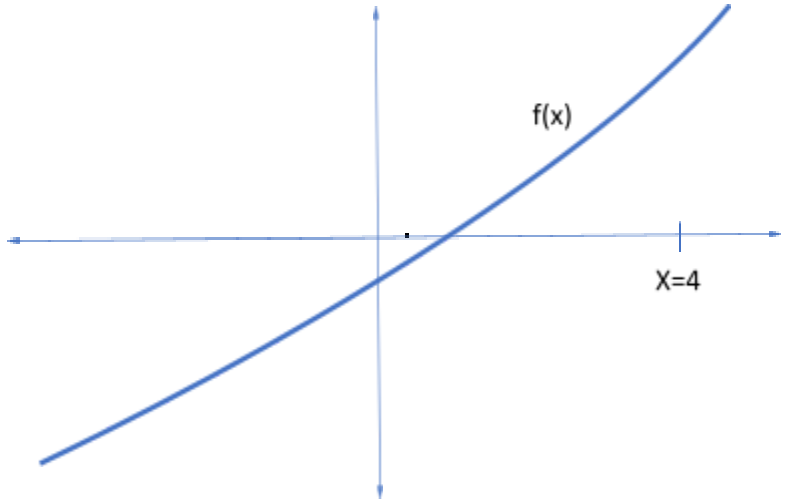
Identifying Domains of Composite Functions

- We must first know what it means to find the "domain" of a function.
- The domain of a function is all possible x values.
- On a (x,y) coordinate plane, the horizontal axis is the x-axis. These are values that are independent of y-axis.

EXAMPLE:



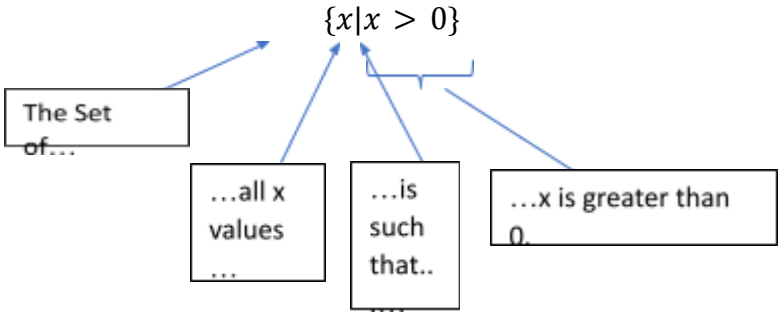
The domain of the graph f(x) is: **all of the possible values** between x = 0 and 4.

We can indicate this in "interval notation", Domain = [0, 4]. In our case, we use brackets because we are including the values x = 0 and x = 4. If those two values were not included, then we would write Domain = (0, 4).

Interval notation recap:

1. It looks like an ordered pair
2. We use Brackets $[x_1, x_2]$ to indicate values are included
3. We use Parenthesis (x_1, x_2) to indicate values are not included
4. Also, we can use both brackets and parenthesis at the same time to indicate that while one value is included in the domain, the other is not.
    a. $(x_1, x_2]$ – is indicating x_1 is not part of the domain, while all values in between x_1 *and* x_2 are part of the domain. Further, the value of x_2 is included in the domain.

Another way we can indicate what the domain of a function is, is by using "set-builder notation". An example of set builder notation, along with explanation is located below:



Set Builder Notation Recap:

1. The brackets indicated the variable we are dealing with
2. The bar in between the x's means "such that"
3. The final part of the notation is the argument for the domain
4. Verbally, the notation sounds like "The set of all values of x is such that x is greater than 0."

- In order to identify the domain of a composite function, f(g(x)), we must first identify the "inside function" and the "outside function".
- In the case of f(g(x)), g(x) is the inside function, and f(x) is the outside function.
- We must then determine what the domain of the inside function is, (g(x)).
- Once we determine the domain of the inside function, we must then determine if the domain of g(x) invokes any restrictions on the outside function, (f(x)).

Example:

$h(x) = \frac{1}{\sqrt{x}}$ -For this problem, we must identify an inside function, and an outside function.

-Call the inside function g(x), and the outside function f(x).

Let:

$f(x) = \frac{1}{x}$, and $g(x) = \sqrt{x}$ -This allows me to say f(g(x)) = h(x)

Now:

Identify the domains of f(x) and g(x) separately.

The domain for g(x), the inside function, is all values greater than or equal to 0.

Interval Notation: [0,∞). We do not include infinity, because it is not a number, but an idea.

Set Builder Notation: $\{x|x \geq 0\}$. This simply states that x is such that x is all values greater than or equal to 0.

The domain for f(x), the outside function, is all real numbers excluding 0.

Interval Notation: $(-\infty, 0) \cup (0, \infty)$. This simply means that x is all possible values excluding infinity and 0.

Set Builder Notation: $\{x|x \neq 0\}$. Which means x is such that x cannot equal 0.

Now that we have identified the domain of f(g(x)) we need to figure out if the outside function puts any domain restrictions on the inside function. We know that f(x) cannot equal 0, and we know that g(x) can never equal a negative number. From this we can infer that the domain of h(x)=f(g(x)) is all values greater than 0.

Interval Notation: (0, ∞)

Set Builder Notation: $\{x|x > 0\}$