FLT. Formulas of numbers A, B, C

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To my wonderful women - grandmother, mother, wife, and friend.

1. **Theorem.** In a hypothetical equality $A^n + B^n - C^n = 0$ in a number system with a prime base $n > 2$, mutually prime natural numbers A, B, C (mod n^k) starting from k = 1 equal to the numbers $A_1^{n\wedge(k-1)}$, $B_1^{n\wedge(k-1)}$, $C_1^{n\wedge(k-1)}$ (mod n^k), where k can be arbitrarily large.

Properties of Equality (1*):

2. Designations: A_1, A_2, … A_k – one-, two-, ... k-significant endings of the number A in the numeral system with base n.
3. The numbers A, B, C can be represented as: $A = A°n^k+A_k$, $B = B°n^k+B_k$, $C = C°n^k+C_k$, where the base n = 10.
4. **Key Lemma.**. The last two members in Newton Binom $(A°n^k+A_k)^n$ are $nA°n^k(A_k)^{n-1} + A_k^n$. From this can be seen, the numbers A^n, B^n, C^n (mod n^{k+1}) are unambiguous functions of the numbers A, B, C (mod n^k).
5. If $A + B$ (mod n) > 0, then factors $A + B$ and R in the decomposition of the degrees $A^n+B^n= (A+B)R$ are mutually prime; If $A + B$ (mod n) = 0, then $R = 0$ (mod n).
Therefore, if A, B, C (mod n) are not zero, then factors in the equalities
6. $C^n = A^n+B^n = (A+B)R$, $A^n = C^n - B^n = (C - B)P$, $B^n = C^n - A^n = (C - A)Q$ are degrees:
7. $A+B = c^n$, $R=r^n$. $C-B = a^n$, $P = p^n$, $C-A = b^n$, $Q = q^n$, and, since according to Fermat's little theorem,
8. $A^{n-1}{}_1 = B^{n-1}{}_1 = C^{n-1}{}_1 = A_1{}^{n-1}{}_1 = B_1{}^{n-1}{}_1 = C_1{}^{n-1}{}_1 = 1$ (mod n), then P_1, Q_1, R_1 (and p, q_1, r_1) in 6* is 1 (mod n), and according to 7*
9. $P = Q = R = 01$ (mod n^2). And from equalities 6* we get a system of equations with unknown A, B, C:
10. $A_1{}^n{}_2 + B_1{}^n{}_2 = (A + B)_2$, $C_1{}^n{}_2 - B_1{}^n{}_2 = (C - B)_2$, $C_1{}^n{}_2 - A_1{}^n{}_2 = (C - A)_2$ (mod n^2). Where do we find:
11. $A_2 = A_1{}^n{}_2 = a_1{}^n{}_2 * p_1{}^n{}_2$, $B_2 = B_1{}^n{}_2 = b_1{}^n{}_2 * q_1{}^n{}_2$, $C_2 = C_1{}^n{}_2 = c_1{}^n{}_2 * r_1{}^n{}_2$. And from 1* we have:
12. $A^n{}_3 + B^n{}_3 - C^n{}_3 = 0$ (mod n^3), where $P_3 = p_1{}^{nn}{}_3$, $Q_3 = q_1{}^{nn}{}_3$, $R_3 = r_1{}^{nn}{}_3$ are equal to 001, and the equalities 10* already give a system of equalities (and not just one!) with new unknowns A, B, C:
13. $A_1{}^{nn}{}_3 + B_1{}^{nn}{}_3 = (A + B)_3$, $C_1{}^{nn}{}_3 - B_1{}^{nn}{}_3 = (C - B)_3$, $C_1{}^{nn}{}_3 - A_1{}^{nn}{}_3 = (C - A)_3$ with the solution:
14. $A_3 = A_1{}^{n\wedge2}{}_3$, $B_3 = B_1{}^{n\wedge2}{}_3$, $C_3 = C_1{}^{n\wedge2}{}_3$, (compare with 10* taking into account 11*).
15. Next, we return to point 10* and repeat the operations 10*-14*, but now with the 3-digit endings $A_1{}^{n\wedge2}{}_3$, $B_1{}^{n\wedge2}{}_3$, $C_1{}^{n\wedge2}{}_3$ of the numbers A, B, C (where the factors $p_1{}^{n\wedge2}{}_3 = q_1{}^{n\wedge2}{}_3 = r_1{}^{n\wedge2}{}_3 = 001$) and find the 4-digit endings A_4, B_4, C_4 (i.e., modulo n^4).
16. And so it goes on INFINITELY, receiving of arbitrarily large A, B, C (mod n^k) and, consequently, of the numbers A, B, C, themselves, from which the impossibility of equality 1* follows.
17. We also see that, starting from k=1, A, B, C are $A_1^{n\wedge(k-1)}$, $B_1^{n\wedge(k-1)}$, $C_1^{n\wedge(k-1)}$ (mod n^k).

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May 2025. Mezos, France.

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https://docs.google.com/document/d/1OguuCS_hvZTmokTiQujaMRCXNTMPOdO6A2v_0k26leM/edit?tab=t.0