

Video Title	Tower of Hanoi
Topic	Problem solving
Aim(s)	To explore the Hanoi Tower problem using an increasing number of blocks and try to develop a sense of how each case leads into the other. Also to recognise patterns in the problem and identify their cause.
Length	45min
Camp Location	
Facilitators	
N. of students	
Date	
Resources needed	Paper (one sheet between two students), Scissors/Ruler and somewhere to write working (blackboard, whiteboard, paper, etc.)
Preparations	Students should watch the video where the preparation required to follow the video is explained.

Video time	What facilitator does	What learners do	Comments
00:00 - 00:26	General VMC Video Introduction		
00:27 - 00:41	Video Introduction		
00:42 - 01:00	Materials required		
01:01 - 02:16	Preparing materials for the problem		
02:17 - 03:17	Problem 1 explanation: 3-Block Tower		
VIDEO PAUSE Problem 1 3-Block Tower	Allow the students to attempt the problem, providing guidance if necessary. If students complete the problem in more than the minimum of 7 moves, encourage them to try to refine their method.	Attempt to move the 3-block tower from the left to right platform following the rules outlined in the video. Try to do this in the fewest number of moves possible.	This problem is not particularly difficult once students understand the rules of the game since there are not many options for how to make each move. The most common error is for students to start by moving the smallest block to the middle platform, rather than the rightmost; if they do they might accidentally move the tower to the middle platform in 7 moves instead of the rightmost.
03:28 - 04:31	Solution to Problem 1		
04:32 - 05:00	Problem 2 explanation: 4-Block Tower		

VIDEO PAUSE Problem 2 4-Block Tower	Allow the students to attempt the problem, providing guidance if necessary. If students complete the problem in more than the minimum of 15 moves, encourage them to try to refine their method.	Attempt to move the 4-block tower from the left to right platform following the rules outlined in the video. Try to do this in the fewest number of moves possible.	With 4 blocks, there are more options for moves so the problem becomes more difficult. The hint included in the video may be useful for students at this point. If students need additional instruction, you could prompt them to focus on moving the top 3 blocks before worrying about the fourth (you will not be able to move the fourth block unless you have the other three block all on one platform, so students can use the solution to the previous problem).
05:11 - 05:21	Hint for Problem 2		
VIDEO PAUSE Problem 2 4-Block Tower (with hint)	<i>(as above)</i>	<i>(as above)</i>	<i>(as above)</i>
05:31 - 06:10	Solution to Problem 2		
06:11 - 07:36	Exploring the relationship between Problems 1 and 2		
07:37 - 07:55	Problem 3 explanation: 5-Block Tower		

VIDEO PAUSE Problem 3 5-Block Tower	Allow the students to attempt the problem, providing guidance if necessary. If students complete the problem in more than the minimum of 15 moves, encourage them to try to refine their method. Hopefully, having seen how to relate the 3-block puzzle to 4, students will work out that the smallest number of moves required is 31 without actually moving the blocks. It is up to the you if you would like them to find the moves to complete the puzzle also. <i>(see notes)</i>	Attempt to move the 5-block tower from the left to right platform following the rules outlined in the video. Try to do this in the fewest number of moves possible.	The actual moves to the solution are not included in the video however you would start by moving the smallest block to the rightmost platform, then moving the 4 smallest blocks all to the middle platform in a similar manner to Problem 2. Then, move the largest block across and “un-do” the first 15 moves (beginning by moving the smallest block to the empty left platform).
08:05 - 08:44	Solution to Problem 3		
08:45 - 09:05	Problem 4 explanation: Spotting a pattern		

VIDEO PAUSE Problem 4 Pattern?	Help students to identify a pattern in the numbers. It may be useful to prompt them to think about how we solved Problems 2 and 3 using the solutions to problems 1 and 2. In other words, we found the 4th and 5th terms in the sequence using the 3rd and 4th respectively.	Attempt to identify a pattern in the sequence of minimum moves to solve the Hanoi Tower problem for increasing numbers of blocks: 1,3,7,15,31... Use this pattern to predict the minimum number of moves to solve the problem with 6,7 or 8 blocks.	There are really a couple of solutions to this problem, depending on how the students approach it. If they think about how the numbers are generated in Problems 2 and 3, they might notice that each term is twice the previous, plus 1. The video explains how this is analogous to how Problems 2 and 3 were solved. If students just try to spot a pattern in the numbers, they might notice that each term is one less than a power of 2, or that the gaps between the numbers double each time (2,4,8, etc). These two observations are expressing the same thing, that the pattern is related to exponents of 2. Students are unlikely to spot the connection between these two approaches but should be able to follow along with the explanation in the video. They may benefit from the video being paused during this and the steps being reiterated. It is mentioned that this is a “proof by induction”; that is, one where we use an assumption about a term in a sequence to prove that the assumption is true for the next term. The video doesn’t explicitly point out the “initial case” for $n = 1$ but this is already apparent in explanation of the problem (in fact, up to $n = 5$).
	09:16 - 10:06	Solution to Problem 4 (“doubling and adding 1” method)	
	10:07 - 11:24	Showing “doubling and adding 1” is the same as “one less than a power of 2”	
	11:25 - 11:37	End of main video	

11:38 - 14:00	Construction a 3D Hanoi Tower: The video ends with a demonstration of how to make a 3D tower using a cardboard box. This can be used to start the lesson if you would like students to create a tower, or used as a final activity, or not used at all. There are plenty of other ways to create 3D towers; students could fairly easily use different sized boxes or books, or they could create a tower using paper, cardboard, wood, or any other materials they have. They should not feel restricted to using the instructions in the video; it would be great to see any ideas that they come up with. You can send pictures or videos of Hanoi Towers which students create to: contactus@virtualmathscamp.com We would love to see what you and your students create.
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