

Roll No.....
Total No. of Questions: [9]

Total No. of Printed Pages: [2]

B.Tech. (Mech., Agriculture) (Semester-1st)
MATHEMATICS – I (CALCULUS, LINEAR ALGEBRA)
Subject Code: BMATH2101
Paper ID: [19110009]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

- Q1. (a). State the Rabbe's Test.
(b). Write the relation between Beta and Gamma function.
(c). Check the validity of the Rolle's theorem for function $f(x) = |x|$ on the interval $[-1, 1]$.
(d). Define Rank of Matrix.
(e). State the Rank – Nullity theorem.

- (f). Discuss the convergence of series $\sum (\sqrt[3]{n^3 + 1} - n)$
(g). State and prove the Cauchy's second theorem on limits for sequence.

(h). If $v = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, Show that $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + \frac{1}{2} \cot v = 0$.

- (i). State the Taylor's theorem (with Lagrange's form of reminder).
(j). Find the Rank of the matrix $A = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 2 \ 1 \ 2]$.

Section – B

(5 marks each)

Q2. Prove that $\nabla \cdot \left\{ r \nabla \left(\frac{1}{r^3} \right) \right\} = \frac{3}{r^4}$

Q3. Expand \tan^x in the powers of $\left(x - \frac{\pi}{4} \right)$ upto first four terms.

Q4. If $z = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$. Prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + y^2 \frac{\partial^2 z}{\partial y^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} = (1 - 4 \sin^2 z) \sin 2z.$$

Q5. State and prove the Cayley-Hamilton Theorem.

Q6. Using the Cauchy's Integral test, discuss the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \quad p > 0.$$

Section – C

(10 marks each)

Q7. Find the volume of the solid formed by the revolution of the curve $y^2(2a-x) = x^3$ about its asymptotes.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q8. Find the Characteristic equation of the matrix A and verify that it is satisfied by A and hence A^{-1} .

Q9. If s_1 and s_2 are positive and $s_{n+1} = \sqrt{s_n s_{n-1}}$, prove that the sequences $s_1, s_3, s_5, \dots; s_2, s_4, s_6, \dots$ are the one increasing and the other decreasing and show that their common limit is $(s_1 s_2)^{\frac{1}{3}}$.