Unusual behavior of as fed prices

Imagine you are sitting at your computer balancing rations with your favorite feed formulation program. Assume your program uses linear programming to find rations that have minimum as fed cost, and you have just calculated a least-cost balanced ration. Now, if you force a new ingredient into the ration − leaving everything else the same − and recalculate the least-cost balanced ration, the as fed price of the new ration will increase. Right? … Well, not always. There are special circumstances under which the as fed price of the new ration may actually decrease!

Computer feed formulation programs prominently display the as fed price of rations and seem to suggest that rations with low as fed prices are the most economical. Why use expensive rations, the argument goes, when cheaper rations are available? If you think about it, however, we shouldn't always be looking for rations with the lowest as fed price. For example, if you mix 1 ton of a ration with as fed price $160/ton with 1 ton of water, you will get a ration with as fed price $80/ton. This is not a better ration, only wetter, and you will have to use twice as much of it to get the same amounts of the nutrients.

Computer feed formulation programs use linear programming to find balanced rations of minimum cost. Some programs hold the as fed weight of the ration constant, and some hold the dry matter weight of the ration constant (and some do not restrict the weight at all). Puzzling results can occur when minimizing the cost of a ration balanced with constant weight. In some cases you may actually be able to lower the as fed price of a ration by forcing a feed into the ration! The answer, of course, is that by forcing in the feed you also make the ration wet enough to lower the as fed price, as shown by the following example.

A simple linear program

What can happen when finding rations with minimum as fed prices and constant weight will be illustrated by a simple linear program with two variables and no constraints other than constant dry matter weight. We assume the following feed values.

	Dry matter	As fed price	As fed amount
Feed 1	90 %	100 $/ton	X tons
Feed 2	30 %	40 $/ton	Y tons

<u>Problem A</u> **Find numbers $X \geq 0$ and $Y \geq 0$ that**

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Minimize:      Z = 100X + 40Y      (as fed cost)
Subject to:    0.9X + 0.3Y = 100   (dry matter weight)
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Problem A is solved by noting that

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z = 100X + 40Y
  = 100(100 - 0.3Y)/0.9 + 40Y
  = (10000 + 6Y)/0.9
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is minimum when Y = 0 tons and X = 100/0.9 tons. The minimum cost is Z = 100X + 40Y = 10000/0.9 $, and the as fed price is (100X + 40Y)/(X + Y) = 100 $/ton. The solution has 0 tons of Feed 2, even though Feed 2 has the smaller as fed price!

Now watch what happens when we force 10 tons of Feed 2 into Problem A.

Problem B **Find numbers X ≥ 0 and Y ≥ 0 that**

```
Minimize:      Z = 100X + 40Y
Subject to:    0.9X + 0.3Y = 100
                          Y ≥ 10
```

Problem B is solved by noting that

```
z = 100X + 40Y = (10000 + 6Y)/0.9
```

is minimum when Y = 10 tons and X = 97/0.9 tons. The minimum cost is Z = 100X + 40Y = 10060/0.9 $, and the as fed price is (100X + 40Y)/(X + Y) = 10060/106. These results are summarized below.

	Cost Z	As fed price
Problem A	**$11111.11**	**100.00 $/ton**
Problem B	**$11177.78**	**94.91 $/ton**

When 10 tons of Feed 2 are forced into the ration, the cost of the ration increases, as it should, but the as fed price of the ration decreases.

Similar results can be shown for rations balanced for minimum dry matter cost and constant as fed weight. We conclude that whether a ration is balanced with constant as fed weight or constant dry matter weight, there are special situations where forcing a feed into the ration can lower the as fed price of the ration.

Finally, when choosing which of two feeds to include in a ration, don't base your decision solely on their as fed prices unless their moisture contents are approximately equal.