ARKANSAS MATHEMATICS STANDARDS Support Document

Grades 6-8

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GRADE 6 MATH STANDARDS & NOTES

Number Concepts & Computations

Rational Numbers

Students use fractions, decimals, integers, and absolute values to represent real-world situations.

6.NCC.1: Explain positive and negative integers as being opposite values or directions and the meaning of 0 in a real-world context.

Teacher Notes:

- Learning focuses on understanding rational numbers in the real number system. Rational numbers focus on negative numbers and their relationship to positive numbers in real-world contexts. To support conceptual understanding, use horizontal and vertical number lines to illustrate the positive and negative integers in context.
- Learning experiences build an understanding that the value of zero indicates a specific location on the number line, that zero is its own opposite, and that the meaning of zero is determined by the real-world context.
- This standard connects with 6.NCC.2.

Examples:

- Temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.

Glossary: integer, number line, conceptual understanding, rational number, real number system

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6.NCC.2: Find and plot rational numbers on horizontal and vertical number lines in real-world and mathematical problems.

Teacher Notes:

- Learning focuses on negative numbers and their relationship to positive numbers and absolute value. The emphasis of the standard connects with 6.NCC.1 focusing on number line usage. The learning in this standard helps students move from number lines to coordinate grids.
- Rational numbers include decimals, fractions, and integers.

Glossary: plot, rational numbers, number line, absolute value, fraction, integer

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6.NCC.3: Compare rational numbers, using inequalities ($<$, $>$, $\leq$, $\geq$, $\neq$) and order on a number line.

Teacher Notes:

- Learning focuses on negative numbers and their relationship to positive numbers, understanding the ordering and absolute value of rational numbers.
- Learning experiences should support mathematical rigor by understanding, comparing, and interpreting rational numbers in real-world contexts.
- To support conceptual understanding of inequalities and ordering numbers, use various models such as horizontal and vertical number lines, thermometers, checkbooks, sea level, etc.

Examples:

- Interpret $-4 > -9$ as a statement that -4 is located to the right of -9 on a number line oriented from left to right.
- Write $-3^{o}C > -7^{o}C$ to express the fact that -3°C is warmer than -7°C.

Glossary: rational number, inequality, number line, absolute value

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6.NCC.4: Interpret the absolute value of numbers for positive or negative quantities in a real-world context.

Teacher Notes:

- Learning focuses on negative numbers and their relationship to positive numbers and the introduction of the meaning of absolute value.
- Students should be able to interpret absolute value as magnitude and model on a number line which leads to the understanding of finding distance in 6.GM.6.

Examples:

- For an account balance of -25 dollars, write |-25| = 25 to describe the size of the debt in dollars.
- Recognize that an account balance less than -40 dollars represents a debt greater than 40 dollars.

Glossary: absolute value, number line

6.NCC.5: Convert between fractions, decimals, and percents in real-world and mathematical problems.

Teacher Note:

- Learning focuses on recognizing that rational numbers can be represented in different forms.
- Students understand that a fraction bar indicates division and can use the following formats interchangeably: $\frac{a}{b} = a \div b$.

Example:

- $38\% = 0.38 = \frac{38}{100}$

Glossary: fraction, percent, rational number

Rational Number Operations

Students extend previous knowledge of operations to decimals and fractions, involving positive rational numbers.

6.NCC.6: Interpret and represent quotients of fractions.

Teacher Notes:

- Learning focuses on using fraction models, such as a number line, area model, or tape diagram, to interpret, represent, and solve problems with the division of fractions.

- To support conceptual understanding, interpreting the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division is needed to understand why $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ and the division of fractions can be considered multiplying by the multiplicative inverse.
- Fractions include all forms (i.e., less than one, greater than one, and mixed numbers).

Glossary: fraction, quotient, multiplicative inverse, tape diagram, number line

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6.NCC.7: Solve problems involving the division of fractions in real-world and mathematical problems.

Teacher Notes:

- Learning focuses on using fraction models to interpret, represent, and solve problems with the division of fractions. This standard connects with 6.NCC.6.
- Learning includes fractions in all forms - less than one, greater than one, and mixed numbers.

Glossary: fraction, conceptual understanding

6.NCC.8: Divide multi-digit numbers fluently in real-world and mathematical problems.

Teacher Notes:

- Learning focuses on fluency built from conceptual understanding and previous opportunities to develop strategies to solve multi-digit division problems. This understanding is extended to include partial quotients and the standard algorithm, long division.
- The U.S. standard algorithm is one tool for solving problems but should not be the only tool students have to solve these problems. Students should have flexibility in choosing a strategy that works best for the given problem so that it may be solved efficiently. Mastery is expected by the end of sixth grade.
- For more information about fluency, see the Fluency Support Document.

Glossary: mastery, quotient, standard algorithm, conceptual understanding

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6.NCC.9: Use any standard algorithm to fluently add and subtract multi-digit decimals and fractions in real-world and mathematical problems.

Teacher Note:

- Learning focuses on fluency built from conceptual understanding and previous opportunities to develop strategies to solve multi-digit addition and subtraction problems and is connected to 6.NCC.10. This understanding is extended to include the standard algorithm with mastery expected by the end of sixth grade.
- The U.S. standard algorithm is one tool for solving problems but should not be the only tool students have to solve these problems. Students should have flexibility in choosing a strategy that works best for the given problem so that it may be solved efficiently.

Glossary: Standard algorithm, fraction, mastery, fluency, conceptual understanding

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6.NCC.10: Use any standard algorithm to fluently multiply and divide multi-digit decimals and fractions in real-world and mathematical problems.

Teacher Note:

- Learning focuses on fluency built from conceptual understanding and previous opportunities to develop strategies to solve multi-digit multiplication and division problems and is connected to 6.NCC.9. This understanding is extended to include the standard algorithm with mastery expected by the end of sixth grade.
- A standard algorithm can be viewed as, but should not be limited to, the traditional recording system and using the multiplicative inverse when dividing fractions.

Glossary: Standard algorithm, fraction, multiplicative inverse, mastery, fluency

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Common Factors and Multiples

Students use factors and multiples to solve problems.

6.NCC.11: Solve real-world and mathematical problems with the greatest common factor of two whole numbers less than or equal to 100.

Teacher Notes:

- Learning focuses on finding factors and multiples of given numbers and the introduction of the distributive property as the application of factors.
- Students will understand the meaning of a factor as a whole number that can divide another number without a remainder.
- Learning includes understanding the process of prime factorization and using organized lists to find the greatest common factor.

Glossary: greatest common factor, factor, whole number, distributive property, prime factorization

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6.NCC.12: Solve real-world and mathematical problems with the least common multiple of two whole numbers less than or equal to 12.

Teacher Notes:

- Learning focuses on finding factors and multiples of given numbers by understanding the meaning of a multiple as a whole number that is the product of a whole number and a factor.

Glossary: least common multiple, multiple, whole number, distributive property, product, factor

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6.NCC.13: Use the distributive property and the greatest common factor to rewrite the sum of two whole numbers, 1 through 100.

Teacher Note:

- Learning focuses on finding factors and multiples of given numbers and the introduction of the distributive property as the application of factors.

Example:

- Express $36 + 8$ *as* $4(9 + 2)$.

Glossary: distributive property, greatest common factor, sum, whole number, factor

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Proportional Relationships

Ratio & Rates

Students understand ratio concepts and use proportional reasoning to solve problems.

6.PR.1: Use precise ratio language and notation to describe a ratio as a relationship between two quantities.

Teacher Note:

- Learning focuses on representing ratios in various forms and comparing types of ratios.
- To support conceptual understanding, connect students' knowledge of fractions and equivalent fractions to ratios and equivalent ratios to multiplicative reasoning.
- Different forms of ratio notation (a: b, $\frac{a}{b}$, a to b) are expected.

Examples:

- The ratio of boys to girls is 4 to 1 (or 4:1, $\frac{4}{1}$) because for every 4 boys there is one girl.
- For every one cupcake sold, there were three cookies sold.

Glossary: ratio, conceptual understanding, equivalent, fraction, proportion

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6.PR.2: Calculate unit rates to include unit pricing and constant speed.

Teacher Note:

- Learning focuses on representing ratios in various forms (e.g., tables, tape diagrams, double number lines, and equations) and comparing types of ratios.
- To support conceptual understanding, build on students' understanding of equivalent fractions and multiplication and division.
- Introduction to rate is expected as the comparison of quantities with two different units and distinguishing between a ratio and a rate. Expectations for unit rates in this grade are limited to non-complex fractions.

Example:

- A store sells 5 caps for $75. What is the cost per cap?
- Billy bikes 24 miles in 3 hours. How many miles per hour does he travel?

Glossary: rate, unit rate, double number line, equation, fraction, tape diagram, conceptual understanding, ratio

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6.PR.3: Give examples of unit rates as a ratio that compares two quantities with different units of measure, limited to non-complex fractions.

Teacher Note:

- Learning focuses on representing ratios in various forms, comparing types of ratios, and the concept of a unit rate as a special kind of ratio.
- To support conceptual understanding, build on students' understanding of equivalent fractions and multiplication and division.

Examples:

- This recipe has a ratio of 2 cups of sugar to 3 cups of flour, so there is $\frac{2}{3}$ a cup of sugar for each cup of flour.
- For every 5 raffle tickets, the cost is $1, so the cost of each raffle ticket is $0.20.

Glossary: unit rate, ratio, equivalent

6.PR.4: Create various representations to compare ratios and find missing values to solve real-world and mathematical problems.

Teacher Note:

- Learning focuses on representing ratios in various forms such as tables, tape diagrams, double number lines, and equations and comparing types of ratios.

Glossary: ratio, tape diagram, double number line, equation, equivalent

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6.PR.5: Find a percent of a quantity as a rate per 100 and solve problems involving finding the whole when given a part and the percent.

Teacher Note:

- Learning focuses on representing ratios in various forms such as tables, tape diagrams, double number lines, and equations and understanding that a percent is a rate over 100.

Examples:

- What is 60% of 500?
- 75 is 30% of what number?

Glossary: percent, rate, ratio, tape diagram, double number line, equation, equivalent

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Algebra

Expressions

Students extend their understanding of arithmetic to algebraic expressions.

6.ALG.1: Read and write expressions in real-world or mathematical problems in which letters stand for numbers.

Teacher Notes:

- Learning focuses on understanding, writing, and evaluating numerical and algebraic expressions.
- To support conceptual understanding of variables, provide opportunities for students to explore the different uses

of variables, such as in a formula, as a constant, as a variable quantity, and as related to the context.
- Learning includes translating verbal expressions into algebraic and/or numerical expressions and vice versa.

Examples:
- Ten less than four times a number can be written as $4x - 10$.
- Seven times the quantity of eight plus a can be written as $7(8 + a)$.

Glossary: expression, constant, variable

6.ALG.2: Use mathematical terms to identify parts of an expression, including the names of operations, terms, factors, coefficients, variables, and constants.

Teacher Notes:
- Learning focuses on understanding, writing, and evaluating numerical and algebraic expressions and includes the understanding of the structure of expressions.

Glossary: term, expression, factors, coefficients, variables, constant, conceptual understanding

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6.ALG.3: Write and evaluate expressions for given values of variables, using order of operations, including expressions with whole number exponents.

Teacher Notes:
- Learning focuses on understanding, writing, and evaluating numerical and algebraic expressions and explaining the process of simplifying expressions.
- Exponents and exponential notation, for numbers other than the powers are 10, are introduced at this grade level. Learning will build upon previous knowledge of the powers of 10.
- Expressions that arise from formulas used in a real-world context are expected.

Examples:
- Evaluate: $5x + 2^2$ for $x = 3$
- Use the formula $V = s^3$ to find the volume of a cube with side length $s = 4$.

Glossary: evaluate, expression, variables, order of operations, exponent

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6.ALG.4: Generate equivalent expressions by applying the associative, commutative, distributive, and identity properties.

Teacher Notes:
- Learning focuses on understanding, writing, and evaluating numerical and algebraic expressions and explaining the process of simplifying expressions.
- Learning builds upon the previous understanding of the properties of operations and is extended to variables.
- The properties of operations are listed in the Appendix.

Examples:

- Apply the distributive property to the expression $4(x + 2y)$ to produce the equivalent expression $4x + 8y$.
- Rewrite the expression $2x + 8y$ using the distributive property to produce the equivalent expression $2(x + 4y)$.

Glossary: equivalent, expression, associative property, commutative property, distributive property, identity property, properties of operations

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6.ALG.5: Identify when two expressions are equivalent by using properties of operations including like terms.

Teacher Notes:

- Learning focuses on understanding, writing, and evaluating numerical and algebraic expressions and is connected to 6.ALG.4.
- Substitution should be used to check for equivalency of two expressions.
- Learning includes the same properties listed in 6.ALG.4.

Example:

- Apply combining like terms to $y + y + y + y$ to produce the equivalent expression $4y$.
- Determine if $y + y + y + y$ is equivalent to $4y$ by substituting 4 for y.

Glossary: expression, equivalent, like terms, properties of operations, substitution

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Equations & Inequalities

Students focus on reasoning about and solving equations and inequalities.

6.ALG.6: Use substitution to determine if a given value in a specified set makes an equation or inequality true.

- Include the following inequality symbols: $<$, $>$, $\leq$, $\geq$, $\neq$.

Teacher Note:

- Learning focuses on conceptual understanding and providing a foundation for solving equations and inequalities, the focus needs to be on understanding the meaning of solving an equation or inequality, not the procedure.
- The term inequality is new and expected at this grade level. Lower grades have used the term comparison symbols. The less than or equal to ($\leq$), greater than or equal to ($\geq$), and not equal to ($\neq$) symbols are new learning for this grade level and build upon previous learning using the less than ($<$), and greater than ($>$) symbols.
- Students should understand the difference between an equation and inequality and know that equations have one value that makes them true, whereas inequalities have a range of values that makes them true.

Examples:

- What value of x makes the equation $x + 4 = 7$ true?
- What are possible solutions that make the inequality $x - 8 \leq 12$ true?

Glossary: substitution, equation, inequality, solution

6.ALG.7: Write and solve one-step equations in real-world and mathematical problems, involving positive rational numbers and zero.

Teacher Note:

- Learning focuses on the meaning of an equation or inequality using prior knowledge and reasoning to solve them.
- Students are expected to write equations and solve them using a variety of strategies.
- While subtraction and division should be used when selecting problems for this standard, problems involving negative numbers, negative variables, a variable in the denominator, and complex fractions are beyond the expectation of this standard.

Glossary: equations, rational numbers, inequality, complex fractions, variable

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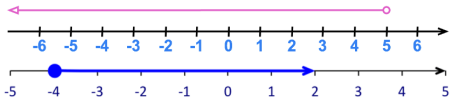
6.ALG.8: Write, solve, and graph one-step inequalities in real-world and mathematical problems.

Teacher Note:

- Learning focuses on the meaning of an equation or inequality using prior knowledge and reasoning to solve them.
- Learning includes the following inequality symbols: $<$, $>$, $\leq$, $\geq$, $\neq$.
- Learning is limited to graphing on a number line and should include inequalities that represent included (closed circle) and not included (open circle) values. Students should use substitution to verify if the graph of the inequality is correct.

Examples:

- Solve $x + 3 < 8$ and graph the solution.
- Solve $2x \geq -8$ and graph the solution.



Glossary: graph, inequality, number line, substitution

Geometry & Measurement

Area, Volume, & Surface Area

Students solve problems involving area, volume, and surface area.

6.GM.1: Find the area of triangles, quadrilaterals, and polygons by composing or decomposing to solve real-world and mathematical problems.

Teacher Note:

- Learning focuses on using prior knowledge and skills to find area, volume, and surface area and applying those concepts to solve real-world and mathematical problems.
- To support conceptual understanding, the focus needs to be on finding the area of triangles and other polygons from the known formula for a quadrilateral and deriving the formula for a triangle.
- Learning includes triangles, squares, rectangles, trapezoids, parallelograms, rhombuses, kites, pentagons, hexagons, and irregular composite shapes.

Glossary: area, polygon, trapezoid, quadrilateral, decompose, composite shape, conceptual understanding

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6.GM.2: Apply the formulas $V = lwh$ and $V = Bh$ to find the volume of right rectangular prisms with fractional edge lengths to solve real-world and mathematical problems, including solving for an unknown dimension.

Teacher Note:

- Learning focuses on using prior knowledge and skills to find area, volume, and surface area and applying those concepts to solve real-world and mathematical problems.
- Lower case b represents the length of the base of a two-dimensional figure, and capital B represents the area of the base of a three-dimensional figure. Memorization of the formula is not expected.

Glossary: volume, area, edge, length, right rectangular prism

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6.GM.3: Construct nets of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid, using the nets to find the surface area of these prisms.

Teacher Note:

- Learning focuses on using prior knowledge and skills to find area, volume, and surface area and applying those concepts to solve real-world and mathematical problems. This standard is connected to 6.GM.1 and should build from the conceptual understanding of 6.GM.1.
- Learning does not include formalized formulas for surface area but should include known area formulas from previous learning.

Glossary: net, surface area, area, volume, rectangular prism, triangular prism, pyramid, conceptual understanding

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Coordinate Plane System

Students graph points in all four quadrants.

6.GM.4: Find and graph pairs of rational numbers in all four quadrants of the coordinate plane in real-world and mathematical problems.

Teacher Note:

- Learning focuses on extending students' knowledge of graphing in the coordinate plane to using coordinates to find distances. This standard connects to 6.NCC.2.
- Learning includes recognizing the signs of all ordered pairs on the coordinate plane: Quadrant I (+, +), Quadrant II (-, +), Quadrant III (-, -), Quadrant IV (+, -).

Glossary: graph, rational number, quadrants, coordinate plane, coordinate, ordered pair

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6.GM.5: Draw polygons in the coordinate plane when given coordinates for the vertices.

Teacher Note:

- Learning focuses on extending students' knowledge of graphing in the coordinate plane to using coordinates to find

distances with an emphasis on naming the vertices with coordinates.
- Coordinates in all four quadrants are expected, as this standard is connected to 6.GM.4.

Glossary: polygon, coordinate plane, coordinates, vertices (vertex), quadrant

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6.GM.6: Use coordinates to calculate vertical and horizontal distances between points with the same x-coordinate or the same y-coordinate to solve real-world and mathematical problems.

Teacher Note:
- Learning focuses on extending students' knowledge of graphing in the coordinate plane to using coordinates to find distances. Remind students that distance cannot be negative.
- Learning includes using distance to find side lengths of polygons. This standard is connected to 6.GM.5.

Example:
- My house is at (-3, 5), the shopping mall is at (-3, -2), and the school is at (3, -2). What is the distance between my house and the shopping mall? The shopping mall and the school?

Glossary: coordinates, x-coordinate, y-coordinate, coordinate plane, graph, length, polygon

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Conversions

Students apply measurement knowledge to solve real-world problems.

6.GM.7: Convert measurements within and between the metric and customary measurement systems to solve real-world and mathematical problems.

Teacher Note:
- Learning focuses on representing ratios in various forms and comparing types of ratios.
- This standard builds upon the conceptual understanding developed in 6.PR.2 and 6.PR.4 and focuses on converting units using ratio reasoning.

Example:
- How many millimeters are in 4 feet. Use the following conversion factors: 1 inch = 25.4 millimeters and 12 inches = 1 foot.

$$\frac{12\ inches}{1\ foot} \bullet \frac{4\ feet}{1} = 48\ inches \quad 48\ inches \bullet \frac{25.4\ millimeters}{1\ inch} = 1219.2\ millimeters \text{ in } 4\ feet$$

Glossary: ratio, conversion factor, conceptual understanding

Statistics & Probability

Statistical & Nonstatistical

Students recognize that data collected to answer a statistical question can be analyzed by their distributions.

6.SP.1: Identify the difference between statistical and non-statistical questions and write simple statistical questions that allow variable responses.

Teacher Note:

- Learning focuses on identifying questions that will result in data and writing questions used to conduct surveys and collect data.

Example:

- "What are the heights of all the students in the classroom?" is a statistical question. "How tall am I?" is a non-statistical question.

Glossary: statistical question, data

Measures of Center

Students explore mean, median, and mode.

6.SP.2: Calculate and interpret any measure of center (mean, median, and mode) of a numerical data set.

Teacher Note:

- Learning focuses on understanding that data can be analyzed by their distribution. Measures of center are new learning at this grade level. This standard is connected to 6.SP.3 and 6.SP.4.
- Students should understand that each measure of center summarizes all data within a numerical data set.

Glossary: measures of center, mean, median, mode, data set, data

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6.SP.3: Determine which measure of center (mean or median) is more appropriate to describe the center of data and justify the choice.

Teacher Note:

- Learning focuses on summarizing and describing data distributions by understanding that mean is appropriate to use with symmetrical distribution and median is appropriate for non-symmetrical distributions and/or distributions with extreme data values.

Glossary: measures of center, mean, median, data

6.SP.4: Describe how the mean or median is affected by outliers of a numerical data set.

Teacher Note:

- Learning focuses on understanding that data can be analyzed by their distribution. This standard is connected to 6.SP.2 and 6.SP.3.

Glossary: mean, median, outlier, data set, data

Measures of Variation

Students explore range and interquartile range.

6.SP.5: Calculate and interpret the measure of variation [range and interquartile range (IQR)] of a numerical data set.

Teacher Note:

- Learning focuses on understanding that data can be analyzed by their distribution. Measures of variation are new learning at this grade level. This standard is connected to 6.SP.6.
- Students should understand that each measure of variation describes the data spread with a single number.

Glossary: measure of variation, range, interquartile range, data set, data

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6.SP.6: Determine which measure of variation (range or interquartile range) is more appropriate to describe the shape; justify the choice.

Teacher Note:

- Learning focuses on summarizing and describing data distributions. This standard is connected to 6.SP.5.

Glossary: measure of variation, range, interquartile range, data

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Numerical Data

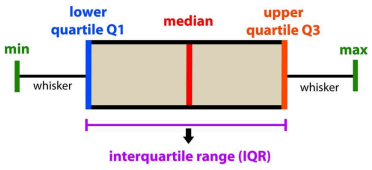
Students summarize and describe distributions.

6.SP.7: Represent numerical data on a number line, histogram, and box plot.

Teacher Note:

- Learning focuses on understanding that data can be interpreted and displayed using various visual representations.
- Learning includes understanding that data are organized in graphs for the purpose of analyzing the data and selecting the appropriate graph for the given data.
- While data can be represented in a dot plot, this representation is not required at this grade level.

Example:



Glossary: data, number line, histogram, box plot

6.SP.8: Calculate the relative frequency of an interval of data values when given a histogram.

Teacher Note:

- Learning focuses on understanding that data can be interpreted and displayed using various visual representations.
- Relative frequency is new learning at this grade level.

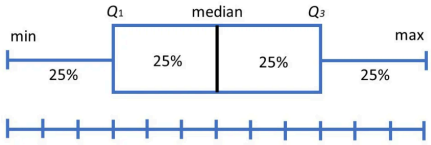
Glossary: data, relative frequency, histogram, interval

6.SP.9: Interpret a box plot to answer statistical questions about a data set.

Teacher Note:

- Learning focuses on understanding that data can be interpreted and displayed using various visual representations.
- Interpreting data from a box plot is new learning for this grade level.

Example:



Glossary: box plot, statistical question, data set

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GRADE 7 MATH STANDARDS & NOTES

Number Concepts & Computations

Rational Numbers

Students model and compute with rational numbers.

7.NCC.1: Represent addition and subtraction of rational numbers in real-world contexts using a variety of forms.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. This standard is connected to 7.NCC.2 and builds on previous learning from 6.NCC.1 - 6.NCC.4.
- Students should use a variety of forms such as number lines, color counters, or technology to represent the addition and subtraction of rational numbers.
- Formalized rules and mastery is expected by the end of seventh grade.

Glossary: rational numbers, mastery, fraction, integer, number line

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7.NCC.2: Model and describe additive inverse in real-world situations to show opposite quantities combine to make 0.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals.
- This standard is connected to 7.NCC.1 and starts the foundation for building conceptual understanding of rational number operations that leads to adding rational numbers with opposite signs.
- Learning focuses on modeling combining positive and negative numbers to make 0.

Examples:

- A hydrogen atom has 0 charge because its two constituents are oppositely charged.
- Kevin is playing football and lost 5 yards in the first play. He gained 5 yards in the second play. He has a net gain of 0 yards after the second play. 5 + (-5) = 0.

Glossary: additive inverse, fraction, rational number

7.NCC.3: Demonstrate in real-world contexts the distance between two rational numbers on the number line as the absolute value of their differences.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. This standard is connected to 7.NCC.1 and 7.NCC.2 and should be taught before 7.NCC.1 to build conceptual understanding.

- Subtraction should be thought of in terms of addition using the additive inverse.

Examples:

- The distance between -5 and 6 is 11. |-5| + |6| = 11. -5 and 6 are 11 units apart on the number line.
- Subtraction as adding the additive inverse: $p - q = p + (-q)$

Glossary: absolute value, additive inverse, conceptual understanding, difference, integer, number line, rational number

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7.NCC.4: Convert a rational number in fraction form to decimal form and recognize that the decimal form of a rational number terminates in 0s or eventually repeats.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers. This standard is the foundation for the introduction of irrational numbers in eighth grade and builds on the learning of 6.NCC.5. Rational numbers include integers, positive and negative fractions, and positive and negative decimals.
- Students should be able to sort decimals as either terminating or repeating.

Glossary: rational number, fraction, terminating decimal, repeating decimal, integer, irrational number

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7.NCC.5: Interpret the products and quotients of rational numbers by describing real-world contexts.

Teacher Note:

- Learning focuses on understanding and computing rational numbers by extending the multiplication and division of whole numbers learned from previous grades. Rational numbers include integers, positive and negative fractions, and positive and negative decimals.
- Learning focuses on understanding that division is the inverse of multiplication.
- Division by 0 is undefined.
- Emphasize if *p* and *q* are integers, then $(-(\frac{p}{q})) = \frac{(-p)}{q} = \frac{p}{(-q)}$.
- Formalized rules and mastery are expected by the end of seventh grade.

Examples:

- Bob owes $7.25 to each of five friends. How much does Bob owe altogether? $5(-7.25) = (-36.25)$ or $5(-7) + 5(-0.25) = (-35) + (-1.25) = (-36.25)$
- If the temperature on Sunday was 0 °F and 5 days later the temperature was -40 °F, what does -8 represent? How was -8 calculated?

Glossary: product, quotients, rational number, fraction, integer, inverse, mastery, whole number

---

Rational Number Operations

Students apply all properties and operations to all rational numbers.

7.NCC.6: Apply properties of operations as strategies to fluently add, subtract, multiply, and divide rational numbers.

Teacher Note:

- Learning focuses on understanding and computing rational numbers using the commutative, associative, distributive, and identity properties. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Mastery of this standard is expected by the end of seventh grade.
- A table of the properties of operations is located in the Appendix.

Glossary: properties of operations, fluency, fraction, integer, mastery, rational number

---

7.NCC.7: Use addition and subtraction with rational numbers in any form to solve multi-step problems in real-world and mathematical contexts.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Mastery of this standard is expected by the end of seventh grade.
- Learning focuses on applying the rules formalized in 7.NCC.1 - 7.NCC.6 to multi-step problems.
- This standard needs to be taught in conjunction with 7.NCC.8.
- Complex fractions are expected at this grade level.

Example:

- You and five friends are going to Magic Springs for the day. Each person has to purchase their own entry ticket and pay $10 for a tube rental. If their total entrance cost was $349.95, what is the cost of one entry ticket?
- Solve $\frac{3}{4} + 7 - 0.65$

Glossary: rational numbers, fraction, complex fraction, integer, mastery

---

7.NCC.8: Use multiplication and division with rational numbers in any form to solve multi-step problems in real-world and mathematical contexts.

Teacher Note:

- Learning focuses on understanding and computing with rational numbers and applying the rules formalized in 7.NCC.1 - 7.NCC.6 to multi-step problems. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Mastery of this standard is expected by the end of seventh grade. This standard needs to be taught in conjunction with 7.NCC.7.
- Complex fractions are expected at this grade level.

Examples:

- You and five friends are going to Magic Springs for the day. Each person has to purchase their own entry ticket and pay $10 for a tube rental. If their total entrance cost was $349.95, what is the cost of one entry ticket?
- Solve: $\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \div \frac{2}{3}$

Glossary: rational numbers, complex fractions, integer, mastery

---

7.NCC.9: Apply operations with rational numbers involving the order of operations, involving nested grouping symbols.

Teacher Note:

- Learning includes the order of operations in the following order: Parentheses or grouping symbols, exponents, multiplication and division (from left to right), and addition and subtraction (from left to right).
- Grouping symbols: parentheses, brackets, divide (fraction) bar.

Example:

- Simplify $3\left[2 - 4(3^2 - 2^{3)}\right] = (-6)$
- Simplify $\frac{4(5+4)}{3(6-2)} = 3$

Glossary: rational numbers, order of operations, nested grouping symbols

---

Proportional Relationships

Ratio & Rates

Students analyze and use unit rates to solve problems.

7.PR.1: Determine the unit rate (constant of proportionality) from tables, graphs, equations, diagrams, or verbal descriptions of proportional relationships.

Teacher Note:

- Learning focuses on building upon previous knowledge of ratios and unit rates and introduces the relationship between unit rate and proportionality.
- Students learn that the unit rate (constant of proportionality) is the numerical coefficient represented by k in the equation $y = kx$ and builds to slope represented by m in the equation $y = mx + b$ in eighth grade.
- Constant of proportionality is represented as k in seventh grade and can be found using the equation $k = \frac{y}{x}$.

Glossary: unit rate, constant of proportionality, proportional, ratio

---

7.PR.2: Calculate unit rates in real-world contexts that include complex fractions.

Teacher Note:

- Learning focuses on building upon previous knowledge of ratios and unit rates and introduces the relationship between unit rate and proportionality.

Example:

- If Joe runs $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

Glossary: unit rate, complex fractions, ratio

7.PR.3: Solve multi-step ratio and percent problems in a real-world context, including percent error and percent increase and decrease.

Teacher Note:

- Learning focuses on extending the learning of understanding that a percent is a rate over 100 in sixth grade to solving ratio and percent problems involving proportional relationships.
- Percent problems include simple interest, tax, tips, commissions, discounts, markup/markdowns, sales, and/or original price as well as percent error and percent increase/decrease.

Glossary: percent, rate, ratio

Constant of Proportionality

Students analyze proportional relationships and solve multi-step ratio and percent problems.

7.PR.4: Determine whether two quantities represent proportional relationships by using equivalent ratios in a table and by graphing on a coordinate plane.

Teacher Note:

- Learning focuses on building upon previous knowledge of ratios and unit rates and introduces the relationship between unit rate and proportionality. This standard is connected to 7.PR.5 -7.PR.7 and begins building the foundation for linear functions in eighth grade.
- Students should recognize equivalent pairs from a table as a proportion. Cross-multiplication is not expected with this standard.
- Students should recognize that a proportional relationship can be represented on a coordinate plane as a straight line that goes through the origin.

Glossary: coordinate plane, equivalent, origin, proportional, ratio, unit rate

---

7.PR.5: Compare two different proportional relationships represented in different forms.

Teacher Note:

- Learning focuses on building upon previous knowledge of ratios and unit rates and introduces the relationship between unit rate and proportionality.

Glossary: proportional, ratio, unit rate

7.PR.6: Create equations in the form of $y = kx$ from tables, verbal descriptions, or graphs.

Teacher Note:

- Learning focuses on building upon previous knowledge of ratios and unit rates to use unit rates (constant of proportionality) to create equations. This standard extends the learning of 7.PR.1 by using the constant of proportionality to write equations builds to slope represented by m in the equation $y = mx + b$ in eighth grade.

Glossary: constant of proportionality, graph, ratio, unit rate

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7.PR.7: Given a graph with a proportional relationship, explain the meaning of a point $(x,\ y)$ on the graph, including the origin $(0,\ 0)$ and the unit rate $(1,\ r)$.

Teacher Note:

- Learning focuses on extending the learning of graphing proportional relationships in 7.PR.4. Students should be able to explain the meaning of a point on a graph in the context of the given situation.

Glossary: graph, proportional, unit rate

Algebra

Expressions

Students apply properties of operations to create equivalent expressions.

7.ALG.1: Generate and justify equivalent expressions, using properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients within mathematical and real-world problems.

Teacher Note:

- Learning focuses on applying previous learning of the properties of operations from sixth grade and extending the knowledge to create and justify equivalent expressions. Coefficients are limited to rational numbers that include integers, positive/negative fractions, and decimals.
- Students are expected to apply properties of operations to calculate numbers in any form and to convert between forms as appropriate.
- Substituting numerical values for variables is a strategy/tool to determine whether expressions are equivalent.

Glossary: equivalent, expression, properties of operations, factor, linear expressions, rational, coefficients, integers, rational number, variable

---

Equations & Inequalities

Students apply previous knowledge of equations and inequalities to two-step problems.

7.ALG.2: Model and solve fluently two-step equations in real-world or mathematical problems.

Teacher Note:

- Learning focuses on using equations and inequalities to solve problems by reasoning about the quantities and builds on previous learning from sixth grade. All forms of rational numbers should be included as values in problems. Mastery is expected by the end of seventh grade.

Glossary: equation, rational number

7.ALG.3: Create, solve, and graph two-step inequalities in real-world and mathematical problems in the forms $px \pm q > r$, $px \pm q < r$, $px \pm q \geq r$, and $px \pm q \leq r$.

Teacher Note:

- Learning focuses on using equations and inequalities to solve problems by reasoning about the quantities and builds on previous learning from sixth grade. All forms of rational numbers should be included as values in problems.
- Learning includes graphing the solution set of the inequality and interpreting the solution in the context of the problem.

Glossary: inequality, graph, rational number

Relationships between Quantities

Students use understanding of algebraic expressions and equations to represent relationships between two quantities.

7.ALG.4: Write an equation to express two quantities in terms of the dependent and independent variables.

Teacher Note:

- Learning focuses on extending students' knowledge of proportional relationships to writing proportional equations and recognizing that a change in the independent variable creates a change in the dependent variable. This standard was previously in sixth grade.

Glossary: dependent variable, independent variable, proportional

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7.ALG.5: Describe the relationship between the dependent and independent variables in an equation using tables and graphs, relating these to the equation.

Teacher Note:

- Learning focuses on extending students' knowledge of proportional relationships to writing proportional equations and recognizing that a change in the independent variable creates a change in the dependent variable. This standard was previously in sixth grade.

Glossary: dependent variable, independent variable, proportional

---

Geometry & Measurement

Area, Volume, & Surface Area

Students solve problems involving area, volume, and surface area.

7.GM.1: Describe the proportional relationship between the circumference and diameter of a circle.

Teacher Note:

- Learning focuses on using prior knowledge of proportional relationships to develop an understanding that the relationship between the diameter and circumference is proportional and that the constant of proportionality is pi (π).

Glossary: proportional, circle, circumference, constant of proportionality

---

7.GM.2: Use area and circumference formulas of a circle to solve real-world and mathematical problems.

Teacher Note:

- Learning focuses on building conceptual understanding, by building on previous learning of area in sixth grade. Students should derive the formulas for circumference and area using models.
- While applying the formulas to solve problems is the expectation, memorization of the formulas is not expected.

Glossary: circle, area, circumference

7.GM.3: Apply the formulas for the volume and surface area of right rectangular prisms, rectangular pyramids, triangular prisms, and triangular pyramids to solve real-world and mathematical problems.

Teacher Note:

- Learning focuses on extending the learning from sixth grade (6.GM.1 - 6.GM.3) to students formulating formulas for volume and surface area of three-dimensional shapes and applying them to solve problems.
- Memorization of the formulas is not expected.

Glossary: rectangular prism, triangular prism, pyramid, volume, surface area, three-dimensional

---

Cross Sections

Students describe cross sections of three-dimensional figures.

7.GM.4: Describe the two-dimensional figure that results from slicing a three-dimensional figure parallel and perpendicular to the base.

- Three-dimensional figures include: right rectangular prisms, triangular prisms, and cylinders.

Teacher Note:

- Learning focuses on describing cross sections that result when a three-dimensional figure is sliced. Students will understand that a cross section refers to the two-dimensional face that results when a three-dimensional figure is sliced.

Glossary: two-dimensional, three-dimensional, rectangular prism, triangular prism, cylinder

---

Triangles & Angles

Students solve problems using various angle properties of lines.

7.GM.5: Solve multi-step problems involving supplementary, complementary, vertical, and adjacent angles to include solving for an unknown angle in a figure.

Teacher Note:

- Learning focuses on angles that are formed by intersecting lines, vertical angles and adjacent angles, and their relationships. This standard builds on previous learning in fourth grade (4.GM.3) with the introduction of the vocabulary terms supplementary and complementary.
- Students are expected to write equations to represent angle problems to find an unknown angle.

Glossary: supplementary, complementary

Scale

Students understand and use scale factor.

7.GM.6: Calculate the scale factor, compute the actual lengths from the scale in a drawing, and reproduce a scale drawing using another scale.

Teacher Note:

- Learning focuses on reading scale drawings, calculating the scale factor, computing actual lengths from a scale drawing, and being able to reproduce a scale drawing using a scale.
- Students should understand that scale drawings are proportional to one another and that angle measures remain the same.

Glossary: scale factor, scale drawing, length

Statistics & Probability

Numerical Data

Students interpret and organize data.

7.SP.1: Interpret data displayed in a histogram and box plot to answer questions about the data.

Teacher Note:

- Learning focuses on comparing centers, spreads, and shapes of distribution of two data sets in histograms and box plots. This standard builds on previous learning from sixth grade standards (6.SP.7 and 6.SP.8) and should be taught with 7.SP.3.

Glossary: histogram, box plot

7.SP.2: Recognize, create, and interpret categorical data in a circle graph.

Teacher Note:

- Learning focuses on recognizing, creating, and interpreting data in a circle graph, understanding that a circle graph consists of sectors and each sector represents a category. This is a new standard and new learning.

7.SP.3: Graph two numerical data sets and compare their variability.

- Variability includes: range, interquartile range, or mean absolute deviation.

Teacher Notes:

- Learning focuses on extending the understanding of graphing one data set to graphing two data sets to compare the shapes of distribution. This standard builds on previous learning from sixth grade standards (6.SP.7 and 6.SP.8) and should be taught with 7.SP.1.
- Mean absolute deviation is new learning for seventh grade.

Glossary: variability, range, interquartile range, mean absolute deviation, data set, graph

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7.SP.4: Select an appropriate measure(s) of center or variability and draw valid comparative inferences for two data sets.

Teacher Note:

- Learning focuses on comparing two data sets from a random sample by using measures of center or variability to make inferences about a population and is a natural learning progression from 7.SP.1. This standard and 7.SP.6 bring out the justification and reasoning behind the data discovered in those two standards.

Glossary: measure of center, measure of variation, data set, inference, variability

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Sampling & Population

Students understand sampling and use samples to make inferences.

7.SP.5: Distinguish between a random and non-random sample.

Teacher Note:

- Learning focuses on determining if a sample is representative of the population by determining if the sample is an unbiased sample (random) or a sample based on factors other than random chance (non-random).

Glossary: random sample, inference

7.SP.6: Use a random sampling of a population to draw valid inferences and generalizations of populations.

Teacher Note:

- Learning focuses on comparing two data sets from a random sample by using measures of center or variability to make inferences about a population and is a natural learning progression from 7.SP.1. This standard along with 7.SP.4 bring out the justification and reasoning behind the data discovered in those two standards.

Glossary: random sample, inference

Probability

Students understand theoretical and experimental probability for simple experiments.

7.SP.7: Determine the sample space of a simple experiment and use the sample space to determine the theoretical probability of a given set of outcomes.

- Simple experiments include: tossing a fair coin, spinning a fair spinner, rolling a fair dice, picking a random marble from a bag, and selecting a random card from a deck.

Teacher Note:

- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur. Seventh grade is limited to determining the probability of simple experiments. This standard builds the foundation for finding the probability of compound experiments in eighth grade.

Glossary: sample space, theoretical probability, experimental probability, probability

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7.SP.8: Recognize that probabilities in a simple experiment can be qualitative descriptors of likelihood: impossible (0), unlikely, neither likely nor unlikely, likely, or certain (1).

Teacher Note:

- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur.
- Learning should include the understanding that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Glossary: probability, likely event, unlikely event

7.SP.9: Determine experimental probabilities in simple experiments and represent as fractions, decimals, and percents.

Teacher Note:

- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur and can be represented in a variety of ways.

Glossary: experimental probability, fraction, percent

7.SP.10: Use theoretical probability of an event in a simple experiment to predict the number of times that an event will occur for a large number of experiments.

Teacher Note:

- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur and can be used to make predictions for a large number of experiments. Emphasis should be given to the relationship between experimental and theoretical probability.

Glossary: theoretical probability, experimental probability

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GRADE 8 MATH STANDARDS & NOTES

Number Concepts & Computations

Rational & Irrational Numbers

Students understand relationships among numbers and the real number system.

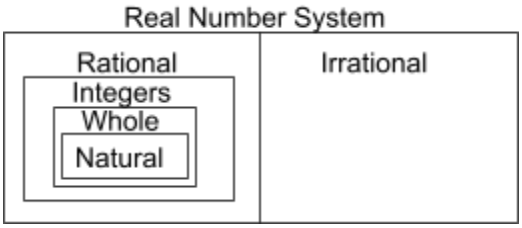
8.NCC.1: Describe relationships in the real number system (rational and irrational).

- Number relationships include: decimal expansion for rational and irrational numbers, square roots of nonperfect squares, and cube roots of nonperfect cubes.

Teacher Notes:

- Learning focuses on expanding students' knowledge of the real number system to include irrational numbers and understanding that every number has a decimal expansion.
- Learning includes converting a repeating decimal expansion into a rational number.

Examples:



- Convert $.\overline{285714}$ to $\frac{2}{7}$.

Glossary: real number system, rational number, irrational number, repeating decimal, square root

---

8.NCC.2: Compare the size of irrational numbers and locate them on a number line by finding the rational approximations.

Teacher Note:

- Learning focuses on the understanding that an estimation of an irrational number can be used to compare irrational numbers and locate them on a number line.

Example:

- Compare $\sqrt{5}$ and $\sqrt{7}$. Students should know that $\sqrt{5}$ and $\sqrt{7}$ are between whole numbers 2 and 3 because $\sqrt{4}$ is 2 and $\sqrt{9}$ is 3.

Glossary: irrational number, number line, whole number

8.NCC.3: Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Teacher Note:

- Learning focuses on building upon the knowledge of whole number exponents learned in sixth grade to understand the properties of integer exponents.
- Eighth grade is the first time students raise negative numbers to a power and see negative exponents. Students should be able to recognize that negative numbers raised to an even power produce different products when parentheses are used.
- Algebraic expressions and formalizing the rules are not expected until Algebra I.

Examples:

- -4^2 and $(-4)^2$ have products of -16 and 16 respectively.
- $4^2 \bullet 4^{-5} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

Glossary: integer, exponent, equivalent, expression, whole number

---

8.NCC.4: Write very large and very small numbers in scientific notation using positive and negative exponents.

Teacher Note:

- Learning focuses on applying knowledge of the base ten number system and exponents to rewrite numbers using scientific notation.

Glossary: exponent, scientific notation

8.NCC.5: Compare numbers written in scientific notation to determine how many times larger or smaller one number is than the other, using real-world and mathematical problems.

Teacher Note:

- Learning focuses on applying knowledge of the base ten number system, exponents, and operations with rational numbers to rewrite numbers using scientific notation and solve real-world problems.
- Learning includes understanding the multiplicative relationship between numbers written in scientific notation.

Example:

- Estimate the Sun is 8.6×10^6 miles across and Earth is 7.9×10^4 miles across and determine that the Sun is about 109 times the diameter of Earth.

Glossary: exponent, rational number, scientific notation

8.NCC.6: Solve real-world and mathematical problems by performing operations with numbers written in standard and scientific notation.

Teacher Note:

- Learning focuses on applying knowledge of the properties of exponents learned in 8.NCC.3 to solve problems written in either scientific notation, standard form, or both. Choosing units of appropriate size for measurements of very large or very small quantities is expected.

Glossary: exponent, rational number, scientific notation

Rational Number Operations

Students work with square and cube roots.

8.NCC.7: Solve equations in the form of $x^2 = p$ or $x^3 = p$ where p is a positive rational number.

Teacher Note:

- Learning focuses on the understanding of square roots and cube roots of positive rational numbers and using square root and cube root symbols to represent solutions to equations.

Example:

- $x^2 = 63;\ x = \sqrt{63}$
- $x^2 = 9;\ x = 3,\ -3$
- $x^3 = 47;\ x = \sqrt[3]{47}$
- $x^3 = 64;\ x = 4$

Glossary: rational number, square root

8.NCC.8: Evaluate square roots of perfect squares and cube roots of perfect cubes.

Teacher Note:

- Learning focuses on the understanding of square roots and cube roots of positive rational numbers.

Example:

- $\sqrt{9} = \pm 3$
- $\sqrt[3]{64} = 4$

Glossary: evaluate, square root, cube root, solution

Functions

Proportional & Linear Relationships

Students understand slope using previous learning of proportional relationships.

8.FN.1: Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Teacher Note:

- Learning focuses on building on previous learning in sixth and seventh grades with proportional relationships to compare graphs, tables, and equations in standard 8.FN.4. Unit rate or slope is referred to as the constant of proportionality in seventh grade.
- Interpreting slope in the context of the relationship is expected.

- Equations are limited to slope-intercept form in eighth grade with mastery expected. Other forms are mastered in higher grade level courses.

Glossary: unit rate, slope, constant of proportionality, graph, mastery, proportional

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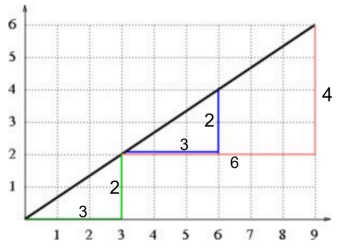
8.FN.2: Explain, using similar right triangles, how the slope of a line is the same between two points on a non-vertical line or non-horizontal line.

- Slope includes: positive, negative, horizontal (zero), and vertical (undefined).

Teacher Notes:

- Learning focuses on deriving the equation $y = mx + b$ building on learning from the seventh grade of equations in the form $y = mx$ and discovering that m is the slope and b is the y-intercept.
- Equations are limited to slope-intercept form in eighth grade with mastery expected. Other forms are mastered in higher grade-level courses.

Example:



$\frac{4}{6}=\frac{2}{3}$

$m=\frac{2}{3}$

$y=\frac{2}{3}x$

Glossary: similar, slope, y-intercept

Functions

Students understand that a function is a rule that assigns each input exactly one output.

8.FN.3: Determine whether a relation is a function or not when given a function map, table, graph, equation, or set of ordered pairs.

Teacher Note:

- Learning focuses on the definition of a function and identifying functions from various forms. An informal discussion of function notation and an introduction to domain (input) and range (output) vocabulary is needed at this time. However, student assessment is not required.

Glossary: function, domain, range, function notation, ordered pair

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8.FN.4: Compare the rate of change (slope) and y-intercept (initial value) of two linear functions each represented in different forms.

- Functions are represented algebraically, graphically, numerically in tables, or by verbal descriptions.

Teacher Note:

- Learning focuses on comparing the different functions in similar contexts to make comparisons of function properties.
- Equations are limited to slope-intercept form in eighth grade with mastery expected. Other forms are mastered in higher grade-level courses.

Example:

- Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Glossary: rate of change, slope, y-intercept, function, mastery

---

8.FN.5: Distinguish between linear and nonlinear functions by comparing graphs and equations.

Teacher Note:

- Learning focuses on recognizing equations in the form of $y = mx + b$ will lead to graphs of straight lines and are linear functions.
- Equations are limited to slope-intercept form in eighth grade with mastery expected. Other forms are mastered in higher grade-level courses.

Glossary: linear equation, function, graph, mastery

8.FN.6: Determine the rate of change (slope) and y-intercept (initial value) from tables, graphs, equations, and verbal descriptions of linear relationships.

Teacher Note:

- Learning focuses on discovering the rate of change and y-intercept when presented with a linear relationship. Learning includes determining the slope of a line using the rise to run ratio.
- Equations are limited to slope-intercept form in eighth grade with mastery expected. Other forms are mastered in higher grade-level courses.

Glossary: rate of change, slope, y-intercept

8.FN.7: Interpret and explain the meaning of the rate of change (slope) and y-intercept (initial value) of a linear relationship in a real-world context.

Teacher Note:

- Learning focuses on building upon the knowledge of 8.FN.6 to interpret the meaning of slope and y-intercept in the context of the situation and includes creating linear equations.

Glossary: rate of change, slope, y-intercept

8.FN.8: Analyze a graph by describing the functional relationships between two quantities.

Teacher Note:

- Learning focuses on describing the characteristics of linear and nonlinear functions and includes determining if the graph is linear or nonlinear and/or where the function is increasing or decreasing.

Glossary: linear, nonlinear, function, graph

8.FN.9: Sketch a graph that exhibits qualitative features of a function described verbally.

Teacher Note:

- Learning focuses on sketching graphs that show the qualitative features of a function when given descriptions verbally. This standard should be taught in conjunction with 8.FN.8 and should include linear and nonlinear functions.

Glossary: function, linear, nonlinear, graph

Algebra

Equations & Inequalities

Students solve linear equations and inequalities.

8.ALG.1: Analyze and solve one-variable linear equations with rational coefficients containing solutions with one, zero, or infinitely many solutions.

Teacher Note:

- Learning focuses on extending the learning of solving one-variable equations and inequalities with a variable on one side in seventh grade to solving one-variable equations and inequalities with the same variable on both sides of the equation or inequality. This standard is the foundation for future work with linear equations and should be taught in conjunction with 8.ALG.2.
- Explanation of whether the solution has one, zero, or infinitely many solutions is expected.

Examples:

- $\frac{1}{4}(4a + 5) = -a + 6$
- $6 - 4p = 5 - \frac{1}{3}(3.5p - 2.5)$
- $\frac{1}{2}(6x - 8) = \frac{1}{4}(12x - 16)$

Glossary: linear equation, rational, coefficient, variable, inequality, solution

---

8.ALG.2: Analyze and solve one-variable linear inequalities with rational coefficients.

Teacher Note:

- Learning focuses on extending the learning of solving one-variable equations and inequalities with a variable on one side in seventh grade to solving one-variable equations and inequalities with the same variable on both sides of the equation or inequality. This standard is the foundation for future work with linear inequalities and should be taught in conjunction with 8.ALG.1.

Examples:

- $3y - 4 > -8 + 7y$
- $\frac{1}{3}r + 7 \leq \frac{2}{3}r - 14$

Glossary: variable, linear, rational, coefficient, inequality

---

Systems of Equations

Students will solve systems of equations.

8.ALG.3: Analyze and solve systems of linear equations in the form $y = mx + b$ in real-world or mathematical contexts, graphically and algebraically.

Teacher Note:

- Learning focuses on solving a system of equations and explaining the solution must be a solution to each equation in the system. Systems of equations with one, zero, or infinitely many solutions are expected. Mastery of solving by substitution and graphing is expected, however, elimination can be introduced, but mastery of elimination is not expected until Algebra I.

Glossary: linear equation, mastery, solution

Geometry & Measurement

Area, Volume, & Surface Area

Students solve problems involving area, volume, and surface area.

8.GM.1: Apply the formulas for the volume and surface area of cylinders, cones, and spheres to solve real-world and mathematical problems.

Teacher Note:

- Learning focuses on extending the learning of volume of rectangular and triangular prisms and pyramids in seventh grade to three-dimensional shapes related to circles. To support conceptual understanding, students should discover the formulas through experiments or hands-on activities.

Glossary: surface area, cylinder, cone, sphere, volume, conceptual understanding, rectangular prism, triangular prism, pyramid

---

Cross Sections

Students describe cross sections of three-dimensional figures.

8.GM.2: Describe the two-dimensional figure that results from slicing a three-dimensional figure parallel and perpendicular to the base.

- Three-dimensional figures include: pyramids, cones, and spheres.

Teacher Note:

- Learning focuses on extending the learning of describing cross sections formed when rectangular prisms, triangular prisms, and cylinders in seventh grade to describing the cross sections formed when pyramids, cones, and spheres are sliced parallel and perpendicular to the base. This is a new standard for eighth grade.

Glossary: pyramid, cone, sphere, rectangular prism, triangular prism, two-dimensional, three-dimensional

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Pythagorean Theorem

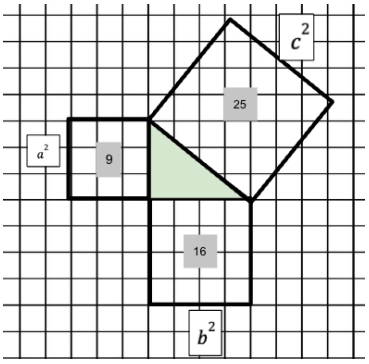
Students explore right triangles and apply the Pythagorean Theorem.

8.GM.3: Model or explain an informal proof of the Pythagorean Theorem and its converse.

Teacher Note:

- Learning focuses on understanding the relationship between the sides of a right triangle to understand the formula $a^2 + b^2 = c^2$ and then apply the formula to solve problems. Students are not expected to formally prove the Pythagorean Theorem until high school Geometry but should be exposed to various models and be able to explain how the models support the theorem.
- Students are expected to understand if a triangle has sides of length a, b, and c and if $a^2 + b^2 = c^2$ then the angle opposite the side of length c is a right angle as the converse of the Pythagorean Theorem.

Example:



Glossary: Pythagorean Theorem

8.GM.4: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles.

Teacher Note:

- Learning focuses on understanding the relationship between the sides of a right triangle to understand the formula $a^2 + b^2 = c^2$ and then apply the formula to solve problems. Application is limited to two-dimensional figures.

Glossary: Pythagorean Theorem, two-dimensional

8.GM.5: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Teacher Note:

- Learning focuses on understanding the relationship between the sides of a right triangle to understand the formula $a^2 + b^2 = c^2$ and then apply the formula to find the distance on a coordinate plane.

Glossary: Pythagorean Theorem, coordinate plane

Transformations & Congruence on a Coordinate Plane

Students use concrete models, diagrams, or geometry to understand congruence and similarity.

8.GM.6: Given a figure, draw a congruent figure on a coordinate plane resulting from a rotation, reflection, or translation.

Teacher Note:

- Learning focuses on the understanding that rotations, reflections, and translations of two-dimensional figures result in congruent figures and using transformations to develop a conceptual understanding of the definition of congruence. Reflections are limited to the x-axis and y-axis, and rotations are limited to 90-degree clockwise and counterclockwise increments and include rotations around the origin and around a point on the figure.
- Correct labeling notation for the image and preimage is expected.

Glossary: congruent figures, coordinate plane, origin

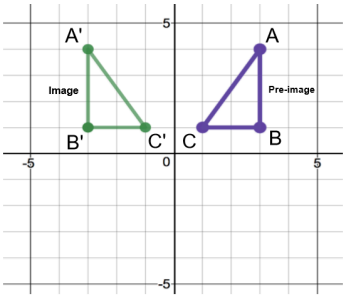
8.GM.7: Identify a single transformation used to transform one figure onto another on a coordinate plane.

- Rotations include: 90°, 180°, and 270°

Teacher Note:

- Learning focuses on describing the transformation of one figure that creates a congruent or similar figure on a coordinate plane. Reflections are limited to the x-axis and y-axis. Rotations should include clockwise, counterclockwise, around the origin, and around a point on the figure.
- Correct labeling notation is expected.

Example:



Triangle ABC is reflected over the y-axis to create Triangle A'B'C'

Glossary: coordinate plane, similar

8.GM.8: Given two congruent figures, describe a sequence of transformations that maps one figure to another.

Teacher Notes:

- Learning focuses on describing a sequence of transformations of one figure to another on a coordinate plane. Rules or coordinates are not expected with this standard. This standard should be taught in conjunction with 8.GM.6.

Glossary: congruent figures, coordinate plane

8.GM.9: Perform a given sequence of transformations of a figure on the coordinate plane, including rotations, reflections, translations, and dilations.

- Rotations include: 90°, 180°, and 270°

Teacher Notes:

- Learning focuses on applying a sequence of transformations to a given figure to create a congruent or similar figure on the coordinate plane giving the new coordinates using correct labeling notation.

Glossary: coordinate plane, congruent figure, similar

8.GM.10: Describe the effects of rotations, reflections, translations, and dilations on two-dimensional figures using coordinates.

- Rotations include: 90°, 180°, and 270°

Teacher Note:

- Learning focuses on extending 8.GM.7 from a single transformation to a sequence of transformations. Reflections are limited to the x-axis and y-axis. Clockwise and counterclockwise rotations are expected.

- Correct labeling notation is expected.

Glossary: two-dimensional, coordinates

8.GM.11: Given two similar two-dimensional figures, describe a sequence of transformations that exhibits similarity, including rotations, reflections, translations, and dilations.

Teacher Note:

- Learning focuses on understanding that similar figures can be created by a sequence of transformations.

Glossary: similar, two-dimensional

Statistics & Probability

Bivariate Data

Students investigate patterns of association to bivariate data.

8.SP.1: Construct scatter plots using bivariate data; determine if the data displays a linear or nonlinear pattern and positive, negative, or no association.

Teacher Note:

- Learning focuses on graphing and interpreting bivariate data, looking for patterns of clustering, outliers, positive or negative, or no association, and linear or nonlinear association.

Glossary: scatter plot, bivariate data, association, outlier

8.SP.2: Construct straight lines to approximately fit data displaying a linear association when presented in scatter plots.

Teacher Note:

- Learning focuses on understanding that a straight line can model a linear relationship in a scatter plot that displays a linear association. The use of linear regression is not expected at this grade level.

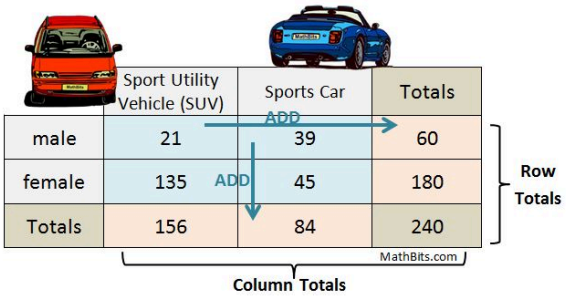
Glossary: scatter plot, association

8.SP.3: Construct and interpret a relative frequency table, using data from two categorical variables collected from the same subject.

Teacher Note:

- Learning focuses on extending the knowledge from sixth grade where students calculated relative frequency from a histogram to collecting data and creating a relative frequency table to organize data between two categorical variables. Students are expected to interpret the data and recognize patterns of association.

Example:



Glossary: relative frequency, histogram

Probability

Students understand theoretical and experimental probability for compound experiments using organized lists, tables, or tree diagrams.

8.SP.4: Determine the sample space and use the sample space to determine the theoretical probability of a given set of outcomes for compound experiments, using organized lists, tables, or tree diagrams.

- Compound experiments include a combination of two different experiments.

Teacher Note:

- Learning focuses on extending the learning of probability for simple experiments to learned in seventh grade, to determine the theoretical probability of a compound event using organized lists, tables, or tree diagrams.

Glossary: theoretical probability, experimental probability, sample space, tree diagram

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8.SP.5: Determine theoretical and experimental probabilities of compound experiments.

Teacher Note:

- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur. This standard extends the learning from seventh grade probability of simple experiments to find the probability of compound experiments. Emphasis should be given to the relationship between experimental and theoretical probability.

Glossary: theoretical probability, experimental probability

8.SP.6: Use theoretical probability of an event in a compound experiment to predict the number of times that event will occur for a large number of experiments.

Teacher Note:

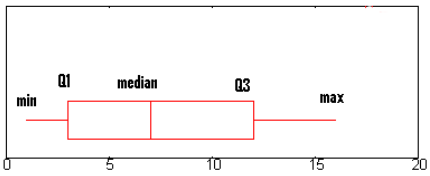
- Learning focuses on understanding that probability is a mathematical way to calculate how likely an event will occur and can be used to make predictions for a large number of experiments. This standard extends the learning

from seventh grade probability of simple experiments to find the probability of compound experiments. Emphasis should be given to the relationship between experimental and theoretical probability.

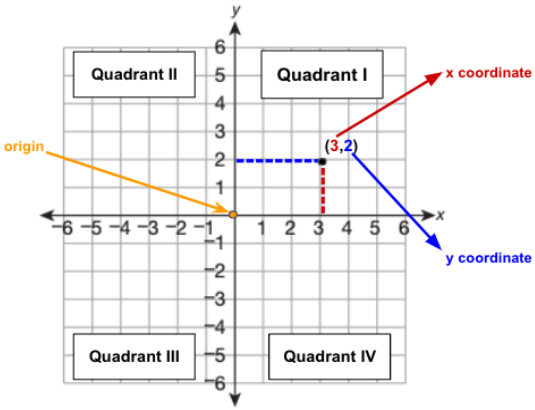
Glossary: theoretical probability, experimental probability

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K - 8 GLOSSARY

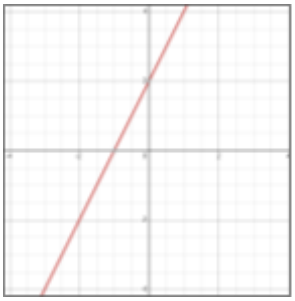
Absolute Value	A number's distance from 0 on the number line.
Addend	Any of the numbers added to find a sum.
Additive Comparison	Compare two amounts by asking how much more or less is one amount than the other.
Additive Inverse	A number that when added to another number gives a sum of 0. Example: $1 + (-1) = 0$.
Algorithm	An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.
Area	The measure of the size of the interior of a figure, expressed in square units.
Association	The relationship between two variables.
Associative Property of Addition	A property of numbers that states that the sum of a set of numbers is the same, regardless of how the numbers are grouped. Example: (4 + 8) + 3 = 4 + (8 + 3)
Associative Property of Multiplication	A property of numbers that states that the product of a set of numbers is the same, regardless of how the numbers are grouped. Example: $(4 \bullet 8) \bullet 3 = 4 \bullet (8 \bullet 3)$
Attributes	Characteristics or properties of an object.
Axis	A vertical or horizontal number line, both of which are used to define a coordinate grid. The horizontal axis is the x-axis, and the vertical axis is the y-axis. The plural of axis is axes.
Benchmark Fraction	A common fraction used when comparing other fractions (e.g., $\frac{1}{2}$, $\frac{1}{4}$).
Bivariate Data	Data that has two variables.
Box Plot	A way to show the interquartile range, mean, and median of a data set. Also known as a box and whisker plot. 
Cardinality	The understanding that when you count items, the number word applied to the last object counted represents the total amount.
Circle	A two-dimensional figure for which all points are the same distance from its center. A circle is identified by its center point and radius.
Circumference	The perimeter of a circle, which is the distance around a circle.
Coefficient	A number or variable by which a variable is multiplied. Examples: $3x + 7$, 3 is the coefficient; $y = mx + b$, m is the coefficient.
Commutative Property of Addition	A property of numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., the sum remains the same. Example: $5 + 9 = 9 + 5$

Commutative Property of Multiplication	A property of numbers that states that the product of two factors is unaffected by the order in which the factors are multiplied, i.e., the product remains the same. Example: $5 \bullet 9 = 9 \bullet 5$
Complementary Angles	Two angles (adjacent or nonadjacent) whose sum is 90 degrees.
Complex Fraction	A fraction in which the numerator, denominator, or both are fractions themselves. Example: $\frac{\frac{1}{2}}{\frac{2}{3}}$ or $\frac{2}{\frac{3}{4}}$
Composite	A whole number with at least one whole number factor besides 1 and itself.
Composite Shape	A shape that can be divided into more than one of the basic shapes. Also called complex or compound shapes.
Computational Fluency	To have efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand, and can explain these methods as well as produce accurate answers efficiently.
Conceptual Understanding	Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and how it works. Conceptual understanding can be accomplished with and without manipulatives.
Cone	A three-dimensional figure that has a circular base and one vertex. A cone has two faces, the circular base and the lateral face.
Congruent Figures	Geometric figures that have the same size and the same shape. Congruent figures may have different orientations. Example: Congruent angles have the same degree measure. Congruent line segments have the same length.
Constant	A fixed value in an expression or equation. Example: 3 is the constant in the expression $4x + 3$.
Constant of Proportionality	The value of the ratio of two proportional quantities, equivalent to unit rate. Represented as *k* in the formula *y* = *kx*.
Conversion Factor	A number used to change one set of units to another by multiplying or dividing.

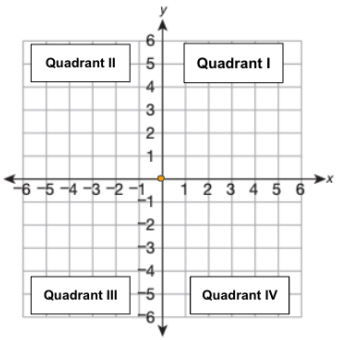
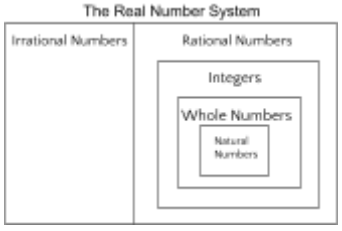
Coordinate Plane	A plane divided by perpendicular number lines creating four quadrants. The perpendicular number lines represent the axes, and where they intersect represents the origin (0,0). Points can be identified using coordinates (x, y) found within the quadrants (example below). 
Coordinates	An ordered pair of numbers that identifies a point on a grid, coordinate plane, or map written as (x, y).
Counting Back	A strategy for finding the difference using backward counting. Example: if a stack of books has 12 books and someone borrows 4 books to read, how many books are left? A student may start at 12 and count back for spaces or numbers saying 12…11, 10, 9, 8; there are 8 books left in the stack.
Counting On	A strategy for finding the number of objects in a group without having to count every member of the group. Example: if a stack of books has 8 books and 3 more books are added to the top, it is unnecessary to count the stack all over again. One can find the total by *counting on* pointing to the top book and saying "eight," following this with "nine, ten, eleven." There are eleven books now.
Cube	A three-dimensional figure with exactly six congruent, square faces.
Cylinder	A three-dimensional figure with two circular bases that are parallel and congruent. It has three faces, the two circular bases and the lateral face.
Data	Information collected and used to analyze a specific concept or situation.
Data Set	A set of numbers collected to be examined, considered, and used to help decision-making.
Decompose	Breaking a quantity or shape into smaller quantities/units/shapes in order to assist computation.
Denominator	The number in a fraction written below the fraction line that divides the number above the fraction line.
Dependent Variable	The output variable in a function; the variable whose value depends on the input or independent variable.
Difference	The distance between two values; result of a subtraction problem.
Dilation	A nonrigid transformation that enlarges or reduces a geometric figure by a scale factor relative to a point.
Distributive Property	When a single-term expression is being multiplied by a sum or difference, the single-term expression can be multiplied by each term before finding the sum or difference. Example: $3(7 + 5) = 3 \bullet 7 + 3 \bullet 5$ $w(5 - 2) = 5w - 2w$

Dividend	A number that is being divided by another number (divisor).
Divisor	The number by which another number is being divided.
Domain	The set of input (x) values for a function.
Double Number Line	Two number lines used when quantities have different units to easily see there are numerous pairs of numbers in the same ratio.
Edge	The line segment where a base and a lateral face, a base and lateral surface(s), two lateral faces, or two lateral surfaces of a three-dimensional figure intersect.
Equation	A statement that has one number or expression equal to another number or expression, such as $8 + 3 = 11$ or $2x - 3 = 7$.
Equivalent	Equal in value.
Evaluate	Calculate or find the value of a mathematical expression.
Expanded form	A multi-digit number is expressed in expanded form when it is written as a sum of the single-digit multiples of powers of ten. Example: $643 = 600 + 40 + 3$ $(3 \bullet 100,000) + (4 \bullet 10,000) + (7 \bullet 1,000) + (2 \bullet 100) + (6 \bullet 10) + (5 \bullet 1) = 347,265;$ $(3 \bullet 100) + (4 \bullet 10) + (7 \bullet 1) + (2 \bullet 0.1) + (6 \bullet 0.01) + (5 \bullet 0.001) = 347.265$ $(3 \bullet 100) + (4 \bullet 10) + (7 \bullet 1) + (2 \bullet \frac{1}{10}) + (6 \bullet \frac{1}{100}) + (5 \bullet \frac{1}{1000}) = 347.265$
Expanded Form (Exponents)	Expressing exponential expressions using multiplication without an exponent. Example: $x^5 = x \bullet x \bullet x \bullet x \bullet x$
Experimental Probability	The ratio of the number of times an event occurs to the total number of trials or times the activity is performed.
Exponent	A symbol that is written above and to the right of a number, *p* in exponential notation a^p, to show how many times the number is to be multiplied by itself.
Expression	A mathematical phrase consisting of numbers, variables, and operations. Algebraic expressions contain variables, constants, and algebraic operations. Numerical expressions contain only numbers and algebraic operations.
Fluency	There are three types of *fluency*. All of them require students to be accurate, efficient, and flexible. The types are defined as follows: Basic fact fluency - Operations with single-digit numbers. Students add, subtract, multiply, and divide accurately, efficiently, and flexibly with single-digit numbers. Computational fluency - Computation with the four operations. Students are able to accurately, efficiently, and flexibly compute problems using the four operations across all number types (whole numbers, fractions, etc.) without regard for magnitude of the numbers. Procedural fluency - Includes more than the four operations. "Ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another." (NCTM 2014, 2020)
Factor	1. A number or quantity that when multiplied with another number or quantity, produces a given number or expression. Example: 5 and 2 are factors of 10. 2. To break down into the terms that multiply to make the quantity to be factored.

Fraction	A number expressible in the form $\frac{a}{b}$ where *a* is a whole number and *b* is a whole number. The word fraction in these standards(K-5) always refers to a non-negative number. This includes all forms of fractions: fractions less than one, fractions greater than one (improper fractions), and mixed numbers.
Function	A rule or relationship in which there is exactly one output value for each input value.
Function notation	$f(x)$ is a way to represent a function, named *f*, where the input is represented by *x* and the output (y-value) is represented by $f(x)$. Example: $f(x) = 3x$ *f*(*x*) = 3*x* is the same as $y = 3x$.
Graph (Verb)	To show or plot information on a number line or coordinate plane.
Greatest Common Factor	The greatest whole number that divides two numbers.
Hierarchy	A system of ranking or organizing things.
Histogram	A graph used to represent the frequency distribution of data points of one variable.
Identity Property	The property asserts that when adding an identity (0) or multiplying an identity (1) with a number (n), the end result will be n. Example: Additive $\rightarrow n + 0 = n; 0 + n = n$ Multiplicative $\rightarrow n \bullet 1 = n; 1 \bullet n = n$
Independent Variable	A variable whose values do not depend on changes in other variables.
Inequality	A numerical sentence containing one of the symbols: >, <, ≥, ≤, or ≠ to indicate the relationship between two quantities. Example: $8 - 2 > 6 \div 3; 7v \leq 49; 5 \neq 2 + 2$
Inference (Statistical)	Deriving logical conclusions about a statistical population based on samples.
Integers	Natural numbers, their opposites, and zero. Integers can be positive or negative and do not contain fractions or decimals.
Interquartile Range	A measure of variation in a set of numerical data; the interquartile range is the distance between the first and third quartiles of the data set.
Interval	Includes all the numbers that come between two particular numbers.
Inverse (Operation)	An operation that is the opposite of, or undoes, another operation. Addition and subtraction are inverse operations as are multiplication and division.
Irrational Number	A number that cannot be expressed as a fraction $\frac{p}{q}$ for any integers *p* and *q*; irrational numbers have decimal expansions that neither terminate nor become periodic.
Least Common Multiple	The smallest positive whole number that is exactly divisible by each member of a given set of positive whole numbers.
Length	The distance from one end of an object to the other end.
Like Terms	Terms in an equation or expression whose variables are raised to the same power. Constants are also considered like terms. Example: $7x$ and $2x$ are like terms and $4y^2$ and $3y^2$ are like terms.
Likely Events	A chance event with a probability between 0.5 and 1; the closer the probability is to being 1, the more likely the event is to occur.
Line Plot	A method of visually displaying a distribution of data values where each data value is shown as an X or mark above a number line. Also known as a dot plot.

Linear Equation	An algebraic equation in which the variables are of the first degree (raised only to the first power). The graph of such an equation is a straight line. 
Linear Expression	An algebraic statement where each term is either a constant or a variable raised to the first power.
Mass	The amount of matter in an object. Often measured by the amount of material it contains which causes it to have weight. However, mass is not to be confused with weight. Weight is determined by the force of gravity on an object while mass is not. For example, an watermelon on Jupiter would have a greater weight than one on Earth because Jupiter's gravity is stronger than Earth's. The mass of the watermelon would be the same on both planets.
Mastery	Knowledge and skill that allows you to do, use, or understand something consistently well.
Mean	A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.
Mean Absolute Deviation	A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.
Measurement Division	Division situation where the number of groups is unknown.
Measures of Center	Statistical measures that are intended to provide numerical representations of the center of a set of numerical data, also called measures of central tendency.
Measures of Variation	Statistical measures that are intended to provide numerical representations of the variability of a set of numerical data.
Median	A measure of center in a set of numerical data; the median of a list of values is the value appearing at the center of a sorted version of the list – or the mean of the two central values if the list contains an even number of values.
Mode	A measure of center in a set of numerical data; the most common value in list of values.
Multiple	The result of multiplying a number by an integer. Example: 12 is a multiple of 4 because $4 \bullet 3 = 12$.
Multiplicative Comparison	Compare two amounts by asking how many times larger or smaller is one amount than the other.
Multiplicative Inverse	A number that, when multiplied by the given number, gives 1 as the product. By multiplicative inverse definition, it is the reciprocal of a number. Example: $\frac{1}{5} \bullet 5 = \frac{5}{5} = 1$; therefore 5 is the multiplicative inverse of $\frac{1}{5}$.
Natural Numbers	Counting numbers 1, 2, 3, 4, 5, 6…
Nested Grouping Symbols	More than one set of grouping symbols within a math problem. Example: $3[2 - 4(3^2 - 2^3)] = (-6)$
Net	Two-dimensional representation of a three-dimensional figure that can be folded up into a three-dimensional figure.
Number Line	A graph that represents the real numbers as ordered points on a line. A number line may be either horizontal (left and right) or vertical (up and down). Starting at zero, the positive numbers progress to the right (or up) and the negative numbers progress to the left (or down).

Numerator	The number in a fraction that is above the fraction line and that is divided by the number below the fraction line.
Order of Operations	A set of rules that define which procedures to perform first in order to evaluate a given expression.
Ordered Pair	A set of two numbers named in an order that matters; represented by (x, y) such that the first number, x, represents the x-coordinate and the second number, y, represents the y -coordinate when the ordered pair is graphed on the coordinate plane; each point on the coordinate plane has a unique ordered pair associated with it.
Ordinal Number	A number defining a position in a series.
Origin	The point in a Cartesian coordinate system where axes intersect.
Outlier	An observation or data point that lies an unusual distance from other values in the data.
Parallel Lines	Two or more distinct lines in the same plane that never intersect, these lines are always equidistant. In the coordinate plane, non-vertical parallel lines have equal slopes.
Partition	Divide or separate into smaller shapes or quantities.
Partitive Division	Division situation where the size of the groups is unknown.
Percent	A number expressed in relation to 100; represented by the symbol %.
Perimeter	The distance around a closed figure.
Perpendicular	Intersecting lines or planes that form right angles.
Place Value	The value of the place of a digit in a numeral; the relative worth of each number that is determined by its position.
Plane Figure	Two-dimensional shapes.
Plot	To place a point(s) on a coordinate plane.
Polygons	A closed two-dimensional figure made up of straight sides.
Prime	A whole number greater than 1 with only two factors. The factors of a prime number are 1 and the number itself.
Prime Factorization	A method of writing a composite number as a product of its prime factors.
Prism	A three-dimensional (solid) figure that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). Prisms are named by the shape of their bases.
Probability	A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).
Product	The resulting quantity when two or more factors are multiplied. (The answer to a multiplication problem.)
Properties of Operations	Rules that apply to the operations with numbers. (See Table 1 below.)
Proportional	When two quantities have the same ratios.
Pyramid	A three-dimensional figure whose base is a polygon and the lateral faces are triangles that share a common vertex.
Pythagorean Theorem	The mathematical relationship stating that in any right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs ($a^2 + b^2 = c^2$).

Quadrant	One of the four sections of a coordinate plane separated by horizontal and vertical axes; they are numbered I, II, III, and IV, counterclockwise from the upper right. 
Quadrilaterals	A polygon with four sides and four angles.
Quotient	The resulting quantity when one quantity (dividend) is divided by another quantity (divisor). The answer to a division problem.
Random Sample	A sample obtained by a selection from a population, in which each element of the population has an equal chance of being selected.
Range	1. The difference between the lowest and highest values from a set of data. 2. The set of output (*y*) values for a function.
Rate	A ratio that relates quantities of different units. Example: miles per hour, price per pound, students per class, heartbeats per minute.
Rate of Change/Slope	The ratio of the vertical change compared to the horizontal change between two points on a coordinate plane. Slope is often expressed as $\frac{rise}{run}$ or $\frac{change\ in\ y}{change\ in\ x}$.
Ratio	A comparison of two quantities, *r* and *s,* which can be written: ● $\frac{r}{s}$, where *r* is the numerator and *s* is the denominator ● *r* : *s* ● *r* to *s*
Rational Number	A number that can be written as a fraction of two integers $\frac{a}{b}$, where $b \neq 0$. Rational numbers can be integers, fractions, and repeating and terminating decimals.
Ray	A part of a line that has one endpoint and continues infinitely in one direction or on one side of that point. A ray is identified by two points: first its endpoint and then another unique point on the ray.
Real Number System	The real number system is the set of numbers containing all of the rational numbers and all of the irrational numbers. Example: 
Rectangular Array	A set of quantities arranged in rows and columns.

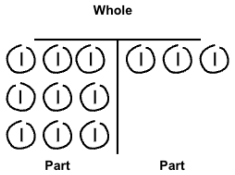
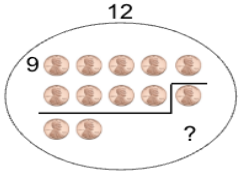
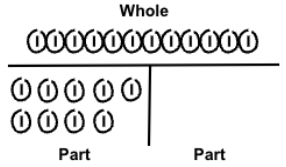
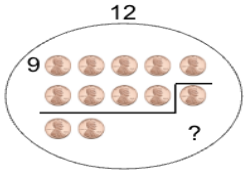
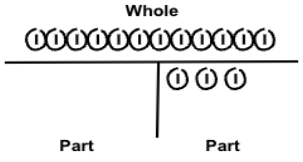
Rectangular Prism	A three-dimensional figure whose six faces are rectangles.
Reflection	A transformation in which every point and its images are on opposite sides and the same distance from a fixed line.
Relative Frequency	The ratio of the observed frequency of some outcome to the total outcomes of a random experiment.
Repeating Decimal	A decimal form of a rational number that, after a certain point, one digit or a set of digits repeat themselves an infinite number of times. Repeating digits are designated with an ellipsis or a bar above them. Example: 0.3333... , $0.\overline{3}$, or 4.00757575…, $4.00\overline{75}$
Rotation	A transformation in which each point is moved by the same angle measure in the same direction along a circular path about a fixed point.
Sample Space	The set of all possible outcomes for a probability experiment. Sample spaces can be displayed as diagrams, lists, and tables.
Scale Drawing	A drawing with dimensions at a specific ratio relative to the actual size of the object.
Scale Factor	The ratio between the corresponding measurements of an object and a representation of that object. A whole number scale factor represents an enlargement, and a fractional scale factor represents a reduction.
Scatter Plot	A two-variable data display where points are plotted to show the relationship (correlation) between two variables.
Scientific Notation	A form of writing a number as the product of a power of 10 and a decimal number such that the absolute value of the decimal number is greater than or equal to one and less than ten.
Similar	Two figures are similar if and only if all corresponding angles are congruent and lengths of all corresponding sides are proportional.
Slope/Rate of Change	The ratio of the vertical change compared to the horizontal change between two points on a coordinate plane. Slope is often expressed as $\frac{rise}{run}$ or $\frac{change\ in\ y}{change\ in\ x}$.
Solution	Any value(s) that make an equation, inequality, or open sentence true.
Sphere	A three-dimensional figure that consists of a set of points in space that are equidistant from a fixed point called the center.
Square Root	The square root of a number is the factor that we can multiply by itself to get that number. The symbol for square root is $\sqrt{\ }$. Finding the square root of a number is the opposite of squaring a number. For example $\sqrt{25} = \pm 5$, since $5 \bullet 5 = 25$ and $(-5) \bullet (-5) = 25$.
Standard Algorithm	One of the conventional algorithms used in the U.S. based on place value and properties of operations for addition, subtraction, multiplication, and division.
Statistical Question	A question that anticipates variability in the data.
Subitize	Quickly see how many objects are in a group without counting. There are two types of subitizing: perceptual and conceptual. Perceptual - instantly see how many objects without counting, usually done with up to 5 objects. Conceptual - quickly seeing small groups of objects as parts of a whole, usually done with more than 5 objects.
Substitution	1. Use of a numerical value to replace a variable. 2. A strategy for solving systems of equations that include solving for one variable and using that solution to find the other variable.
Sum	The resulting quantity when two or more addends are combined. The answer to an addition problem.
Supplementary Angles	Two angles(adjacent or nonadjacent) for which the sum of their measures is 180°.
Surface Area	The sum of the areas of all the faces of a three-dimensional (solid) figure or object.

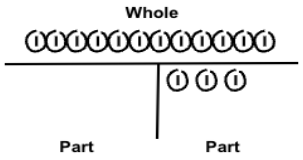
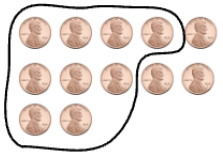
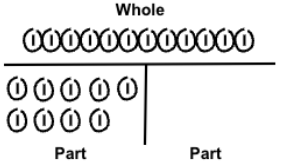
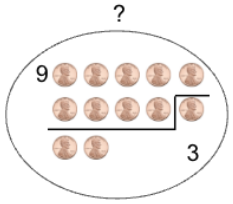
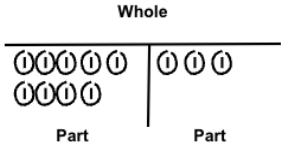
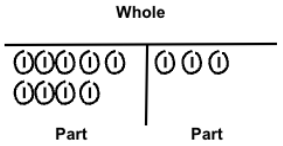
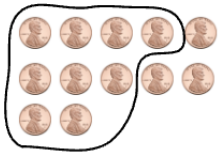
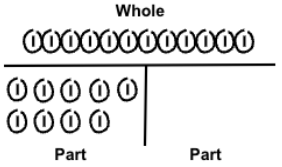
Tape Diagram	A rectangular model that looks like a segment of tape used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
Term	Terms are constants, variables, or the product or quotient of constant(s) and variable(s).
Terminating Decimal	The decimal form of a rational number that ends in 0. Example: $\frac{1}{2} = 0.50$
Theoretical Probability	The ratio of the number of favorable outcomes to the number of possible outcomes.
Three-Dimensional	An object that has three dimensions such as height, width, and depth.
Transformation	A rule that assigns to each point of a figure another point in the plane.
Translation	A transformation where each point is moved in the same direction and the same distance.
Trapezoid	A quadrilateral with *at least* one pair of parallel sides.
Tree Diagram	A diagram that shows the possible outcomes of an event by means of a connected, branching graph.
Triangular Prism	A three-dimensional figure made up of two triangular bases and three rectangular sides or faces.
Two-Dimensional	A flat figure or shape that has two dimensions such as length and width.
Unit Fraction	A fraction where the numerator is 1 and the denominator is a positive integer.
Unit Rate	A comparison of two measurements in which one of the terms has a value of 1.
Unlikely Event	A chance event with a probability between 0 and 0.5; the closer the probability is to being 0, the less likely the event is to occur.
Value	Numerical worth or amount.
Variable	1. A symbol used to represent an unknown or undetermined value in an expression or equation. 2. A quantity that may change within the context of a mathematical problem.
Variability	A measure of the dispersion or spread of data.
Vertex (Plural: Vertices)	A point where two or more line segments meet.
Visual Fraction Model	A graphic representation of fractions, such as a tape diagram, number line diagram, or an area model.
Volume	The amount of space contained in a three-dimensional figure; measured in cubic units.
Whole Numbers	A set of numbers containing all natural numbers and 0 (0, 1, 2, 3…).
X-Coordinate	The first number in an ordered pair representing the point's distance from the y-axis.
X-Intercept	A point where a graph of an equation intersects the x-axis.
Y-Coordinate	The second number in an ordered pair representing the point's distance from the x-axis.
Y-Intercept	A point where a graph of an equation intersects the y-axis.

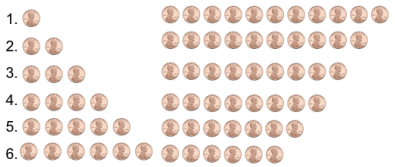
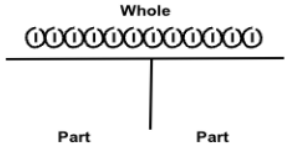
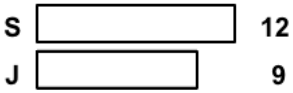
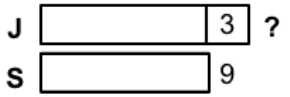
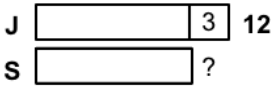
APPENDIX

Problem Types

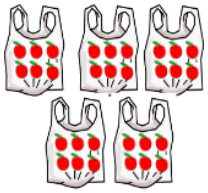
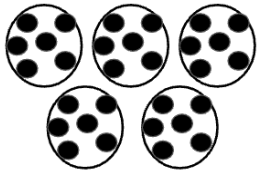
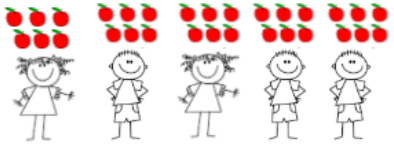
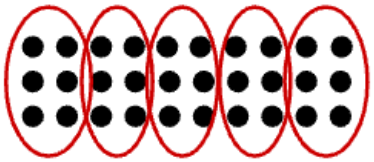
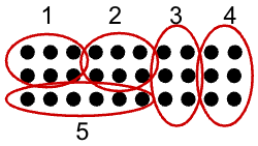
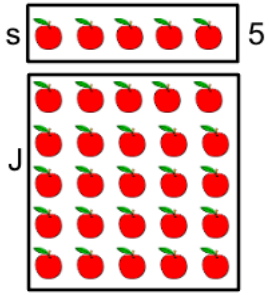
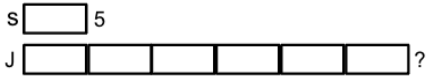
Addition & Subtraction

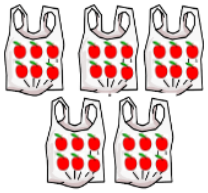
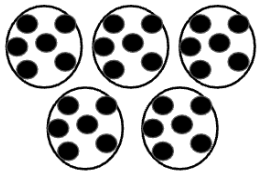
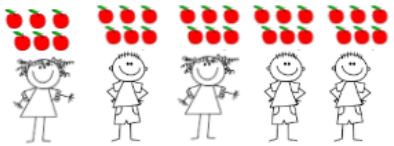
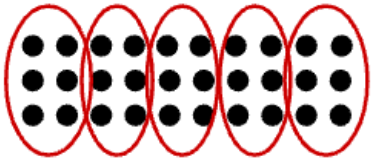
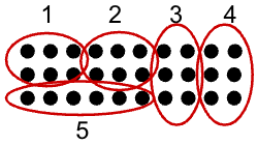
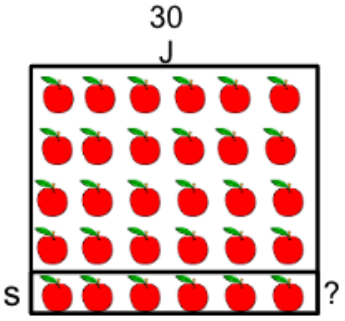
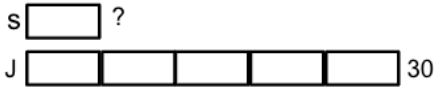
		Problem	Concrete	Representational	Abstract/ Equation(s)
Join	**Result Unknown**	Sarah had 9 pennies. Jacob gave her 3 more. How many pennies does Sarah have altogether?			$9 + 3 = \square$
	Change Unknown	Sarah had 9 pennies. Jacob gave her some more. Now Sarah has 12 pennies. How many pennies did Jacob give her?			$9 + \square = 12$
	Start Unknown	Sarah had some pennies. Jacob gave her 3 more pennies. Now Sarah has 12 pennies. How many did Sarah have in the beginning?			$\square + 3 = 12$

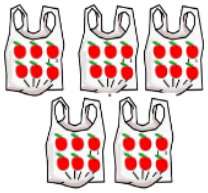
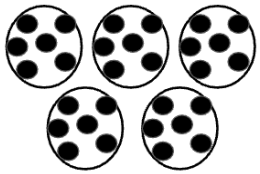
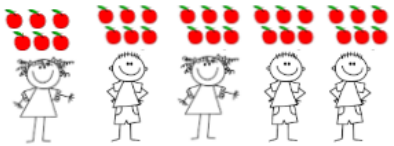
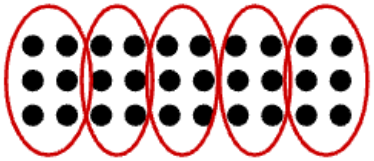
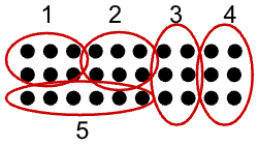
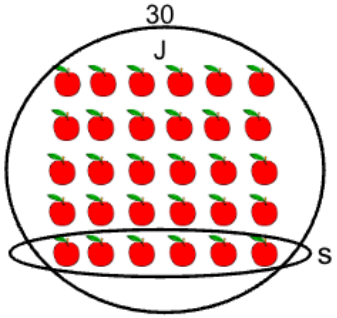
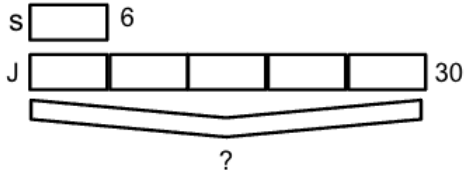
Separate	**Result Unknown**	Sarah had 12 pennies. She gave 3 pennies to Jacob. How many pennies does Sarah have now?			$12 - 3 = \square$
	Change Unknown	Sarah had 12 pennies. She gave some to Jacob. Now Sarah has 9 pennies. How many pennies did she give Jacob?			$12 - \square = 9$
	Start Unknown	Sarah had some pennies. She gave 3 to Jacob. Now Sarah has 9 pennies. How many pennies did she start with?			$\square - 3 = 9$
Part-Part -Whole	**Whole Unknown**	Jacob has 3 pennies and Sarah has 9 pennies. How many pennies do they have altogether?			$9 + 3 = \square$
	Part Unknown	Jacob and Sarah have 12 pennies in a jar. Sarah put in 9 pennies. How many pennies did Jacob put in the jar?			$12 = 9 + \square$

	Both Parts Unknown	There are 12 pennies in the jar. They want to put the pennies into two containers. How many pennies can they put into each container?			$1 + 11 = 12$ $2 + 10 = 12$ $3 + 9 = 12$ $4 + 8 = 12$ $5 + 7 = 12$ $6 + 6 = 12$
Compare	**Difference Unknown**	Sarah has 12 pennies and Jacob has 9 pennies. How many more pennies does Sarah have than Jacob?			$12 - \square = 9$ $9 + \square = 12$
	Larger Unknown	Jacob has 3 more pennies than Sarah. Sarah has 9 pennies. How many pennies does Jacob have?			$9 + 3 = \square$ $3 + 9 = \square$
	Smaller Unknown	Jacob has 3 more pennies than Sarah. Jacob has 12 pennies. How many pennies does Sarah have?			$12 - 3 = \square$ $3 + \square = 12$

Multiplication & Division

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \cdot 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \cdot \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \cdot 6 = 30$ $30 \div 6 = \square$
Comparison	**Product Unknown (Multiplication)**	Sarah picked 5 apples. Jacob picked 6 times as many apples as Sarah. How many apples did Jacob pick?			$5 \cdot 6 = \square$

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \bullet 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \bullet \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \bullet 6 = 30$ $30 \div 6 = \square$
	Set Size Unknown (Partitive Division)	Jacob picked 30 apples. He picked 5 times as many apples as Sarah. How many apples did Sarah pick?			$5 \bullet \square = 30$ $30 \div 5 = \square$

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \cdot 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \cdot \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \cdot 6 = 30$ $30 \div 6 = \square$
	Multiplier Unknown (Measurement Division)	Jacob picked 30 apples. Sarah only picked 6 apples. How many times as many apples did Jacob pick than Sarah?			$\square \cdot 6 = 30$ $30 \div 6 = \square$

Scaffolding Addition and Subtraction

When introducing grade-level addition and subtraction problem types, some students will need additional assistance with engaging in the problem. Considering students in the classroom progress at different rates, educators should use meaningful scaffolds that support grade-level content. Changing the number size is one strategy for helping students. When implementing these strategies, the scaffold aims to implement just-in-time support that compels student thinking toward grade-level content. Remember that teaching skills in context rather than isolation are practical and effective scaffolds for students.

At the end of fourth grade, Arkansas students are expected to be fluent with multi-digit addition and subtraction and the standard algorithm.

Addition		
Grade Level	**Description**	**Example**
K	1 digit + 1 digit	3 + 7
1, 2	1 digit + 1 digit	8 + 7
1	2 digit + 1 digit (no regrouping)	21 + 7
1	2 digit + a multiple of 10	34 + 20
2	2 digit + 1 digit (regrouping)	38 + 6
2	2 digit + 2 digit (no regrouping)	43 + 24
2	2 digit + 2 digit (regrouping)	37 + 28
2	3 digit + 1 digit (no regrouping)	351 + 4
2	3 digit + 1 digit (regrouping)	426 + 8
2	3 digit + 2 digit (no regrouping)	523 + 43

2	3 digit + 2 digit (regrouping)	728 + 76
2, 3	3 digit + 3 digit (no regrouping)	284 + 112
2, 3	3 digit + 3 digit (regroup ones to tens)	338 + 247
2, 3	3 digit + 3 digit (regroup tens to hundreds)	592 + 235
2, 3	3 digit + 3 digit (regroup ones to tens and tens to hundreds)	456 + 275
3	3 digit + 3 digit (regrouping with hundreds)	571 + 718
3	3 digit + 3 digit (regroup all places)	684 + 548
4	4 digit + 3 digit	1,245 + 322
4	4 digit + 4 digit	2,856 + 4,479
4	5 digit + 4 digit	24,732 + 5,201
4	5 digit + 5 digit	56,902 + 38,392
4	6 digit + 5 digit	823,651 + 62,745
4	6 digit + 6 digit	562,884 + 319,023

Subtraction		
Grade Level	**Description**	**Example**
K	1 digit - 1 digit	7 - 3

1	multiple of 10 - multiple of 10	30 - 20
1	2 digit - 10	54 - 10
1, 2	2 digit - 1 digit (within 20; no regrouping)	15 - 3
1, 2	2 digit - 1 digit (within 20; regrouping)	12 - 8
2	2 digit - 1 digit (greater than 20; no regrouping)	38 - 6
2	2 digit - 1 digit (greater than 20; regrouping)	42 - 7
2	2 digit - 2 digit (no regrouping)	43 - 21
2	2 digit - 2 digit (regrouping)	87 - 38
2	3 digit - 1 digit (no regrouping)	357 - 4
2	3 digit - 1 digit (regrouping)	422 - 8
2	3 digit - 2 digit (no regrouping)	583 - 43
2	3 digit - 2 digit (regrouping)	724 - 76
2, 3	3 digit - 3 digit (no regrouping)	284 - 112
2, 3	3 digit - 3 digit (regroup tens to ones)	332 - 217
2, 3	3 digit - 3 digit (regroup hundreds to tens)	532 - 290
2, 3	3 digit - 3 digit (regroup tens to ones and hundreds to tens)	451 - 275
4	4 digit - 3 digit	1,245 - 322

4	4 digit - 4 digit	4,856 - 2,479
4	5 digit - 4 digit	24,732 - 5,201
4	5 digit - 5 digit	56,902 - 38,392
4	6 digit - 5 digit	823,651 - 62,745
4	6 digit - 6 digit	562,884 - 319,023

Table 1: Properties of Operations	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a, there exists –a so that $a + (-a - a) = (-a - a) + aa = 0$.
Associative property of multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Commutative property of multiplication	$a \bullet b = b \bullet a$
Multiplicative identity property of 1	$a \bullet 1 = 1 \bullet a = a$
Existence of multiplication inverses	For every $a \neq 0$, there exists $\frac{1}{a}$ so that $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$.
Distributive property of multiplication over addition	$a \bullet (b + c) = a \bullet b + a \bullet c$

Table 2: Properties of Equality	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$, and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \bullet c = b \bullet c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 3: Properties of Inequality
Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, $b < a$.
If $a > b$, then $a + c > b + c$.
If $a > b$, then $a - c > b - c$.
If $a > b$ and $c > 0$, then $a \cdot c > b \cdot c$.
If $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 4: Inequalities	
>	Greater than
<	Less Than
=	equal
≈	approximately
≠	Not equal to
≤	Less than or equal to
≥	Greater than or equal to
Graphing inequalities	
○	Open circle for non-inclusion symbols: >, <, or ≠
⚫	Closed circle for inclusion symbols: ≤, ≥, or =

Table 5: Exponent Rules				
Product Rule	$a^m \cdot a^n = a^{m+n}$		Zero Exponent Rule	$a^0 = 1$
Quotient Rule	$a^m \div a^n = a^{m-n}$		Identity Exponent Rule	$a^1 = a$
Power of a Power Rule	$(a^m)^n = a^{mn}$		Negative Exponent Rule	$a^{-m} = \frac{1}{a^m}$
Power of a Product Rule	$(ab)^m = a^m b^m$			
Power of Quotient Rule	$(\frac{a}{b})^m = \frac{a^m}{b^m}$			