AP Precalculus 1.8 | Worksheet A | Teacher Explanation & Key

Teaching sequence: (1) factor; (2) identify original denominator exclusions; (3) solve numerator zeros; (4) determine end behavior from degrees; (5) build a sign chart.

A canceled factor creates a hole, not a zero. For ≥ or ≤, include defined zeros only; never include denominator zeros.

1. Find zeros of $f(x) = \frac{x+4}{x-2}$

Step 1 — Factor: Numerator: x + 4; denominator: x − 2.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: 2. Answer: x = -4.

End behavior: lim as x → ∞ = 1; lim as x → −∞ = 1. Compare leading terms to explain the limit.

Check: substitute any sample x-value from a selected interval to confirm its sign.

2. Find zeros of $f(x) = \frac{x^2}{x^2-2x-15}$

Step 1 — Factor: Numerator: x²; denominator: x² − 2x − 15.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -3, 5. Answer: x = 0.

End behavior: lim as x → ∞ = 1; lim as x → −∞ = 1. Compare leading terms to explain the limit.

Check: substitute any sample x-value from a selected interval to confirm its sign.

3. Find zeros of $f(x) = \frac{x^2-3x-4}{x^2-5x-6}$

Step 1 — Factor: Numerator: x² − 3x − 4; denominator: x² − 5x − 6.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -1, 6. Answer: x = 4.

Check: substitute any sample x-value from a selected interval to confirm its sign.

4. Find zeros of f(x) = $\frac{x^2 - 16}{x^2 - 3x - 4}$

Step 1 — Factor: Numerator: x² − 16; denominator: x² − 3x − 4.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -1, 4. Answer: x = -4.

Check: substitute any sample x-value from a selected interval to confirm its sign.

5. Find zeros of f(x) = $\frac{x^3 - 9x}{2x^2 + 5x - 3}$

Step 1 — Factor: Numerator: x³ − 9x; denominator: 2x² + 5x − 3.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -3, 1/2. Answer: x = 0, x = 3.

Check: substitute any sample x-value from a selected interval to confirm its sign.

6. Find zeros of f(x) = $\frac{x^2 - 5x + 6}{x - 3}$

Step 1 — Factor: Numerator: x² − 5x + 6; denominator: x − 3.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: 3. Answer: x = 2.

Check: substitute any sample x-value from a selected interval to confirm its sign.

7. Analyze $f(x) = \frac{x^2 - 3x - 10}{x^2 - 4x}$

Step 1 — Factor: Numerator: $x^2 - 3x - 10$; denominator: $x^2 - 4x$.

Domain: all real x except 0, 4. Zeros: -2, 5.

Horizontal asymptote: $y = 1$. y-intercept: none.

Sign chart: $(-\infty, -2)$: test $x = -3 \rightarrow$ positive; $(-2, 0)$: test $x = -1 \rightarrow$ negative; $(0, 4)$: test $x = 2 \rightarrow$ positive; $(4, 5)$: test $x = 9/2 \rightarrow$ negative; $(5, \infty)$: test $x = 6 \rightarrow$ positive.

Answer for $f(x) < 0$: $(-2, 0) \cup (4, 5)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

8. Analyze $f(x) = \frac{x^2 - 16}{x + 4}$

Step 1 — Factor: Numerator: $x^2 - 16$; denominator: $x + 4$.

Domain: all real x except -4. Zeros: 4.

Horizontal asymptote: none (end behavior is not constant). y-intercept: (0, -4).

Sign chart: $(-\infty, -4)$: test $x = -5 \rightarrow$ negative; $(-4, 4)$: test $x = 0 \rightarrow$ negative; $(4, \infty)$: test $x = 5 \rightarrow$ positive.

Answer for $f(x) > 0$: $(4, \infty)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

9. Analyze $f(x) = \frac{3x - 9}{x^2 - 2x - 15}$

Step 1 — Factor: Numerator: $3x - 9$; denominator: $x^2 - 2x - 15$.

Domain: all real x except -3, 5. Zeros: 3.

Horizontal asymptote: $y = 0$. y-intercept: $(0, 3/5)$.

Sign chart: $(-\infty, -3)$: test $x = -4 \rightarrow$ negative; $(-3, 3)$: test $x = 0 \rightarrow$ positive; $(3, 5)$: test $x = 4 \rightarrow$ negative; $(5, \infty)$: test $x = 6 \rightarrow$ positive.

Answer for $f(x) > 0$: $(-3, 3) \cup (5, \infty)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

10. Graph inequality for f(x) = $\frac{x+2}{x-3}$ >0 0

Step 1 — Factor: Numerator: x + 2; denominator: x − 3.

Step 2 — Mark critical numbers: -2, 3. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -2): test x = -3 → positive; (-2, 3): test x = 1/2 → negative; (3, ∞): test x = 4 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, -2) ∪ (3, ∞).

Graph interpretation: choose the x-values where the curve is above or on the x-axis when equality is allowed; never include a vertical asymptote.

Check: substitute any sample x-value from a selected interval to confirm its sign.

11. Graph inequality for f(x) = $\frac{1-x}{x+4}$ <=0 0

Step 1 — Factor: Numerator: 1 − x; denominator: x + 4.

Step 2 — Mark critical numbers: -4, 1. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -4): test x = -5 → negative; (-4, 1): test x = -3/2 → positive; (1, ∞): test x = 2 → negative. Equality includes numerator zeros when defined.

Answer: (−∞, -4) ∪ [1, ∞).

Graph interpretation: choose the x-values where the curve is below or on the x-axis when equality is allowed; never include a vertical asymptote.

Check: substitute any sample x-value from a selected interval to confirm its sign.

12. Solve inequality for f(x) = $\frac{x+5}{x-2}$ <0 0

Step 1 — Factor: Numerator: x + 5; denominator: x − 2.

Step 2 — Mark critical numbers: -5, 2. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -5): test x = -6 → positive; (-5, 2): test x = -3/2 → negative; (2, ∞): test x = 3 → positive. Equality includes numerator zeros when defined.

Answer: (-5, 2).

Check: substitute any sample x-value from a selected interval to confirm its sign.

13. Solve inequality for f(x) = $\frac{-4}{x+1}$ >=0 0

Step 1 — Factor: Numerator: -4; denominator: x + 1.

Step 2 — Mark critical numbers: -1. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -1): test x = -2 → positive; (-1, ∞): test x = 0 → negative. Equality includes numerator zeros when defined.

Answer: (−∞, -1).

Check: substitute any sample x-value from a selected interval to confirm its sign.

14. Solve inequality for f(x) = $\frac{x^2-x-6}{x-5}$ >0 0

Step 1 — Factor: Numerator: $x^2 - x - 6$; denominator: x − 5.

Step 2 — Mark critical numbers: -2, 3, 5. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -2): test x = -3 → negative; (-2, 3): test x = 1/2 → positive; (3, 5): test x = 4 → negative; (5, ∞): test x = 6 → positive. Equality includes numerator zeros when defined.

Answer: (-2, 3) ∪ (5, ∞).

Check: substitute any sample x-value from a selected interval to confirm its sign.

15. Solve inequality for f(x) = $\frac{x^2+4x+4}{x-4}$ <=0 0

Step 1 — Factor: Numerator: $x^2 + 4x + 4$; denominator: x − 4.

Step 2 — Mark critical numbers: -2, 4. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -2): test x = -3 → negative; (-2, 4): test x = 1 → negative; (4, ∞): test x = 5 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, 4).

Check: substitute any sample x-value from a selected interval to confirm its sign.

16. Solve inequality for f(x) = $\frac{x^2+2x-3}{x^2+3x-10}$ >=0 0

Step 1 — Factor: Numerator: x² + 2x − 3; denominator: x² + 3x − 10.

Step 2 — Mark critical numbers: -5, -3, 1, 2. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -5): test x = -6 → positive; (-5, -3): test x = -4 → negative; (-3, 1): test x = -1 → positive; (1, 2): test x = 3/2 → negative; (2, ∞): test x = 3 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, -5) ∪ [-3, 1] ∪ (2, ∞).

Check: substitute any sample x-value from a selected interval to confirm its sign.

Teacher video resources — links for each problem type

Zeros, intercepts and numerator factors (Questions 1–6): https://www.khanacademy.org/v/graphs-of-rational-functions-zeros

Asymptotes, canceled factors, graphing (Questions 7–11): https://www.khanacademy.org/v/finding-asymptotes-example

Solving rational inequalities by test intervals (Questions 7–16): https://www.khanacademy.org/math/precalculus-tx/xecf67014514b60db%3Arational-functions-equations-and-inequalities/xecf67014514b60db%3Asolving-rational-equations-and-inequalities/v/solving-factored-form-rational-inequalities

Inequalities read directly from a graph (Questions 10–11): https://www.khanacademy.org/math/precalculus-tx/xecf67014514b60db%3Arational-functions-equations-and-inequalities/xecf67014514b60db%3Asolving-rational-equations-and-inequalities/v/solving-rational-inequalities-from-a-graph

AP Precalculus Topic 1.8 lesson video: https://www.youtube.com/watch?v=K90mV6M7aNw

User-provided 1.8A worksheet worked-solutions video (original, not these new numbers): https://drive.google.com/file/d/1wFQk-ySME9DVIRhhr1RQJWGiEyA-HxF2/view

Video links explain the same METHODS, but may use different example values than these original questions.