Dynamic Integrate Climate Economy (DICE) Model:

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Model:

Overview: The purpose of this DICE model is to compute the social cost of carbon.

In summary, the production of the global output Y (t) requires the emissions E (t) which affect the concentration C (t) and, through the increase of the radiative forcing FRAD (t), the temperatures T (t). This increase of the average temperature reduces the global economic net output Q(t). To preserve consumption over time, the model finds the optimal value for the control rate, μ(t), decreasing emissions but coming with the costs Λ(t).

The social cost of carbon is computed along the path, as the ratio of Lagrange multipliers for incremental change in welfare with respect to emission and incremental change in welfare with respect to consumption

- ### The neoclassical fundamental model

Following the Solow Growth Model, the production output Y (t) is given by the Cobb-Douglas function:

$$Y(t) = A_{\text{TFP}}(t)\, K(t)^{\alpha}\, L(t)^{1-\alpha} \qquad (1)$$

where ATFP (t) is the technological progress or total factor of productivity (TFP), K (t) is the capital and L(t) is the labor force proportional to the population.

<u>A discrepancy is considered between Y (t) and Q(t)</u> because of the losses generated by climate damages at the global level:

$$Q(t) < Y(t). \qquad (2)$$

The net output Q(t) is defined as follows:

$$Q(t) = \Omega_{\text{climate}}(t) \cdot Y(t) \qquad (3)$$

Ω_{climate} (t) < 1, is the percentage of the output that is lost because of climate change. It is assumed that output loss ratio due to climate costs is given by:

$$\Omega_{\text{climate}}(t) = \frac{1 - \Lambda(t)}{1 + D(t)} = \Omega_D \cdot \Omega_\Lambda \qquad (4)$$

where D (t) > 0 is the climate damage function and Λ(t) are the abatement costs (the cost of reducing GHG emissions). D (t) measures the losses implied by natural disasters or production disruption, whereas Λ(t) represents for instance the investment costs required to shift from fossil fuel to clean energy sources.

Utilizing traditional macroeconomic modelling:

$$Q(t) = C(t) + I(t). \qquad (5)$$

where C (t) is the consumption, and I (t) is the investment given by:

$$I(t) = s(t)Q(t). \qquad (6)$$

Where, s(t) is saving rate. From (5) and (6):

$$\begin{aligned} C(t) &= (1-s(t))\,Q(t) \\ &= (1-s(t))\,\Omega_{\text{climate}}(t)\,A_{\text{TFP}}(t)\,K(t)^{\alpha}\,L(t)^{1-\alpha} \end{aligned} \tag{7}$$

The knowledge factor A_{TFP}(t), the capital K (t) and the labour force L(t) are defined as follows:

$$\begin{cases} A_{\text{TFP}}(t) = (1+g_A(t))\,A_{\text{TFP}}(t-1) \\ K(t) = (1-\delta_K)K(t-1) + I(t) \\ L(t) = (1+g_L(t))\,L(t-1) \end{cases} \tag{8}$$

where $\mathbf{g_A}$ **(t**) is the growth rate of the technological change, **δ** is the rate of depreciation of capital stock and $\mathbf{g_L}$ **(t)** is the time-varying growth of the population. It is assumed that:

$$g_A(t) = \frac{1}{1+\delta_{g_A}} g_A(t-1) \; ; \; g_L(t) = \frac{1}{1+\delta_{g_L}} g_L(t-1) \tag{9}$$

where δ_{gA} and δ_{gL} are respectively the decline rate of TFP and labor growth.

- Endogenous costs of climate change:

Output loss ratio due to climate costs is given by (4) as:

$$\Omega_{\text{climate}}(t) = \frac{1-\Lambda(t)}{1+D(t)} = \Omega_D \cdot \Omega_\Lambda$$

Climate damages D (t) are represented with a quadratic function of the atmospheric temperature T_{AT} such as:

$$D(t) = a_1 T_{\text{AT}}(t) + a_2 T_{\text{AT}}(t)^2 \tag{10}$$

where T_{AT} (t) is the atmospheric temperature a_1 and a_2 are scale parameters.

Ω_D (t) = $(1 + D(t))^{-1}$ = fraction of output, most commonly GDP in global macroeconomic models, lost because of the increase in temperatures.

The cost of reduction of greenhouse gas emissions, abatement or mitigation costs, are modeled as follows:

$$\Lambda(t) = b_1\mu(t)^{b2} \tag{11}$$

where μ(t) is the endogenous control rate or feedback parameter fixed by an optimization process and b1 and b2 scales and nonlinearity parameters.

Combining Cost (10) and Damage (11) functions:

$$\begin{aligned}\Omega_{\text{climate}}(t) &= \Omega_D \cdot \Omega_\Lambda \\ &= \frac{1}{1+D(t)} \cdot (1-\Lambda(t)) \\ &= \frac{1-b_1\mu(t)^{b_2}}{1+\theta_1 \mathcal{T}_{\text{AT}}(t)+\theta_2 \mathcal{T}_{\text{AT}}(t)^2}\end{aligned} \qquad (12)$$

- Geophysical climate module:

To link between the production of representative goods, the implied increase of GHG concentration in the atmosphere and climate change.

The total emission of GHG $\mathcal{E}(t)$ implied by the production Y (t) at time t follows:

$$\mathcal{E}(t) = (1-\mu(t))\sigma(t)Y(t) + \mathcal{E}_{\text{Land}}(t) \qquad (13)$$

where mitigation policies are translated by the control rate μ(t), $\mathcal{E}_{\text{Land}}$ (t) represents exogenous land-use emissions, and σ (t) is the uncontrolled ratio of GHG emissions to output.

$$\begin{aligned}\sigma(t) &= (1+g_\sigma(t))\sigma(t-1) \\ g_\sigma(t) &= \frac{1}{1+\delta_\sigma} g_\sigma(t-1)\end{aligned} \qquad (14)$$

The DICE uses a three-reservoir model comprising the atmosphere AT, the upper oceans UP and deep ocean LO that can be considered as an infinite sink for carbon. The set of geothermic layers on which this model is based is consequently LC = {AT,UP,LO}. The concentrations between layers follow:

$$\begin{cases}\mathcal{C}_{\text{AT}}(t) = \xi_{1,1}\mathcal{C}_{\text{AT}}(t-1) + \xi_{2,1}\mathcal{C}_{\text{UP}}(t-1) + \mathcal{E}(t) \\ \mathcal{C}_{\text{UP}}(t) = \xi_{1,2}\mathcal{C}_{\text{AT}}(t-1) + \xi_{2,2}\mathcal{C}_{\text{UP}}(t-1) + \xi_{3,2}\mathcal{C}_{\text{LO}}(t-1) \\ \mathcal{C}_{\text{LO}}(t) = \xi_{2,3}\mathcal{C}_{\text{UP}}(t-1) + \xi_{3,3}\mathcal{C}_{\text{LO}}(t-1)\end{cases} \qquad (15)$$

where $\xi_{i,j}$ represents the flow parameters between reservoirs over the step Δ considered.

Let $C = (C_{AT}, C_{UP}, C_{LO}) \in R^3$, the problem becomes:

$$\mathcal{C}(t) = \Phi_{\mathcal{C},\Delta}\mathcal{C}(t-1) + B_{\mathcal{C},\Delta}\mathcal{E}(t) \qquad (16)$$

where the matrices $\Phi_{C,\Delta}$ and $B_{C,\Delta}$ are defied as:

$$\Phi_{\mathcal{C},\Delta} = \begin{bmatrix}\xi_{1,1} & \xi_{1,2} & 0 \\ \xi_{2,1} & \xi_{2,2} & \xi_{3,2} \\ 0 & \xi_{3,2} & \xi_{3,3}\end{bmatrix} \quad \text{and} \quad B_{\mathcal{C},\Delta} = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix} \qquad (17)$$

- GHG and climate change:

The next step consists in linking accumulated GHG and climate change. The relationship between the GHG accumulation and the increase in radiative forcing (Change in energy flux caused by a driver and is calculated at the tropopause or at the top of the atmosphere) F_{RAD} (t) arises from empirical measurements and climate models.

$$\begin{aligned}\mathcal{F}_{\mathrm{RAD}}(t) &= \eta \ln_2\left(\frac{\mathcal{C}_{\mathrm{AT}}(t)}{\mathcal{C}_{\mathrm{AT}}(1750)}\right) + \mathcal{F}_{\mathrm{EX}}(t) \\ &= \eta \ln_2\left(\mathcal{C}_{\mathrm{AT}}(t)\right) - \eta \ln_2\left(\mathcal{C}_{\mathrm{AT}}(1750)\right) + \mathcal{F}_{\mathrm{EX}}(t)\end{aligned} \tag{18}$$

where η is the radiative force equilibrium obtained for carbon doubling.

In this model, the climate system for temperatures is characterized by a multilayer system comprising the atmosphere and the mixed layer. The simplified temperature module is therefore represented by a two-layer system:

$$\begin{cases} \mathcal{T}_{\mathrm{AT}}(t) = \mathcal{T}_{\mathrm{AT}}(t-1) + \left(\frac{1}{C_{\mathrm{AT}}}\right) \cdot \left(\mathcal{F}_{\mathrm{RAD}}(t) - \lambda \mathcal{T}_{\mathrm{AT}}(t-1) - \gamma\left(\mathcal{T}_{\mathrm{AT}}(t-1) - \mathcal{T}_{\mathrm{LO}}(t-1)\right)\right) \\ \mathcal{T}_{\mathrm{LO}}(t) = \mathcal{T}_{\mathrm{LO}}(t-1) + \left(\frac{1}{C_{\mathrm{LO}}}\right) \cdot \gamma\left(\mathcal{T}_{\mathrm{AT}}(t-1) - \mathcal{T}_{\mathrm{LO}}(t-1)\right) \end{cases} \tag{19}$$

where T_{AT} (t) and T_{LO} (t) = the temperatures of the atmospheric and near surface layers at time t,

F_{RAD} = the radiative forcing relative to the GHG concentration,

C_{AT} and C_{LO} = the thermal capacity of the two layers,

γ = the heat exchange coefficient and λ = climate feedback parameter.

This equation can be rewritten with $T = (T_{AT}, T_{LO}) \in R^2$ as:

$$\mathcal{T}(t) = \Phi_{\mathcal{T},\Delta}\mathcal{T}(t-1) + B_{\mathcal{T},\Delta}\mathcal{F}_{\mathrm{RAD}}(t)$$

$$\text{where: } \Phi_{\mathcal{T},\Delta} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} \\ \phi_{2,1} & \phi_{2,2} \end{bmatrix}, \quad B_{\mathcal{T},\Delta} = \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix}$$
$$\text{and:}$$

$$\phi_{1,1} = 1 - \frac{\Delta}{C_{\mathrm{AT}}}(\lambda + \gamma), \quad \phi_{1,2} = \frac{\gamma\Delta}{C_{\mathrm{AT}}}, \quad \phi_{2,1} = \frac{\gamma\Delta}{C_{\mathrm{LO}}}, \quad \phi_{2,2} = 1 - \frac{\gamma\Delta}{C_{\mathrm{LO}}}, \quad \xi_1 = \frac{\Delta}{C_{\mathrm{AT}}} \tag{20}$$

Optimization:

In this model, the constant relative risk aversion utility function of the social planner is:

$$u(c(t), L(t)) = L(t) \cdot \frac{(c(t))^{1-\theta} - 1}{1-\theta} \tag{21}$$

where θ is the measure of social valuation of different levels of consumption or rates of inequality aversion.

Let W be the inter-temporal social welfare to maximize as a function of the control rate μ(t) and the saving rate s (t):

$$W(\mu(t), s(t)) = \sum_{t=1}^{T} \frac{u(c(t), L(t))}{(1+\rho)^t} \qquad (22)$$

where the arguments {μ(t) , s (t)} represent the optimal control rate and saving rate maximizing the inter-temporal welfare.

The other variables being endogenous of the system of equations. The social planner has no power over the exogenous population growth, productivity factor or capital, so the only degree of freedom are these two parameters. The optimal pathways are derived by maximizing the social welfare at each step:

$$W^{\star} = \max_{\mu(t), s(t)} W(\mu(t), s(t)) \qquad (23)$$

For a set of constraints, fixing for example the maximum temperature or the growth constraint for the mitigation ratio, the following optimization problem is defined:

$$\begin{aligned}
\{\mu^{\star}(t), s^{\star}(t)\} \quad &= \quad \arg\max \sum_{t=0}^{T} \frac{u(c(t), L(t))}{(1+\rho)^t} \\
\text{s.t.} \quad & \left\{ \begin{array}{l}
A_{\mathrm{TFP}}(t) = (1 + g_A(t)) A_{\mathrm{TFP}}(t-1) \\
K(t) = (1-\delta_K) K(t-1) + I(t) \\
L(t) = (1 + g_L(t)) L(t-1) \\
\mathcal{E}(t) = (1-\mu(t)) \sigma(t) Y(t) + \mathcal{E}_{\mathrm{Land}}(t) \\
\mathcal{C}(t) = \Phi_{C,\Delta} \mathcal{C}(t-1) + B_{C,\Delta} \mathcal{E}(t) \\
\mathcal{F}_{\mathrm{RAD}}(t) = \eta \log_2(\mathcal{C}_{\mathrm{AT}}(t)) - \eta \log_2(\mathcal{C}_{\mathrm{AT}}(1750)) + \mathcal{F}_{\mathrm{EX}}(t) \\
\mathcal{T}(t) = \Phi_{\mathcal{T},\Delta}(t-1) + B_{\mathcal{T},\Delta} \mathcal{F}_{\mathrm{RAD}}(t) \\
C(t) = (1 - s(t)) \Omega_{\mathrm{climate}}(t) A_{\mathrm{TFP}}(t) K(t)^{\alpha} L(t)^{1-\alpha}
\end{array} \right.
\end{aligned}$$

(24)

where μ (t) ∈ [0, 1] and s*(t) ∈ [0, 1] and potentially subject to a set of constraint.