Course syllabus: Geometric Mechanics

Fall 1 semester in 2024-2025 academic year

Instructors:
Daniel Holmes
Sadashige Ishida (main contact)
Chris Wojtan

Dates and times:
On Mondays and Wednesdays, from Oct. 7th to Nov. 13th. 14:45-16:00.

Location:
Sunstone Building / O1 / Big Office Meeting room (101) (I23.O1.101),
ISTA (Am Campus 1, Klosterneuburg 3400)

Course description & goals
This course is an introduction to geometric mechanics. It is a study of dynamics through the lens of geometry, with connections to many fields e.g. classical and quantum physics, control theory, complex geometry, optimal transport, computer graphics/vision, etc.

Dynamics can be phrased in terms of shapes in finite or infinite-dimensional abstract spaces. We study how this works, with examples from physics, especially fluid dynamics. We will cover selected items from the following list of standard elements:
1. Minimal overview of symplectic geometry and Hamiltonian systems, 2. Poisson geometry and Lie-Poisson systems, 3. Euler-Arnold equation as geodesics, 4. Symmetry and conserved quantities via momentum maps and Noether's theorem, 5. Integrability via symplectic/Poisson reduction and coadjoint orbits, 6. Contact geometry and fluids in equilibrium, 7. Fluids on a non-simply connected domain, 8. Applications to other subjects like differential geometry, optimal transport, and numerical integration.

We take a physicist's approach. That is, we may occasionally prioritize physical or geometric intuition and sacrifice mathematical rigor.

The course contents and the level can be adjusted to the attendees' backgrounds and interests.

[Goals]
In contrast to intense PDE analysis, geometric mechanics offers somewhat qualitative and relaxed views of dynamics. A goal of this course is that participants enjoy learning this spirit, and they can take geometric approaches to problems in their fields.

At a glance, the field of geometric mechanics may seem intimidating as it uses languages from multiple fields. But this feature also helps us reach the research level quickly from the intersection of geometric mechanics and each one's strengths/interests. Hence a little ambitious

goal is that participants understand a fragment of the state of the art and even get ready to form and investigate new questions.

Literature
- Holm, Schmah, Stoica. “Geometric Mechanics and Symmetry: From Finite to Infinite Dimensions”.
- Cannas da Silva. “Lectures on symplectic geometry”.
- Arnold & Khesin. “Topological methods in hydrodynamics”.

Target audience
Anyone interested in geometric mechanics. Researchers in dynamical systems, differential equations, and theoretical physics who want to extend their view of dynamics toward geometry. Researchers in geometry who want to see its power in exploring dynamics.

Preliminaries
- Basics of differential geometry i.e. differential manifolds, tangent maps, exterior calculus.
- Basics of (co)homology theory, (Lie) group theory, and (principal) fiber bundles will be helpful, but not required.
- No prior knowledge in physics like fluid dynamics or Hamiltonian systems is assumed.
- No prior knowledge of PDEs or dynamical systems is assumed.

If you are not confident with your background in these preliminaries but motivated, I will do my best to help you catch up and enjoy the course.

Teaching format
Lectures and participants' presentations on blackboard or whiteboard. Two classes per week.

Assessment form(s)
Pass/Fail. Active participation and homework. The participants may be asked to present their solutions to exercises.

A model timeline

Week 1-2: Overview of symplectic & Poisson geometry and Hamiltonian dynamics
Week 3: Symplectic and Poisson integrators
Week 4: Symmetries and conserved quantities. Momentum maps and Noether’s theorem.
Week 5: Integrability via reduction. Symplectic reduction.
Week 6: Applications in other subjects or backup slots in case we need more time to cover the contents for Weeks 1-5.

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