01 | Foundations & exponential form

Name: _____ Date: _____ Class: _____

Today I am learning to identify, evaluate, graph and interpret exponential functions.

So I can model constant percent change and distinguish it from constant additive change.

I'll know I have learned it when I can justify a model, describe its graph and interpret its parameters.

Essential question: How does multiplying repeatedly change the shape and behavior of a function?

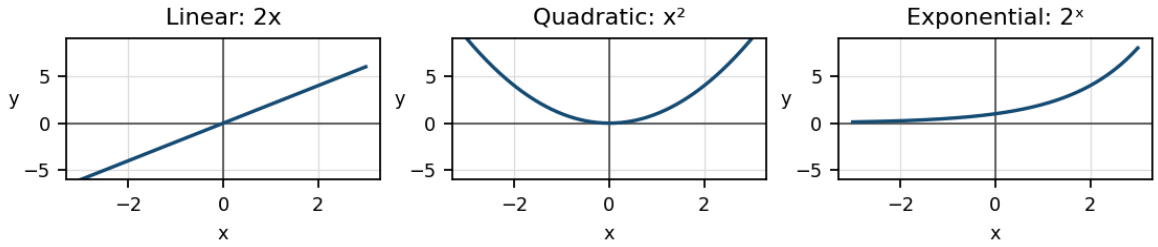
Prerequisite review: $b^0 = 1$; $b^{-1} = 1 \div b$; $b^m \cdot b^n = b^{m+n}$.

An exponential function has form $f(x) = ab^x$, where $a \neq 0$, $b > 0$ and $b \neq 1$.

a = _____ value; b = _____ factor. Domain: _____.

initial; multiplicative (growth or decay); $(-\infty, \infty)$. Since $b^0 = 1$, $f(0) = a$.

Compare the function families: linear $2x$, quadratic x^2, exponential 2^x.



1. I DO — Identify $f(x) = 3(2)^x$. The coefficient is 3, the base is 2, and x is in the exponent. $f(0) = 3$. This is exponential growth because $a > 0$ and $b > 1$.

2. WE DO — Identify a and b for $g(x) = 8(0.6)^x$. Is this growth or decay?

$a = 8$, $b = 0.6$. Decay because $a > 0$ and $0 < b < 1$.

3. YOU DO — Which are exponential in standard form: $4x^{-3}$, $5e^{-x}$, $3 \cdot 4^e$, 7^x? Explain.

$5e^{-x} = 5(1 \div e)^x$ and $7^x = 1(7)^x$. $4x^{-3}$ is a power function; $3 \cdot 4^e$ is constant.

Check 1. Why is $b = 1$ excluded? Check 2. What is $f(0)$ for $f(x) = -4(3)^x$?

1. Outputs would all equal a. 2. −4, since $3^0 = 1$.

02 | Repeated multiplication & exact values

KEY IDEA: For every real x, $f(x + 1) = b \cdot f(x)$. A step of one multiplies the previous output by the _____.

base b. This follows from $ab^{x+1} = b(ab^x)$.

4. I DO — Write $7 \cdot 3 \cdot 3 \cdot 3$ as $f(x) = ab^x$ at a particular x.

Initial value = 7; repeated factor = 3; number of factors = 3.

Therefore $f(x) = 7(3)^x$, $x = 3$, and $f(3) = 189$.

5. WE DO — Write $0.4 \cdot 6 \cdot 0.4 \cdot 0.4$ as a function and identify x.

a = _____; b = _____; x = _____; $f(x)$ = _____.

$a = 6$; $b = 0.4$; $x = 3$; $f(x) = 6(0.4)^x$. $f(3) = 0.384$.

6. YOU DO — Write $(-2) \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as an exponential function at an input.

$f(x) = -2(5)^x$, $x = 4$; $f(4) = -1250$.

7. I DO — Evaluate $f(2)$ for $f(x) = 6(3)^{x+1}$.

Substitute: $6(3)^{2+1} = 6(3)^3 = 6(27) = 162$.

A rational exponent represents a root: 8 raised to one third is $\sqrt[3]{8} = 2$.

8. WE DO — Evaluate $g(3)$ when $g(x) = 2(4)^{x-1}$.

$g(3) = 2(4)^{3-1} = 2(16) = 32$.

9. YOU DO — Evaluate $h(x) = -5(8)^x$ at $x = \frac{1}{3}$. Then evaluate $h(-1)$.

$h(\frac{1}{3}) = -5\sqrt[3]{8} = -10$. $h(-1) = -5 \times \frac{1}{8} = -\frac{5}{8}$.

Check 3. Is $2(3)^3$ equal to 6^3? Explain. Check 4. What is 4^{-2}?

3. No; 54 versus 216; the exponent applies only to 3. 4. $1 \div 16$.

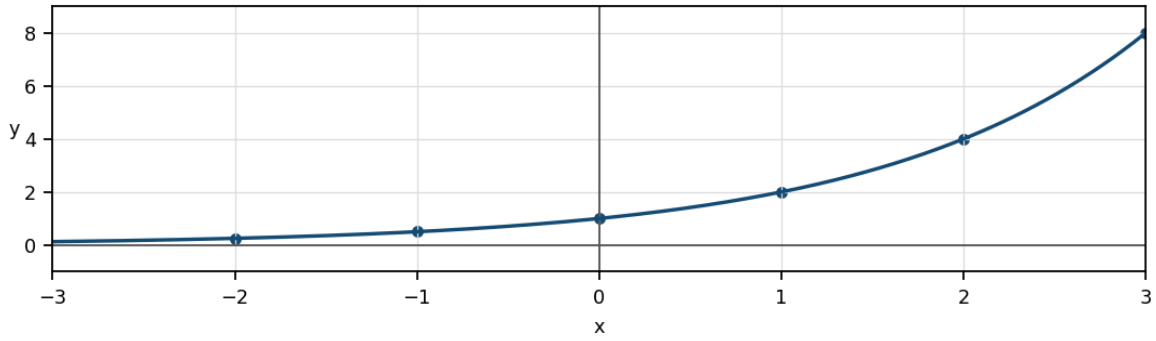
03 | Graphing a positive exponential

Graphing steps: choose inputs, evaluate the function, plot points, and draw a smooth curve. The horizontal asymptote is y = _____ for $f(x) = ab^x$.

0. A nonzero exponential never equals zero.

10. I DO — Graph $f(x) = 2^x$. Use the table and graph below.

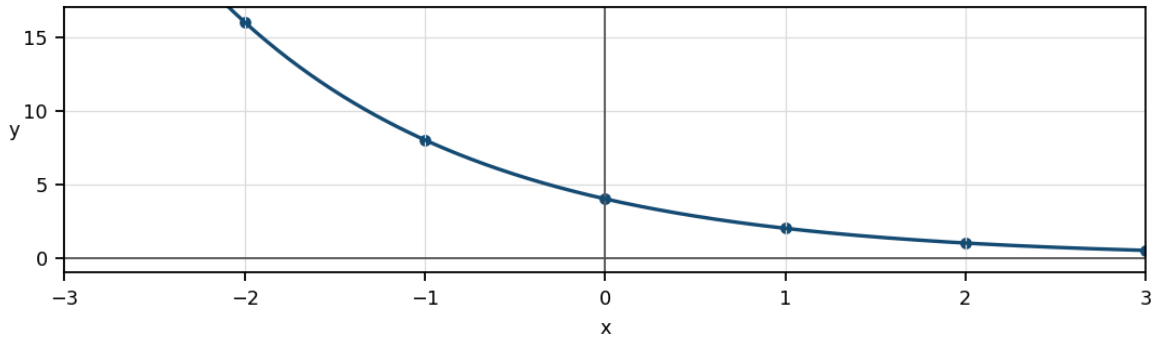
x	−2	−1	0	1	2	3
f(x)	¼	½	1	2	4	8



11. WE DO — Graph $g(x) = 4(½)^x$. Complete the table, then plot the points.

Outputs are 16, 8, 4, 2, 1, ½. Multiply by ½ each step.

x	−2	−1	0	1	2	3
g(x)	16	8	4	2	1	½



12. YOU DO — Give the domain, range, intercepts, asymptote and concavity of g.

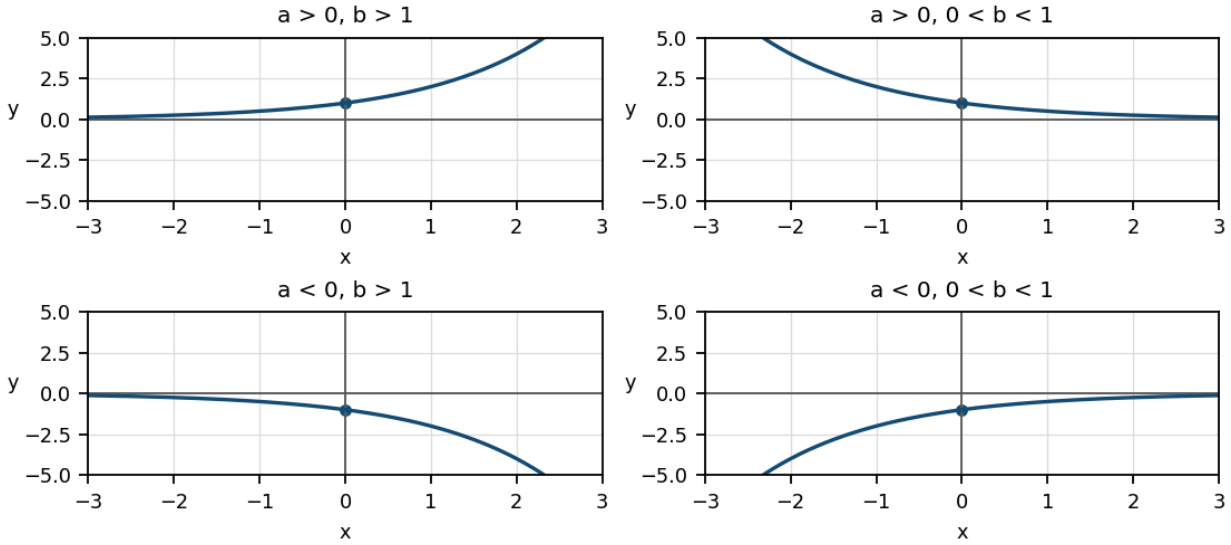
Domain $(-\infty,\infty)$; range $(0,\infty)$; y-intercept (0,4); no x-intercept; y = 0; concave up.

Check 5. Does g ever reach zero? Check 6. Why is g decreasing?

5. No, a positive base raised to a real power is positive. 6. Each unit step halves a positive output.

04 | Four graph shapes & end behavior

The sign of a determines concavity: positive a gives concave _____; negative a gives concave _____. A negative a reflects the positive graph across the x-axis.
up; down. Reflection reverses monotonicity and concavity.



Parameters	Direction	Concavity	Left end	Right end
a > 0, b > 1	Increasing	Up	0	∞
a > 0, 0 < b < 1	Decreasing	Up	∞	0
a < 0, b > 1	Decreasing	Down	0	−∞
a < 0, 0 < b < 1	Increasing	Down	−∞	0

13. I DO — $f(x) = -6(2)^x$ is decreasing, concave down, approaches 0 on the left, and decreases without bound on the right. Domain is all real numbers; range $(-\infty,0)$.

14. WE DO — Describe direction, concavity, range and both end limits of $g(x) = 5(0.9)^x$.

Decreasing; concave up; (0,∞); lim as x → −∞ is ∞, lim as x → ∞ is 0.

15. YOU DO — Describe the same features for $h(x) = -4(0.2)^x$.

Increasing; concave down; (−∞,0); left limit −∞; right limit 0.

Check 7. Are there local extrema? Check 8. Are there inflection points?

7. No, direction never changes (closed intervals can have endpoint absolute extrema). 8. No, concavity never changes.

05 | Tables, ratios & rates of change

For equal input steps of length h, the ratio of outputs is b^h. Average rate of change = change in output ÷ change in input. A pure exponential has a constant _____, not a constant difference.

ratio. For unit steps, the ratio equals b; for other steps take the appropriate root.

16. I DO — Find a model and adjacent average rates of change.

x	−1	0	1	2
f(x)	2	4	8	16

At $x = 0$, $a = 4$.
Ratios: $4 \div 2 = 8 \div 4 = 16 \div 8 = 2$. Thus $f(x) = 4(2)^x$.
Adjacent rates are 2, 4, 8 because $\triangle x = 1$.

17. WE DO — Find g(x), each adjacent ratio and each adjacent average rate of change.

x	−2	−1	0	1	2
g(x)	24	12	6	3	1.5

a = _____;
b = _____;
equation =

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a = 6; b = ½; g(x) = 6(½)ˣ. All ratios ½; rates −12, −6, −3, −1.5.

18. YOU DO — Find an exponential equation using the table. Explain the unit-step factor.

x	0	2	4	6
h(x)	5	20	80	320

Work:

Two-unit ratio = 4, so $b^2 = 4$ and b = 2 (positive base). h(x) = 5(2)ˣ. Adjacent rates: 7.5, 30, 120.

Check 9. If the ratio over 3 units is 27, what is b? Check 10. If a model starts at 12 and doubles each unit, write it.

9. $b = \sqrt[3]{27} = 3$. 10. f(x) = 12(2)ˣ.

AP PRECALCULUS / TOPIC 2.3 / TEACHER KEY

06 | Identify models & additive transformations

For equally spaced inputs: constant first differences suggest linear; constant second differences suggest quadratic; constant non-unit positive ratios suggest pure exponential. These tests show consistency with a model, not proof for all inputs.

19. I DO — Outputs 3, 5, 9, 17, 33 occur at inputs 1, 6, 11, 16, 21. First differences: 2, 4, 8, 16. Second differences: 2, 4, 8. Ratios differ. None of the three pure models fits. Subtracting 1 gives 2, 4, 8, 16, 32, a shifted exponential pattern.

20. WE DO — Classify each row (inputs 0, 1, 2, 3). Justify with differences or ratios.

A	2	5	8	11
B	1	4	9	16
C	3	9	27	81
D	2	3	7	10

Work:

A linear: first differences 3. B quadratic: differences 3,5,7; second differences 2. C exponential: ratios 3. D none: differences 1,4,3; second differences 3,−1; ratios unequal.

For $g(x) = f(x) + k$ with $f(x) = ab^x$, the graph shifts vertically by k; the asymptote becomes $y =$ _____. The ratios of g generally are not constant, but the ratios of $g(x) - k$ are constant over equal input steps.

k. Vertical shifts preserve direction and concavity.

21. I DO — $f(x) = 2^x$; $g(x) = 2^x + 3$. At $x = 0,1,2$, g gives 4,5,7; ratios 1.25 and 1.4 differ. Subtract 3 to recover 1,2,4 with ratio 2. Asymptote $y = 3$.

22. YOU DO — $g(x) = 3(2)^x - 4$. Find $g(0)$, $g(1)$, $g(2)$, the asymptote, range, and whether g has a constant output ratio.

−1, 2, 8; $y = -4$; range $(-4,\infty)$; ratios −2 and 4 differ. Adding 4 restores ratio 2.

Check 11. Does adding a constant change concavity? Check 12. If $h(x) = f(x) + 2$ is pure exponential, which has constant ratios: h or f?

11. No. 12. h only (for nonzero shift and nonconstant pure exponential).

07 | Real-world growth & decay

For positive initial amount a and decimal rate r, growth factor = 1 + r; decay factor = 1 − r. Multiply, do not add the same amount at every step.

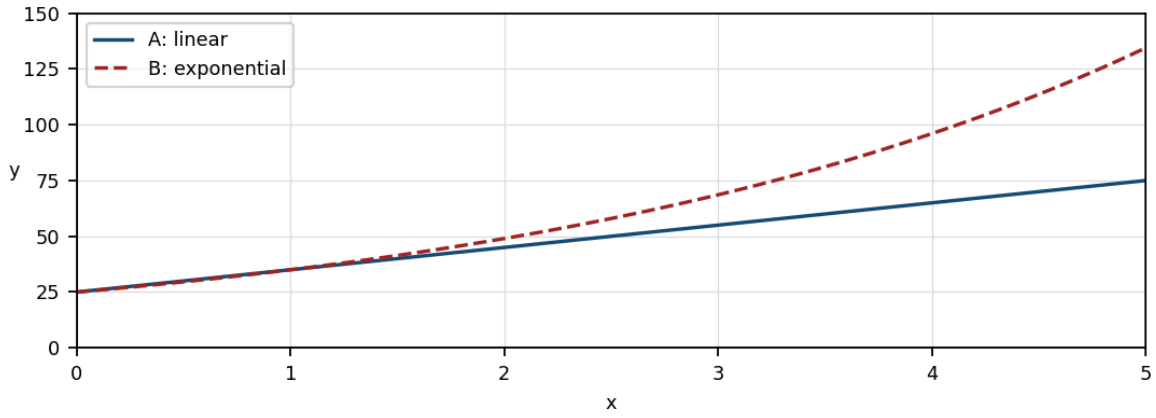
23. I DO — A population starts at 200 and increases 15% annually. $f(t) = 200(1.15)^t$. After 2 years: $200(1.15)^2 = 264.5$ (about 265 individuals).

24. WE DO — A medicine starts at 80 mg and loses 25% each hour. Write a model and find the amount after 3 hours.

$M(t) = 80(0.75)^t$; $M(3) = 80(0.421875) = 33.75$ mg.

25. YOU DO — Compare two clubs. A starts with 25 members and adds 10 each year. B starts with 25 and increases by 40% each year. Complete the table, graph and compare.

Year	0	1	2	3	4	5
A	25	35	45	55	65	75
B	25	35	49	68.6	96.04	134.456



Write both equations; identify years with equal modeled membership, domain and range for years 0–5 (whole years), and maximum modeled membership.

$A(t) = 25 + 10t$; $B(t) = 25(1.4)^t$. Equal at t = 0,1. Domain {0,1,2,3,4,5}; ranges are respective table outputs. Maxima: 75 and 134.456 (about 134). The model allows fractional estimates; actual counts are whole numbers.

Check 13. Which model has constant differences? Check 14. Which has constant ratios?

13. A. 14. B.

08 | Practice: Levels 1 & 2

26. Level 1 — Classify as growth or decay and identify a and b: (a) $9(1.2)^x$; (b) $8(0.35)^x$; (c) $0.5(3)^x$; (d) $12(\frac{3}{4})^x$.

(a) a=9, b=1.2 growth; (b) a=8, b=0.35 decay; (c) a=0.5, b=3 growth; (d) a=12, b=¾ decay. Compare positive b with 1.

27. Level 1 — Write a function and input for each product: (a) $2 \cdot 2 \cdot 2 \cdot 9$; (b) $0.3 \cdot 0.3 \cdot (-5) \cdot 0.3$.

(a) $f(x)=9(2)^x$, x=3, output 72. (b) $g(x)=-5(0.3)^x$, x=3, output −0.135.

28. Level 2 — Find the exact values: (a) $3(2)^{x+2}$ at x = 1; (b) $-4(27)^x$ at $x = \frac{1}{3}$; (c) $6(2)^x$ at x = −2.

(a) $3(2)^3=24$; (b) $-4\sqrt[3]{27}=-12$; (c) $6 \times \frac{1}{4}=1.5$.

29. Level 2 — Describe direction, concavity, range and both end limits for (a) $7(4)^x$; (b) $-2(5)^x$; (c) $6(\frac{1}{4})^x$; (d) $-3(0.6)^x$.

(a) increasing, up, (0,∞), left 0/right ∞. (b) decreasing, down, (−∞,0), left 0/right −∞. (c) decreasing, up, (0,∞), left ∞/right 0. (d) increasing, down, (−∞,0), left −∞/right 0.

30. Level 2 — A device worth $500 loses 18% of its value per year. Write a model and find the value after two years.

$V(t)=500(0.82)^t$; $V(2)=500(0.6724)=\$336.20$.

09 | AP-style challenge & exit ticket

31. MCQ — At x = 0, 2, 4, 6, outputs are 80, 40, 20, 10. Which is the unit-step base?

A. 0.5 B. √2 C. √0.5 D. 2

Justify your answer.

C. $b^2 = 40 \div 80 = 0.5$, so $b = \sqrt{0.5}$.

32. FRQ — A function g has outputs 10, 16, 28, 52 at x = 0,1,2,3. It has form $g(x) = ab^x + k$. (a) Find the first differences. (b) Use their ratios to determine b. (c) Find a and k. (d) Predict g(4) and state the asymptote.

(a) 6,12,24. (b) ratios 2, so b=2 because successive differences of a shifted exponential scale by b. (c) $g(1)-g(0)=a(b-1)=6$, hence a=6; 10=6+k, k=4. (d) $g(4)=6(16)+4=100$; y=4.

WHAT I LEARNED TODAY

The initial value is _____. For pure exponentials the equal-step output _____ is constant. Positive a gives concavity _____. The horizontal asymptote of ab^x is _____; adding k shifts it to _____.

a; ratio; up; y=0; y=k.

Exit 1 — What is the difference between an exponential growth factor and a growth rate?

The factor is 1+r; r is the decimal percent increase, e.g., 1.2 versus 0.2 (20%).

Exit 2 — Find f(x) if f(0)=18 and the output triples every unit. Find f(3).

$f(x)=18(3)^x$; f(3)=18(27)=486.

Exit 3 — A student says $-2(0.5)^x$ is decreasing because its base is less than 1. Explain the error.

The negative coefficient reflects the positive decay curve; outputs increase toward 0 from below (−2,−1,−0.5).

10 | Teaching resources & video links

Suggested use: introduce vocabulary and graphs (pages 1–4), then tables and transformations (pages 5–6), models and AP practice (pages 7–9). Pause videos for students to complete WE DO items.

https://www.youtube.com/watch?v=uYxBtnRBu7g

https://www.youtube.com/watch?v=FzofPpE0ihc&t=2s

https://www.youtube.com/watch?v=iyI2aJwzPsw

Video titles and exact topic alignment could not be verified online. Preview each before assigning it to a particular question.

Teaching tip: compare positive and negative a before classifying monotonicity. Have students read the left and right end behavior separately and justify their claims using repeated multiplication.