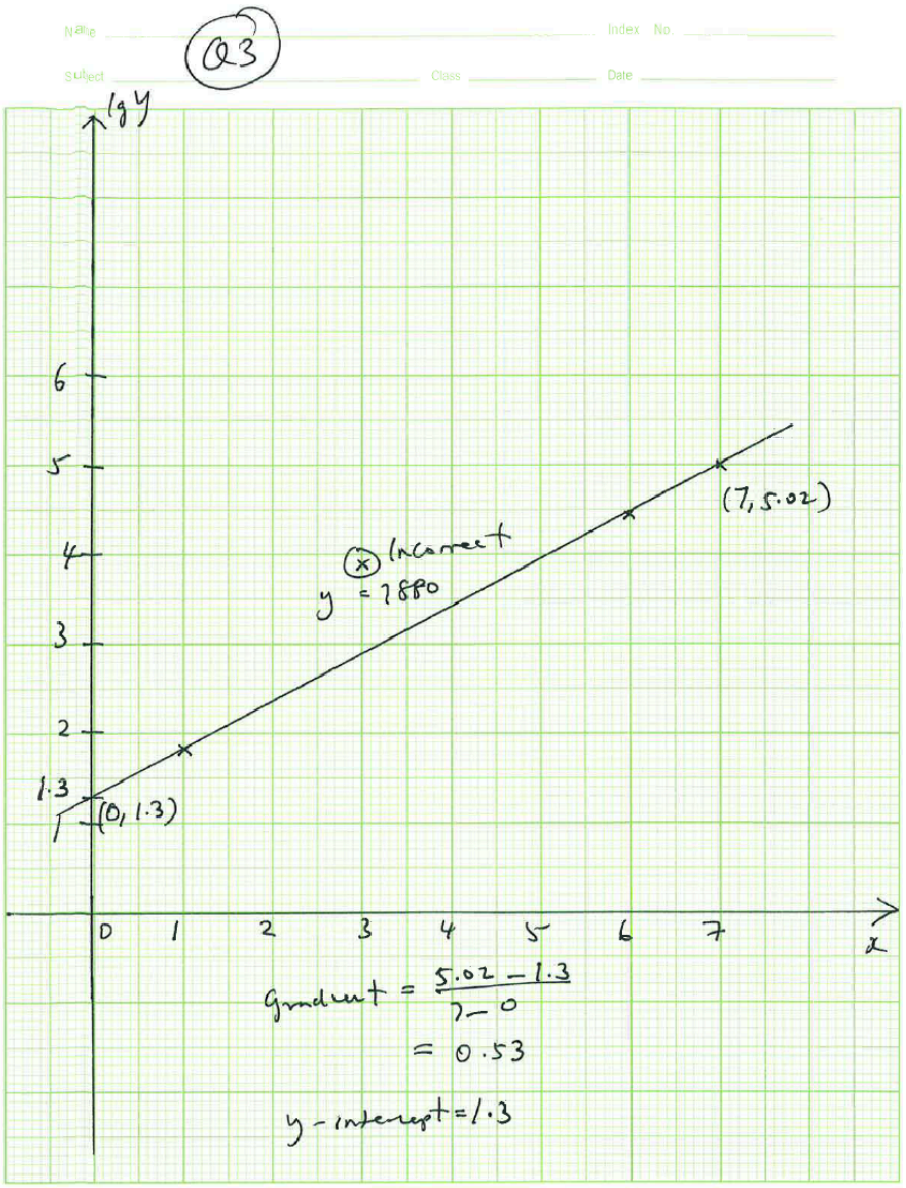
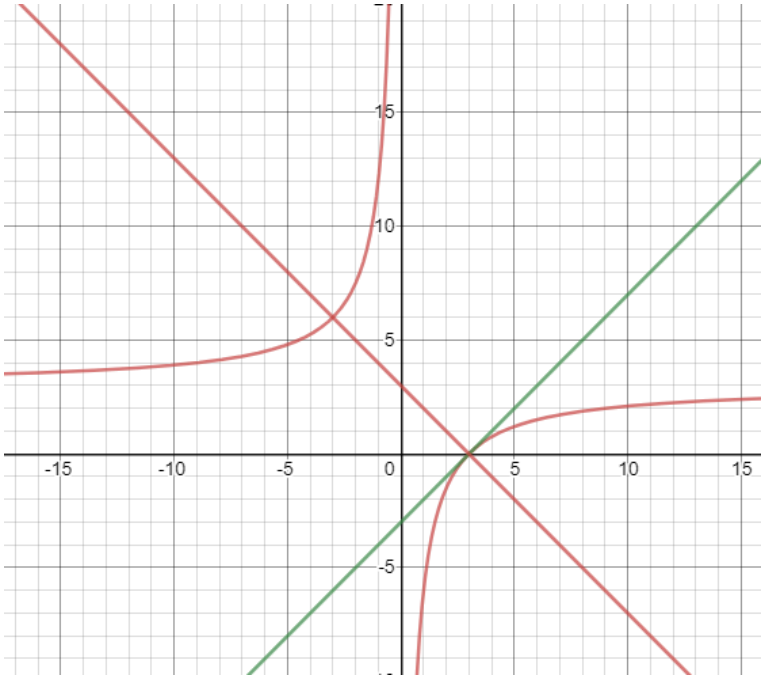
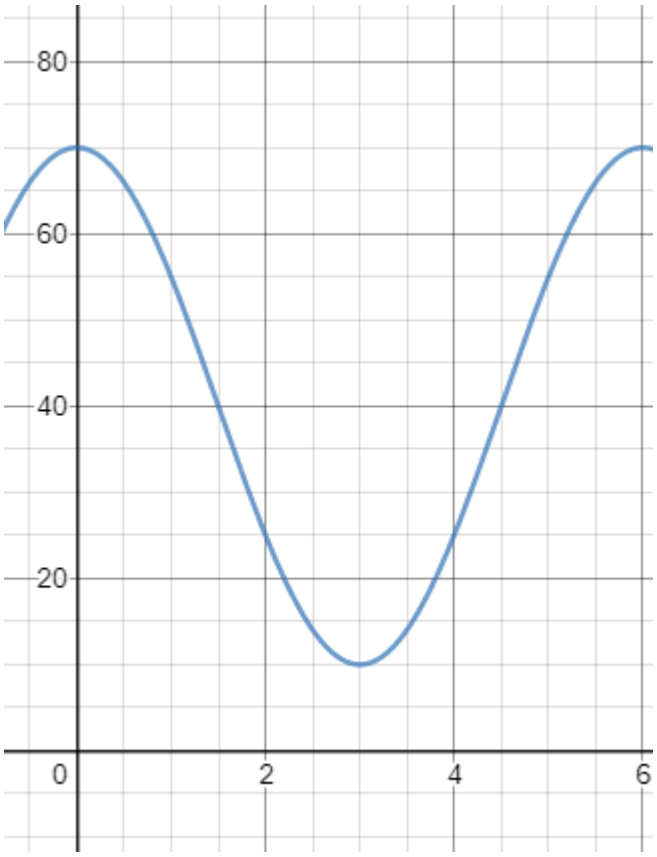
YISS Preliminary Examination 2020
Additional Mathematics – Sec 4E/5N
Marking Scheme
Paper 2

Question	Marking Scheme	Marks
1	$\frac{d}{dx}\left(x^2 lnx - 5x\right) = x^2\left(\frac{1}{x}\right) + lnx(2x) - 5$ $= x + 2xlnx - 5$ (Shown) Since $\int_0^3 x + 2xlnx - 5dx = \left[x^2 lnx - 5x\right]_0^3$ $\int_0^3 xdx + \int_0^3 2xlnxdx - \int_0^3 5dx = \left[x^2 lnx - 5x\right]_0^3$ $\int_0^3 2xlnxdx = \left[x^2 lnx - 5x\right]_0^3 - \int_0^3 xdx + \int_0^3 5dx$ $\int_0^3 2xlnxdx = \left[x^2 lnx - 5x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3 + [5x]_0^3$ $\int_0^3 2xlnxdx = [\{-5.112\} - \{0\}] - \left[\left\{\frac{9}{2}\right\} - \{0\}\right] + [\{15\} - \{0\}]$ $\int_0^3 2xlnxdx = 5.388$ $\int_0^3 xlnxdx = 2.69$ (3 sig. figures)	M1 A1 M1 A1 A1 A1 A1 [7]
2(i)	$lg(1 - 2x) - 2lgx = 1 - lg(2 - 5x)$ $lg(1 - 2x) - lgx^2 = lg10 - lg(2 - 5x)$ $lg\frac{(1-2x)}{x^2} = lg\frac{10}{(2-5x)}$ $\frac{(1-2x)}{x^2} = \frac{10}{(2-5x)}$ $10x^2 = 2 - 9x + 10x^2$ $9x = 2$ $x = \frac{2}{9}$	M1 A1 A1 [3]
2(ii)	$4^z = lg3^{z+1}$ $lg4^z = lg3^{z+1}$ $zlg4 = (z + 1)lg3$ $zlg4 - zlg3 = lg3$ $z = \frac{lg3}{lg4-lg3}$ $z = 3.82$	 M1 A1 A1 [3]
3(i)	$\lg y = \lg A + x \lg b$ Plot $\lg y$ against x	B1 [1]

[2]

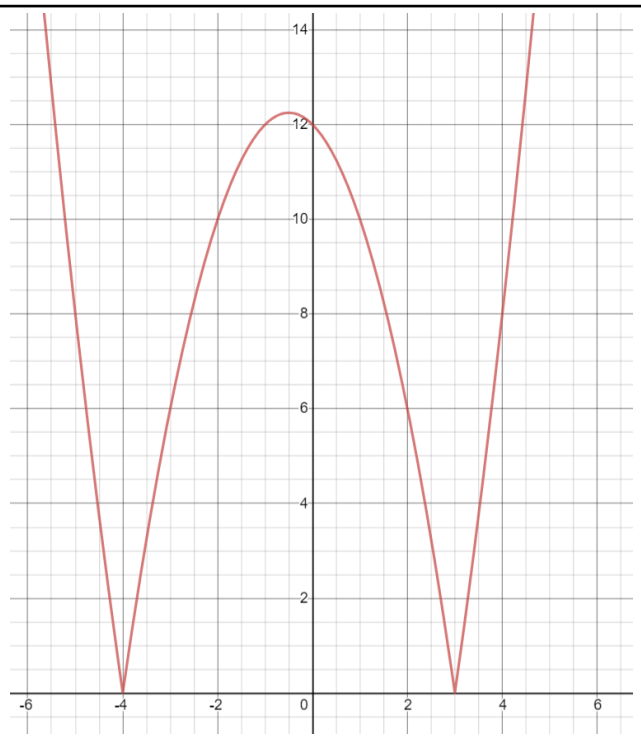
		
3(ii)	Incorrect y is 7880 x | 1 | 3 | 6 | 7 $lg\ y$ | 1.84 | 3.90 | 4.49 | 5.02	G2 B1 B1 [4]
3(iii)	gradient $= 0.53 = \lg b \Rightarrow b = 3.4$ intercept $= 1.3 = \lg A \Rightarrow A = 20$ From the graph $\lg y = 2.9 \Rightarrow y = 794$	B1 B1 B1 [3]
4(i)	$y = 3 - \frac{9}{x}$ $\frac{dy}{dx} = 9x^{-2}$ $\frac{dy}{dx} = \frac{9}{x^2}$	M1 A1

	Gradient of tangent at $A(3,\ 0)$, $\frac{dy}{dx} = 1$ Gradient at normal $= -\ 1$ Equation of normal at point A: $y\ -\ 0 = -\ 1(x\ -\ 3)$ $y = -\ x\ +\ 3$	A1 A1 [4]
4(ii)	Equation of curve: $y\ =\ 3\ -\frac{9}{x}$ - (1) Equation of the normal: $y = -\ x\ +\ 3$ – (2) Solving Equation (1) and (2) simultaneously, we have $3\ -\frac{9}{x} = -\ x\ +\ 3$ $\frac{3x-9}{x} = -\ x\ +\ 3$ $x^2\ -\ 9\ =\ 0$ $x = -\ 3$ (reject $x\ =\ 3$) $y\ =\ 6$ The normal meets the curve at $(-\ 3,\ 6)$. Coordinates of point $B(-\ 3,\ 6)$  	M1 A1 A1 [3]
4(iii)	Length of $AB = \sqrt{(3\ +\ 3)^2\ +\ (0\ -\ 6)^2}$ $= \sqrt{72} = 6\sqrt{2}$ (Exact length)	M1 A1 [2] Accept only EXACT
5(i)	$x^2\ +\ 2x\ -\ 8\ =\ 0$ SOR $= \alpha\ +\ \beta = -\ 2$ POR $= \alpha\beta = -\ 8$	B1 B1 [2]
5(ii)	$(\alpha\ +\ \beta)^3 = (\alpha\ +\ \beta)(\alpha\ +\ \beta)^2$ $= (\alpha\ +\ \beta)\left(\alpha^2\ +\ 2\alpha\beta\ +\ \beta^2\right)$ $= \alpha^3\ +\ 2\alpha^2\beta\ +\ \alpha\beta^2\ +\ \beta\alpha^2\ +\ 2\alpha\beta^2\ +\ \beta^3$	B1 [1]

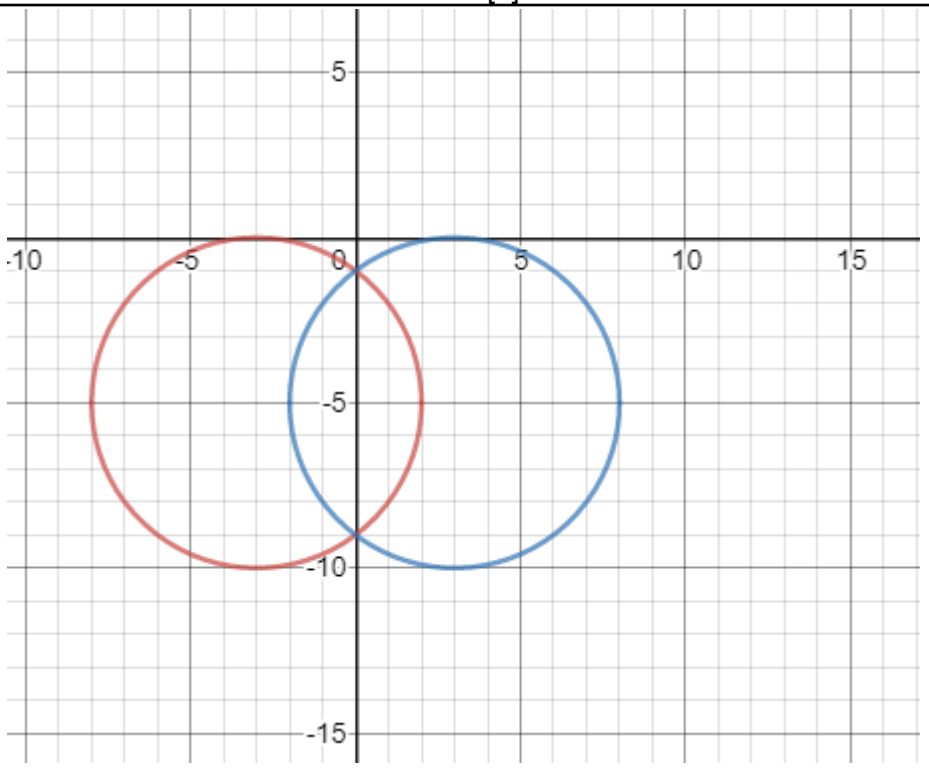
	$= \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	
5(iii)	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\alpha^3 + \beta^3 = (-2)^3 - 3(-8)(-2) = -56$ $\alpha^3\beta^3 = (\alpha\beta)^3$ $= (-8)^3 = -512$	M1 A1 B1 [3]
5(iv)	From (iii), Quadratic equation $x^2 - (-56)x + (-512) = 0$ $x^2 + 56x - 512 = 0$	B1 [1]
6(i)	$p = 30$ $q = \frac{\pi}{3}$ $r = 40$	B1 B1 B1 [3]
6(ii)		G1 P1 L1 [3]
6(iii)	$45 = 30cos\frac{\pi}{3}t + 40$ $cos\frac{\pi}{3}t = \frac{1}{6}$ $\propto = 1.403$ $\frac{\pi}{3}t = 1.403$ or 4.881 $t = 1.34$ *or* 4.66 Duration = $1.34 + (6 - 4.66) = 2.68$ sec	M1 A1 A1 A1 A1 A1 [6]
7(i)	Height of trapezium, AR is given by $sin\theta = \frac{AR}{5}$ $\therefore AR = 5sin\theta$	

	$\therefore OR = OC$ is given by $5cos\theta$ $\therefore$ Perimeter of trapezium $y = 3(5cos\theta) + 5sin\theta + 5$ $y = 15cos\theta + 5sin\theta + 5$ $y = 5(1 + sin\theta + 3cos\theta)$ (Shown)	M1 A1 A1 [3]
7(ii)	Let $5sin\theta + 15cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{(5)^2 + (15)^2}$ $R = \sqrt{250}$ $tan\alpha = \frac{15}{5}$ $\alpha = 71.6^\circ$ $y = \sqrt{250}sin(\theta + 71.6^\circ) + 5$	 B1 B1 B1 B1 [4]
7(iii)	Maximum $y = \sqrt{250} + 5 = 20.8$ cm When $sin(\theta + 71.6^\circ) = 1$ $\theta = 90^\circ - 71.6^\circ = 18.4^\circ$	B1 M1 A1 [3]
8(a)	Area of Area $A = \int_0^h 10x - x^2 dx$ $= \left[5x^2 - \frac{x^3}{3}\right]_0^h$ $= \left(5h^2 - \frac{h^3}{3}\right)$ square units. Area of Area $B = \int_h^{2h} 10x - x^2 dx$ $= \left[5x^2 - \frac{x^3}{3}\right]_h^{2h}$ $= \left(5(2h)^2 - \frac{(2h)^3}{3}\right) - \left(5(h)^2 - \frac{(h)^3}{3}\right)$ $= \left(15h^2 - \frac{7h^3}{3}\right)$ square units $\left(15h^2 - \frac{7h^3}{3}\right) = 2\left(5h^2 - \frac{h^3}{3}\right)$ $5h^2 - \frac{5h^3}{3} = 0$ $5h^2\left(1 - \frac{1}{3}h\right) = 0$ $h = 3$	M1 A1 M1 A1 A1 [5]

[6]

8(b)(i)	 $y = \lvert x^2 + x - 12 \rvert$	Max pt (B1) G1 Turning pt (B1) [3]
8(b)(ii)	Area $= \int_{-2}^{3} \lvert x^2 + x - 12 \rvert dx$ Area $= \left[\frac{x^3}{3} + \frac{x^2}{2} - 12x\right]_{-2}^{3}$ Area $= \left\lvert \left[\frac{(3)^3}{3} + \frac{(3)^2}{2} - 12(3)\right] - \left[\frac{(-2)^3}{3} + \frac{(-2)^2}{2} - 12(-2)\right] \right\rvert$ Area $= \left\lvert \left[-\frac{45}{2}\right] - \left[\frac{70}{3}\right] \right\rvert = \frac{275}{6} = 45.8$ square units	M1 A1 A1 A1 [4]
9(i)	Since AB is the diameter, centre C will be the midpoint of AB. The midpoint of AB is $\left(\frac{7+(-1)}{2}, \frac{-2-8}{2}\right) = (3, -5)$ The centre of the circle is $(3, -5)$. Radius of the circle will be C or BC, $r = \sqrt{(7-3)^2 + (-2+5)^2} = 5$ units	M1 A1 For each coordinate [M1 A1 [4]
9(ii)	Hence, the equation of the circle is $(x-3)^2 + (y+5)^2 = 25$ Standard form or general form $x^2 - 6x + 9 + y^2 + 10y + 25 = 25$ $x^2 + y^2 - 6x + 10y + 9 = 0$	M1 A1 [2]

[7]

9(iii)	 Since the circle is reflected in the y-axis, only the x-coordinate of the centre needs to be changed from positive to negative value. So, the new coordinates of the centre are $C(-3,-5)$. Hence, the equation of the circle is $(x+3)^2+(y+5)^2=25$	B1 [1]
9(iv)	Diagree The highest point on the circle occurs at 5 units above the centre with the x-coordinate to be 3. Hence, the coordinates of H are $(3, 0)$	A1 A1 [2]
10(i)	Let $\frac{10x+1}{(x-2)^2(x+1)}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x+1)}$ $10x+1=A(x+1)(x-2)+B(x+1)+C(x-2)^2$ Let $x=-1$, $10(-1)+1=C(9)$ $C=-1$ Let $x=2$, $10(2)+1=3B$ $B=7$ Let $x=0$, $10(0)+1=-2A+B+4C$ $1=-2A+7-4$ $2A=2$ $A=1$ Hence $\frac{10x+1}{(x-2)^2(x+1)}=\frac{1}{(x-2)}+\frac{7}{(x-2)^2}-\frac{1}{(x+1)}$	M1 A1 A1 A1 A1 [5]
10(ii)	$\int_3^5 \frac{10x+1}{(x-2)^2(x+1)}d=\int_3^5 \frac{1}{(x-2)}dx+\int_3^5 \frac{7}{(x-2)^2}dx-\int_3^5 \frac{1}{(x+1)}dx=[ln(x-2$	

[8]

	$= ln2 + \frac{14}{3}$	M1 A1, A1, A1 A1 [5]
11(i)	$f(x) = x^3 - x^2 + px + q$ $f(-3) = (-3)^3 - (-3)^2 + p(-3) + q = 0$ $-36 - 3p + q = 0$ – Equation (1) $f(2) = (2)^3 - (2)^2 + p(2) + q = -10$ $14 + 2p + q = 0$ – Equation (2) Equation (2) $-$ (1) $50 + 5p = 0$ $p = -10$ $q = 6$	M1 A1 A1 A1 [4]
11(ii)	$f(x) = x^3 - x^2 - 10x + 6$ $f(x) = (x + 3)\left(x^2 - 4x + 2\right)$ $f(x) = 0$ $x = -3$ or $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2}$ $x = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$ $a = 2,\ b = 1$	M1 A1 A1 [3]
11(iii)	$f(x) = 2x + 6$ $x^3 - x^2 - 10x + 6 = 2x + 6$ $x^3 - x^2 - 12x = 0$ $x\left(x^2 - x - 12\right) = 0$ $x(x + 3)(x - 4) = 0$ $\therefore x = 0,\ x = -3,\ x = 4$	M1 A1 A1 [3]