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Total No. of Questions: [09]

B.Sc. Non-Medical (Semester – 4th)

ALGEBRA-II

Subject Code: BSNMS1-407

Paper ID: [22131423]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Let R be a ring and $a \in R$, show that $a.0 = 0.a = 0$.
b. Prove that the unity of a ring is unique.
c. Give an example of a non-commutative ring with unity.
d. Show that every field is an integral domain.
e. State Sylow's second theorem.
f. State Cauchy-Schwartz inequality.
g. Find the eigen values of the matrix $A = [0 \ -1 \ -1 \ 0]$
h. Let V be an inner product space, show that $(0, v) = 0 \ \forall \ v \in V$.
i. Prove that null space of a L.T. $T : V(F) \to W(F)$ is a subspace of vector space $V(F)$.
j. Show that the mapping $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (x - y + z, 2x)$ is a Linear Transformation.

Section – B **(5 marks each)**

Q2. Prove that a commutative ring is an integral domain if and only if $\forall a, b, c \in R \ (a \neq 0)$, $ab = ac \Rightarrow b = c$.

Q3. If in a ring R, $x^2 = x \ \forall \ x \in R$, then show that $2x = 0$ and $x + y = 0 \Rightarrow x = y$.

Q4. Find all the ring homomorphisms from $Z_{20} \to Z_{30}$.

Q5. Let T be a Linear Transformation on V, such that, rank $\left(T^2\right)$ =rank (T), then show that range $(T \cap Ker\ T) = \{0\}$.

Q6. Let S be an orthogonal set of non-zero vectors in an inner product space V. Then prove that S is a linearly independent set.

Section – C **(10 marks each)**

Q7. a) State and prove Cayley's theorem.

b) Prove that Z_n is a commutative ring under addition and multiplication modulo n.

Q8. a) Let V be a vector space of 2×2 matrices over R and $P = [1 \ -1 \ -2 \ 2]$. Let $T: V \to V$ be a linear transformation defined by $T(A) = PA, \ \forall A \in V$. Find basis and dimensions of

(i) Null space of T (ii) Range space of T.

b) Let V be a non-zero inner product space of dimension n. Then show that V has an orthonormal basis.

Q9. a) Verify Rank-Nullity theorem for the L.T. $T : R^2 \to R^2$ defined by $T(x, y) = (x + y, x - y)$.

b) Let $f : R \to R'$ be a ring homomorphism, then prove that Ker (f) is an ideal of ring R.