**Roll No……..** **Total No. of Printed Pages: 1**

**Total No. of Questions: [09]**

**B.Sc. Non-Medical (Semester – 4th)**

**ALGEBRA-II**

**Subject Code: BSNMS1-407**

**Paper ID: [22131423]**

**Time: 03 Hours** **Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A** **(2 marks each)**

Q1. Attempt the following:

a. Let $R$ be a ring and $a \in R$, show that $a.0 = 0.a = 0$.
b. Prove that the unity of a ring is unique.
c. Give an example of a non-commutative ring with unity.
d. Show that every field is an integral domain.
e. State Sylow's second theorem.
f. State Cauchy-Schwartz inequality.
g. Find the eigen values of the matrix $A = [0 \ -1 \ -1 \ 0]$
h. Let $V$ be an inner product space, show that $(0, v) = 0 \ \forall \ v \in V$.
i. Prove that null space of a L.T. $T: V(F) \to W(F)$ is a subspace of vector space $V(F)$.
j. Show that the mapping $T: V_3(R) \to V_2(R)$ defined by
$T(x, y, z) = (x - y + z, 2x)$ is a Linear Transformation.

**Section – B** **(5 marks each)**

Q2. Prove that a commutative ring is an integral domain if and only if $\forall a, b, c \in R$ $(a \neq 0)$, $ab = ac \Rightarrow b = c$.

Q3. If in a ring $R$, $x^2 = x \ \forall \ x \in R$, then show that $2x = 0$ and $x + y = 0 \Rightarrow x = y$.

Q4. Find all the ring homomorphisms from $Z_{20} \to Z_{30}$.

Q5. Let $T$ be a Linear Transformation on $V$, such that, rank $\left(T^2\right)$ =rank $(T)$, then show that range $(T \cap Ker\ T) = \{0\}$.

Q6. Let $S$ be an orthogonal set of non-zero vectors in an inner product space $V$. Then prove that $S$ is a linearly independent set.

**Section – C** **(10 marks each)**

**Q7.** a) State and prove Cayley's theorem.

b) Prove that $Z_n$ is a commutative ring under addition and multiplication modulo $n$.

Q8. a) Let V be a vector space of $2 \times 2$ matrices over $R$ and $P = [1 \ -1 \ -2 \ 2]$. Let $T: V \to V$ be a linear transformation defined by $T(A) = PA, \ \forall A \in V$. Find basis and dimensions of
(i) Null space of $T$ (ii) Range space of T.

b) Let $V$ be a non-zero inner product space of dimension $n$. Then show that $V$ has an orthonormal basis.

**Q9.** a) Verify Rank-Nullity theorem for the L.T. $T: R^2 \to R^2$ defined by $T(x, y) = (x + y, x - y)$.

b) Let $f: R \to R'$ be a ring homomorphism, then prove that Ker $(f)$ is an ideal of ring $R$ .