

Resources - Abstract Algebra II Spring 24

Homework is posted here: <https://www.overleaf.com/read/tnfmgvfmqzwc>

Group Tables: <https://hobbes.la.asu.edu/groups/>

Feedback Notes:  Math 456 F23: Notes from Grading

Weekly Class Notes

Week 1:

Text sections: Chapter 5 & 6

Takeaways: Fundamental Theorem of Finite Abelian Groups

- Review of Semester 1 concepts
 - Element order ($|g^b| = |g|/\gcd(|g|, b)$), Lagrange's Theorem
 - Factor groups, properties of isomorphism and homomorphism, 1st & 4th isomorphism theorems
 - p-groups, Sylow p-subgroups, and Sylow's theorems
 - Inner and Outer Direct Products

Perusall: Syllabus by 11:30pm **Monday 1/22**

Homework 1: Turn in conversation recording and Problem 4 through WISE by 11:30pm **Friday 1/26**

Week 2:

Text sections: Chapter 5 & 6 & 7.1

Takeaways: Fundamental Theorem of Finite Abelian Groups, Introduction to Rings

- Takeaways of FTFAG, elementary divisor and invariant factor decompositions
- Definitions of rings, units, unity, zero divisors, integral domains, and fields
- Examples of rings

Perusall: Dummit & Foote 7.1 (due 11:30pm Monday 1/29)

Homework #2 (due Wednesday 1/31)

Week 3:

Text sections: 7.2 & 7.3

Takeaways:

- Polynomial Rings
- Ring homomorphism
- Factor Rings

[Perusall:](#) Unit Conjecture (due 11:30pm Monday 2/5)

[Homework #3](#) (due Wednesday 2/7)

Week 4:

Text sections: 7.3 & 7.4

Takeaways:

- Ring homomorphism
- Factor Rings & Ideals
- Ring Isomorphism Theorems
- Principal Ideals

[Perusall:](#) Students Unravel Widely Believed Conjecture (due Monday 2/12)

[Homework #4](#) (due Wednesday 2/14)

Week 5:

Text sections: 7.4 & 7.5

Takeaways:

- Prime and Maximal Ideals

[Perusall:](#)

[Homework #5](#) (due Wednesday 2/21)

Week 9:

Text sections: Pinter Ch. 24-26

Takeaways:

- Domains of Polynomials over Fields $F[x]$, Introduction and Properties
- Perspectives on Polynomials: Elements versus Functions on Fields
 - Connections between roots and factors
 - Fundamental Theorem of Algebra

Perusall: Pinter Ch. 24 (due **11:30pm Monday 3/11**)

Individual Celebration of Knowledge **Tuesday 3/12:** [Study Guide](#)

Week 10:

Text sections: Pinter Ch. 26-27

Takeaways:

- Irreducibility tests in \mathbb{Q} and \mathbb{Z}
- Extension Fields

[Homework #8](#) (due **Wednesday 3/20**)

Ooops! Sort of forgot about Class Notes after Spring Break

Week 15:

Text sections: Pinter Ch. 32-33

Takeaways:

- Fundamental Theorem of Galois Theory
- Solvability

Solution to Quadratic: (you fill in)

Solution to Cubic:

$$\begin{aligned}
 x_1 &= -\frac{b}{3a} \\
 &\quad -\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]} \\
 &\quad -\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]} \\
 x_2 &= -\frac{b}{3a} \\
 &\quad +\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]} \\
 &\quad +\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]} \\
 x_3 &= -\frac{b}{3a} \\
 &\quad +\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]} \\
 &\quad +\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}\right]}.
 \end{aligned}$$

Solution to Quartic:

$$\begin{aligned}
 x_1 &= \frac{1}{2} \left(\frac{2b^2 - 3ac + 10a}{\sqrt{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \right) \\
 x_2 &= \frac{1}{2} \left(\frac{2b^2 - 3ac + 10a}{\sqrt{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \right) \\
 x_3 &= \frac{1}{2} \left(\frac{2b^2 - 3ac + 10a}{\sqrt{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \right) \\
 x_4 &= \frac{1}{2} \left(\frac{2b^2 - 3ac + 10a}{\sqrt{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \sqrt{\frac{2b^2 - 3ac + 10a}{4b^2 - 3ac + 10a}} \right)
 \end{aligned}$$

Langlands Conjecture: https://en.wikipedia.org/wiki/Langlands_program

[Homework #12](#) (due **Monday 4/29**)