Exponent rules and prerequisites

Name: _____ Date: _____ Class: _____

Today I am learning to rewrite exponential functions and describe their transformations.

So I can recognize different equations that represent the same graph.

I'll know I have learned it when I can justify equivalent forms, graph transformations and interpret a model.

Essential question: How can changing an exponent produce the same effect as changing a coefficient or base?

Think of a population that doubles each day: moving the starting day changes the initial amount, not the doubling factor.

For positive bases, use these rules: product of like bases: add exponents; power of a power: multiply exponents; negative exponent: take the reciprocal; power of a product or quotient: apply the power to each factor; a reciprocal integer exponent represents a root.

1. I DO · Simplify $2^2 \cdot 2^3$ and $(2^2)^3$.

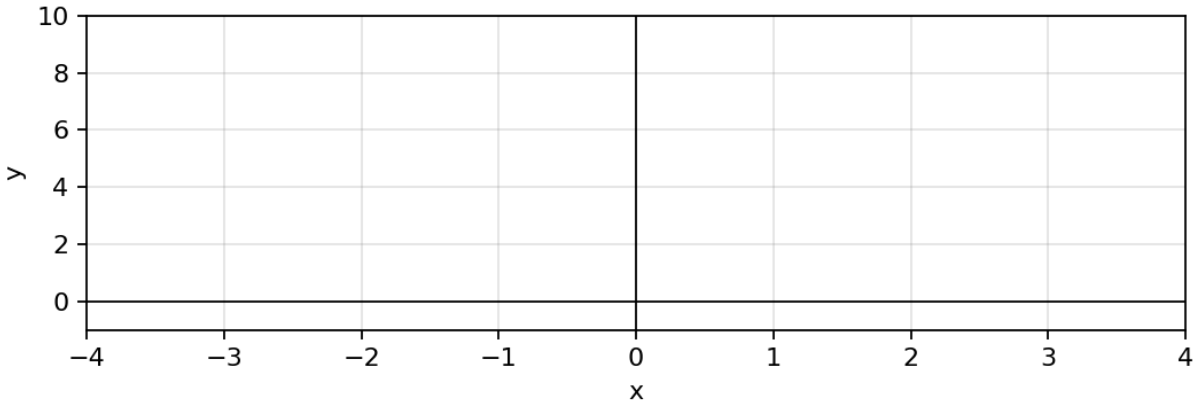
2. WE DO · 3^{-2} = _____; $(2 \cdot 5)^3$ = _____; $\sqrt[3]{27}$ = _____.

3. YOU DO · Simplify $(\frac{6}{3})^2$; evaluate $5^{2.3}$, $3^{\sqrt{7}}$, and $6^{-1.5}$ to three decimals.

4. CHECK · Correct the claim $2^3 \cdot 2^4 = 2^{12}$. What does 7^0 equal?

Parent graphs and reflections

For $f(x) = b^x$, $b > 0$ and $b \neq 1$: domain is all real numbers; range is positive real numbers; y-intercept is (0, 1); horizontal asymptote is $y = 0$. Both growth and decay parent graphs are continuous, concave up, and one-to-one. Growth has $b > 1$; decay has $0 < b < 1$.



5. I DO · For $f(x) = 2^x$, evaluate at $x = -2, -1, 0, 1, 2$.

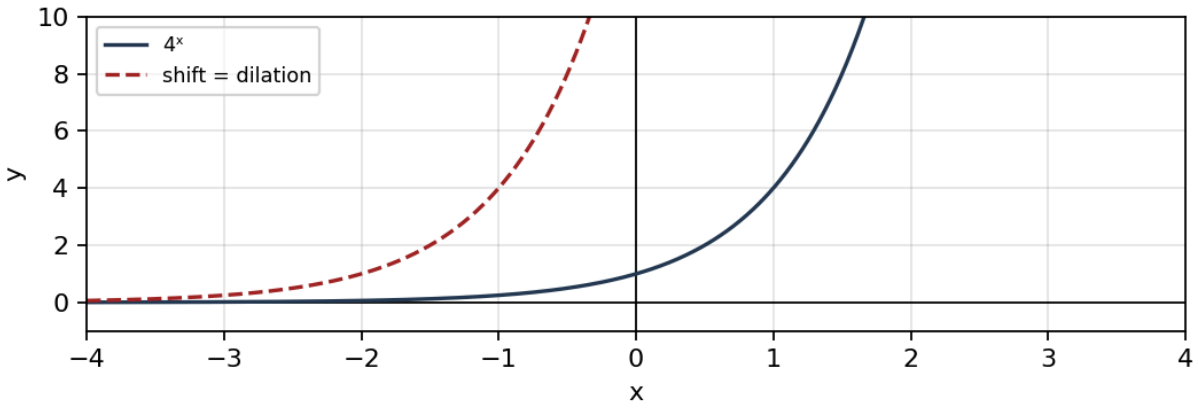
6. WE DO · Complete the same five values for $g(x) = 2^{-x}$. Graph both.

7. YOU DO · Describe the domain, range and asymptote of g. Is g increasing or decreasing?

8. CHECK · Both graphs intersect at _____. Why can neither cross the x-axis?

Horizontal translations = vertical dilations

A horizontal translation changes inputs. A vertical dilation multiplies outputs. For $b > 0$, $b \neq 1$: $b^{x+h} = b^h \cdot b^x$. Thus a left shift of h units (h positive) is a vertical dilation by b^h. A right shift uses a negative exponent. A coefficient already outside the power must also be included.



9. I DO · Rewrite 4^{x+2} as $a \cdot b^x$.
10. WE DO · Rewrite $3^{x-2} = 3^x \cdot$ _____ = _____.
11. YOU DO · Rewrite $4 \cdot 3^{x+2}$. Then describe both transformations.
12. CHECK · If $f(x) = 3^x$, describe $27f(x)$ and $f(x)/9$ using horizontal shifts.

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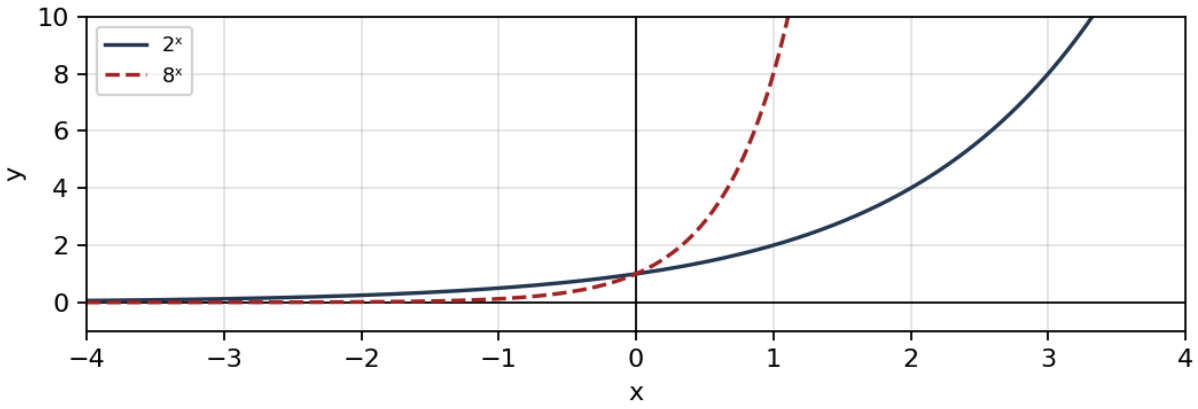
Horizontal dilations = change of base

For $c \neq 0$, $b^{cx} = (b^c)^x$. The horizontal scale factor is $1/|c|$. When $|c| > 1$ the graph compresses; when $0 < |c| < 1$ it stretches. Negative c also reflects across the y-axis. The parent y-intercept stays $(0, 1)$.



13. I DO · Rewrite 2^{3x}.

14. WE DO · Rewrite $9^{\frac{x}{2}}$ = (_____)x = _____.

15. YOU DO · Let f(x) = 5x. Rewrite $(f(x))^{-3}$ and describe transformations.

16. CHECK · Rewrite 2^{-2x} as a single-base power. Does the y-intercept change?

AP PRECALCULUS / TOPIC 2.4 / GUIDED NOTES

Fractional exponents and exact values

A fractional exponent is a root followed by a power: $b^{\frac{m}{n}} = (\sqrt[n]{b})^m$. A negative fractional exponent also requires a reciprocal. Keep exact radicals unless the question asks for a decimal.

17. I DO · $h(x) = 2 \cdot 5^{\frac{x}{2}}$. Find h(1).

18. WE DO · $h(x) = 6 \cdot 2^{\frac{x}{3}}$. Find h(−2).

19. YOU DO · $h(x) = 4 \cdot 4^{\frac{x}{5}}$. Find $h(2)$; $k(x) = 7 \cdot 2^{\frac{x}{3}}$. Find $k(-2)$.

20. CHECK · Is $16^{\frac{1}{2}}$ equal to 8? Explain. Find $27^{\frac{-2}{3}}$.

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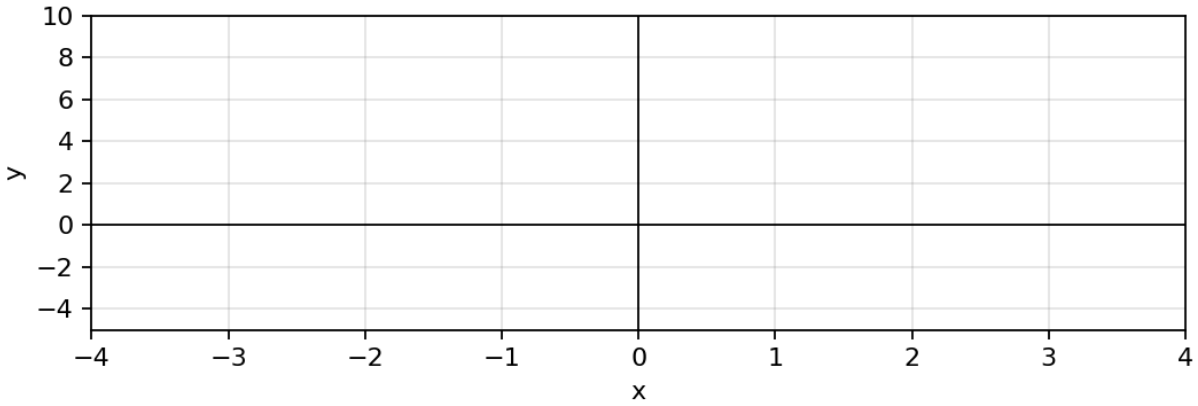
Combined transformations

For $g(x) = A \cdot b^{c(x-h)} + k$, the horizontal asymptote is $y = k$, vertical scale is $|A|$, horizontal scale is $1/|c|$, and shifts are right h and up k (negative values reverse direction). A negative A reflects across the x-axis; negative c reflects across the y-axis. Evaluate $g(0)$ for the y-intercept.



21. I DO · Graph -2^x and $-2^x + 3$. Label intercepts and asymptotes.

22. WE DO · For $g(x) = (1/3)^{x-4} + 2$, identify shifts, asymptote and intercept.

23. YOU DO · Describe $y = -(\frac{1}{2}) \cdot (\frac{1}{2})^x$ relative to 2^x; state range and intercept.

24. CHECK · For $f(x) = 4^x$, write $f(x-3)$, $f(x)+5$, $f(-2x)$, and $-3f(x+1)-2$ as equations.

Practice · Levels 1–3

Show your reasoning. Use exponent rules rather than relying only on a calculator. The following questions are original practice modeled on the source materials.

25. Level 1 · $f(x) = 4^x$. Describe $16f(x)$, $f(x)/16$ and $-3f(x)$.
26. Level 1 · Rewrite 16^x using base 2; rewrite 2^x using base 16.
27. Level 2 · $f(x) = 4^x$. Describe $g(x) = -16^x$.
28. Level 2 · MCQ: Which equals $9 \cdot 4^x$? A: $3 \cdot 16^{\frac{x}{2}}$ B: $3 \cdot 16^{2x}$ C: $9 \cdot 16^{\frac{x}{2}}$ D: $9 \cdot 16^{2x}$.
29. Level 3 · Explain why 2^{x+3} and $8 \cdot 2^x$ coincide for every real x. Why is 2^{x+3} not equivalent?

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AP-style modeling · Calculator active

A pond is covered by algae. Its area (in hundreds of square meters) is recorded below. Assume $A(t) = a \cdot b^t$, where t is days. A model describes exponential change; average rate of change describes a secant slope over an interval.

Days	0	1	2
Area (hundreds m^2)	5	4	3.2
30a. Write two equations using the table.			

30b. Find a and b and write the model.

30c. Find the average rate of change from day 0 to day 2.

30d. Interpret the rate in context.

30e. Use that average rate to estimate the area on day 3 in square meters. Compare with the exponential model.

30f. Is this growth or decay? Justify.

Lesson summary and exit ticket

WHAT I LEARNED TODAY

A horizontal translation is equivalent to a vertical _____. A horizontal dilation is equivalent to a change of _____. A negative exponent indicates a _____. A vertical translation changes the horizontal _____.

31. Basic · Rewrite 2^{x-4} as $a \cdot b^x$.

32. Application · Rewrite $27^{\frac{x}{3}}$ and describe the horizontal scale.

33. Reasoning · A student says -2^x and 2^{-x} have the same graph. Explain the mistake using intercepts and reflection axes.