



Flipping Physics Lecture Notes:

Mechanical Energy of an Object in Orbit

An object in orbit has both gravitational potential energy and kinetic energy.

$$ME_{total} = U_g + KE = -\frac{Gm_{object}m_{planet}}{r} + \frac{1}{2}m_o v_o^2$$

In order to simplify this expression we need to identify that the only force acting on the orbital object is the force of gravity which is directed toward the center of mass of the planet. Therefore we can sum the forces in the in-direction on the orbital object.

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_o m_p}{r^2} = m_o \frac{v_o^2}{r} \Rightarrow \frac{Gm_p}{r} = v_o^2 \Rightarrow v_o = \sqrt{\frac{Gm_p}{r}} \quad (\text{velocity of orbital object})$$

(everybody brought the mass of the object divided by the radius to the party)

Substitute $v_o^2 = \frac{Gm_p}{r}$ into the ME_{total} equation:

$$ME_{total} = -\frac{Gm_o m_p}{r} + \frac{1}{2}m_o \frac{Gm_p}{r} = \frac{Gm_o m_p}{r} \left(-1 + \frac{1}{2} \right) = \frac{Gm_o m_p}{r} \left(-\frac{1}{2} \right) = \boxed{-\frac{Gm_o m_p}{2r}}$$

That's right, the total mechanical energy of an orbital object equals half the universal gravitational potential

energy between the object and the planet:

$$ME_{total} = -\frac{Gm_o m_p}{2r} = \frac{1}{2} \left(-\frac{Gm_o m_p}{r} \right) = \frac{1}{2} U_g$$

I can't help but point out this out about the escape velocity we determined in a previous lesson:

$$v_{escape} = \sqrt{\frac{2Gm_p}{r}} = \sqrt{2} \sqrt{\frac{Gm_p}{r}} = (\sqrt{2}) v_{orbit}$$

That is correct, the escape velocity equals $\sqrt{2}$ times the orbital velocity. I don't know why that is, or why it is interesting, however, it is interesting.