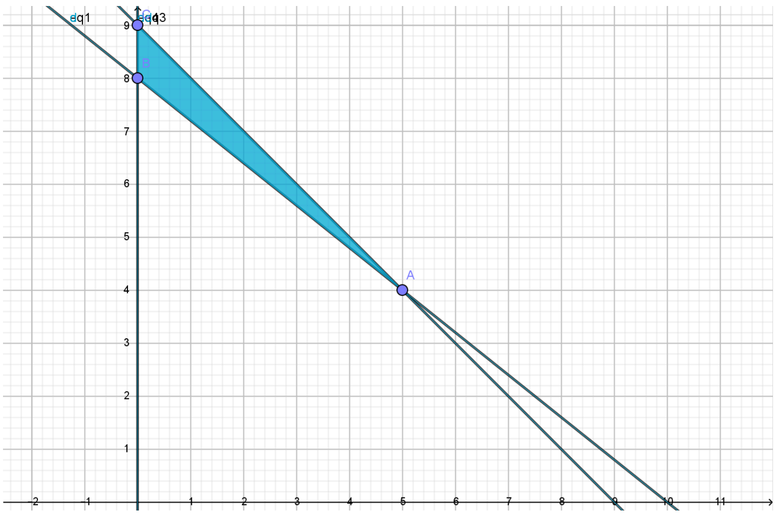
CONSTRAINTS

$$Eq1:\ 40x + 50y \geq 400$$ **0.5 marks**

$$Eq2:\ x + y \leq 9$$ **0.5 marks**

$$Eq3:\ x \geq 0$$

$$Eq4:\ y \geq 0$$



2 marks

$$A = (6, 4)$$

$$B = (0, 8)$$

$$C = (0, 9)$$

$$Objective\ Function;\ F(x, y) = 600,000x + 800,000y$$

$$F(5, 4) = 6,200,000$$ **1 mark**

$$F(0, 8) = 6,400,000$$

$$F(0, 9) = 7,200,000$$

1 mark

$Hence\ the\ minimum\ cost\ is\ 6,200,00,\ hence\ 5\ small\ buses\ and\ 4\ large\ buses\ should\ be\ used$