

Text in black gives basic requirements. Text in blue gives advanced requirements. Every student at the beginning of the exam picks a favourite part out of the four below. Then a question is drawn uniformly and independently at random from each part. A student should answer the basic requirements in all questions and advanced in the question in the favourite topic.

In general, by a *sketch* of a proof we mean understanding statements of consecutive steps of the proof and a general idea how they are proved. A *full proof* means ability to reproduce a complete argument on the board, possibly with some slackness in numerical constants.

Part 1: Basics (Lectures 1, 2, 3)

1. Basic definitions: parameterized problem, XP, fixed-parameter tractability. Examples of different problems and parameterizations. $2^k \cdot (n+m)$ FPT algorithm for Vertex Cover, and any improvement to $c^k \cdot (n+m)$ for some $c < 2$. $k^{O(k)} \cdot (n+m)$ FPT algorithm for Feedback Vertex Set via branching (with proof). [Platypus 1 and 3.1-3.3]
2. Definition of kernelization. Proof that the existence of any kernelization algorithm is equivalent to the problem being FPT. Some easy examples of problems with polynomial kernels, including a quadratic kernel for Vertex Cover. Sunflower Lemma (statement or full proof) and its application for a $O(k^d)$ kernel for d-Hitting Set. [Platypus 2.1, 2.2 and 2.6]
3. Crown Decomposition Lemma (statement and sketch or full proof) and its application to give a $3k$ kernel for Vertex Cover. [Platypus 2.3]
4. Iterative compression: presentation of the technique and its applications in two of the following problems: Vertex Cover, Feedback Vertex Set, Feedback Vertex Set in Tournaments, Odd Cycle Transversal. One of these examples should be the $5^k \cdot \text{poly}(n)$ algorithm for Feedback Vertex Set. [Platypus 4, scanned notes]
5. $3^{|T|} \cdot \text{poly}(n)$ algorithm for Steiner Tree and $2^{|U|} \cdot \text{poly}(|U|+|F|)$ algorithm for Set Cover via dynamic programming on subsets. Two more examples of FPT algorithms obtained by dynamic programming on subsets. [Platypus 6.1]

Part 2: Color coding, algebraic techniques, repsets (Lectures 4, 7, 8, 11)

1. Color coding and random separation with two (three) different applications, including k-Path in time $(2e)^k \cdot \text{poly}(n)$ [Platypus 5.2.1], Subgraph Isomorphism in bounded degree graphs [Platypus 5.3] and long directed cycle in $4^k \cdot \text{poly}(n)$, [Scanned notes, paper]. Statement of derandomization through perfect hash families and universal sets [Platypus 5.6].
2. Inclusion-Exclusion principle and at least one algorithmic application [Platypus 10.1]. Definitions of zeta and Moebius transforms, Inversion formula (statement), Yates' algorithm (computing all values of zeta/Mobius transform in $2^n \cdot \text{poly}(n)$ time) [Platypus 10.2], fast cover product and fast subset convolution [Platypus 10.3], with applications to dynamic programming over tree decomposition [Platypus 11.1]. See also [scanned notes]
3. k-Path in $2^k \cdot \text{poly}(n)$ time by a reduction to detecting monomials linear in a set of variables (proof that the monomial linear in $x_1 \dots x_k$ can be detected in a polynomial which

is k -homogeneous $n^{x_1 \dots x_k}$ by a randomized algorithm in time 2^k) [[scanned notes](#)].

Remark: alternative exposition, by a direct 2^k algorithm for k -path [Platypus 10.4.1] is also permitted.

4. Definition of representative sets. Existence and computation of a small representative set using Gaussian elimination (statement or [full proof](#)) [Platypus 12.3.1]. One ([two](#)) examples of applications of parameterized algorithms (e.g. k -path in $5.19^k \cdot \text{poly}(n)$) [[scanned notes](#)], kernels for d -Hitting Set, d -Set Packing [Platypus 12.3.2, 12.3.3])

Part 3: Treewidth, cut problems and applications of LP (Lectures 5, 6, 9, 10)

1. Definition of treewidth, nice tree decompositions, example of dynamic programming over a tree decomposition, Courcelle's theorem (statement), 4-approximation algorithm for treewidth in FPT time (statement or [full proof](#)). [Platypus 7.1-7.4 and 7.6]
2. Grid minor theorem and its applications for parameterized algorithms. Treewidth and grid minors in planar graphs. Bidimensionality: applications in exact and parameterized algorithms. Baker's technique in parameterized algorithms. [Proof that the treewidth of a planar graph is linear in the radius](#). [Platypus 7.7]
3. Definition of important cuts. Upper bound on the number of important cuts of size at most k (statement and [full proof](#)). Application for a parameterized algorithm for Edge Multiway Cut in time $4^k \cdot \text{poly}(n)$. [Platypus 8.1-8.3]
4. FPT algorithm for Directed Feedback Vertex Set (sketch or [full proof](#)) [Platypus 8.5 and 8.6]
5. Nemhauser-Trotter Theorem ([proof](#)) and its application in kernelization of vertex cover [Platypus 2.5]. LP-guided branching: $4^{k-LP} \cdot \text{poly}(n)$ algorithm for vertex cover [Platypus 3.4]. Also see [[scanned notes](#)]

Part 4: Lower bounds (Lectures 12, 13, 15)

1. Basic definitions: FPT reductions, $W[1]$ - and $W[2]$ -hardness and completeness. Examples of $W[1]$ - and $W[2]$ -problems with corresponding reductions ([including examples with gadgets for choosing edges](#)). [Platypus 13.1, 13.2 and 13.6]
2. Definitions of ETH and SETH. Transferring ETH/SETH lower bounds via reductions. Examples, including non-linear parameter blow-ups. Statement of the sparsification Lemma, refutation of subexponential algorithm for 3SAT in terms of formula size (statement or [full proof](#)). [Proof that SETH implies ETH](#). [Platypus 14.1, 14.2, 14.3.1]
3. Lower bounds for W -hard problems under ETH. Hardness of Clique under ETH (statement and sketch, or [full proof](#)). [Platypus 14.4]
4. Hardness based on SETH: k -DominatingSet: $n^{k-\epsilon}$, Independent Set for graphs with given path decomposition of width p : $1.99^p \cdot \text{poly}(n)$, 2-Orthogonal Vectors: $n^{1.99}$, 1.4999-approximation of Diameter in $n^{1.999}$ time for sparse graphs ($E=O(V)$). Basic requirements: all formulations, at least one reductions; [advanced: at least 2 reductions, including Independent Set](#). [Platypus 14.5 + [scanned notes](#)]