See this page in the course material.

Learning Outcomes

- Identify a power function.
- Describe the end behavior of a power function given its equation or graph.

Three birds on a cliff with the sun rising in the background. Functions discussed in this module can be used to model populations of various animals, including birds. (credit: Jason Bay, Flickr)

Suppose a certain species of bird thrives on a small island. Its population over the last few years is shown below.

Year	2009	2010	2011	2012	2013
Bird Population	800	897	992	1,083	1,169

The population can be estimated using the function [latex]P\left(t\right)=-0.3{t}^{3}+97t+800[/latex], where [latex]P\left(t\right)[/latex] represents the bird population on the island *t* years after 2009. We can use this model to estimate the maximum bird population and when it will occur. We can also use this model to predict when the bird population will disappear from the island.

Identifying Power Functions

In order to better understand the bird problem, we need to understand a specific type of function. A **power function** is a function with a single term that is the product of a real number, **coefficient,** and variable raised to a fixed real number power. Keep in mind a number that multiplies a variable raised to an exponent is known as a **coefficient**.

As an example, consider functions for area or volume. The function for the **area of a circle** with radius [latex]r[/latex] is:

[latex]A\left(r\right)=\pi {r}^{2}[/latex]

and the function for the **volume of a sphere** with radius *r* is:

[latex]V\left(r\right)=\frac{4}{3}\pi {r}^{3}[/latex]

Both of these are examples of power functions because they consist of a coefficient, [latex]\pi [/latex] or [latex]\frac{4}{3}\pi [/latex], multiplied by a variable *r* raised to a power.

A General Note: Power FunctionS

A **power function** is a function that can be represented in the form

[latex]f\left(x\right)=a{x}^{n}[/latex]

where *a* and *n* are real numbers and *a* is known as the **coefficient**.

Q & A

Is [latex]f\left(x\right)={2}^{x}[/latex] a power function?

No. A power function contains a variable base raised to a fixed power. This function has a constant base raised to a variable power. This is called an exponential function, not a power function.

Example: Identifying Power Functions

Which of the following functions are power functions?

$$\begin{array}{c}f\left(x\right)=1\hfill & \text{Constant function}\hfill \\ f\left(x\right)=x\hfill & \text{Identify function}\hfill \\ f\left(x\right)={x}^{2}\hfill & \text{Quadratic}\text{ }\text{ function}\hfill \\ f\left(x\right)={x}^{3}\hfill & \text{Cubic function}\hfill \\ f\left(x\right)=\frac{1}{x} \hfill & \text{Reciprocal function}\hfill \\ f\left(x\right)=\frac{1}{{x}^{2}}\hfill & \text{Reciprocal squared function}\hfill \\ f\left(x\right)=\sqrt{x}\hfill & \text{Square root function}\hfill \\ f\left(x\right)=\sqrt[3]{x}\hfill & \text{Cube root function}\hfill \end{array}$$

Show Solution

All of the listed functions are power functions.

The constant and identity functions are power functions because they can be written as $f\left(x\right)={x}^{0}$ and $f\left(x\right)={x}^{1}$ respectively.

The quadratic and cubic functions are power functions with whole number powers $f\left(x\right)={x}^{2}$ and $f\left(x\right)={x}^{3}$.

The **reciprocal** and reciprocal squared functions are power functions with negative whole number powers because they can be written as $f\left(x\right)={x}^{-1}$ and $f\left(x\right)={x}^{-2}$.

The square and **cube root** functions are power functions with fractional powers because they can be written as $f\left(x\right)={x}^{1/2}$ or $f\left(x\right)={x}^{1/3}$.

Try It

Which functions are power functions?

[latex]\begin{array}{c}f\left(x\right)=2{x}^{2}\cdot 4{x}^{3}\hfill \\ g\left(x\right)=-{x}^{5}+5{x}^{3}-4x\hfill \\ h\left(x\right)=\frac{2{x}^{5}-1}{3{x}^{2}+4}\hfill \end{array}[/latex]

Show Solution

[latex]f\left(x\right)[/latex] is a power function because it can be written as [latex]f\left(x\right)=8{x}^{5}[/latex]. The other functions are not power functions.

Identifying End Behavior of Power Functions

The graph below shows the graphs of [latex]f\left(x\right)={x}^{2},g\left(x\right)={x}^{4}[/latex], [latex]h\left(x\right)={x}^{6}[/latex], [latex]k(x)=x^{8}[/latex], and [latex]p(x)=x^{10}[/latex] which are all power functions with even, whole-number powers. Notice that these graphs have similar shapes, very much like that of the quadratic function. However, as the power increases, the graphs flatten somewhat near the origin and become steeper away from the origin.

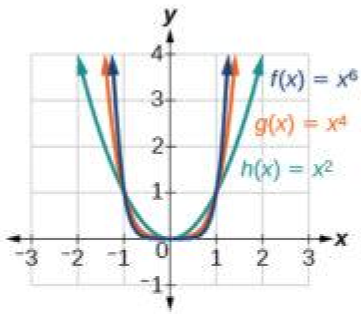
To describe the behavior as numbers become larger and larger, we use the idea of infinity. We use the symbol [latex]\infty[/latex] for positive infinity and [latex]-\infty[/latex] for negative infinity. When we say that "*x* approaches infinity," which can be symbolically written as [latex]x\to \infty[/latex], we are describing a behavior; we are saying that *x* is increasing without bound.

With even-powered power functions, as the input increases or decreases without bound, the output values become very large, positive numbers. Equivalently, we could describe this behavior by saying that as [latex]x[/latex] approaches positive or negative infinity, the [latex]f\left(x\right)[/latex] values increase without bound. In symbolic form, we could write

[latex]\text{as }x\to \pm \infty , f\left(x\right)\to \infty[/latex]

The graph below shows
[latex]f\left(x\right)={x}^{3},g\left(x\right)={x}^{5},h\left(x\right)={x}^{7},k\left(x\right)={x}^{9},\text{a

nd }p\left(x\right)={x}^{11}[/latex], which are all power functions with odd, whole-number powers. Notice that these graphs look similar to the cubic function. As the power increases, the graphs flatten near the origin and become steeper away from the origin.



These examples illustrate that functions of the form [latex]f\left(x\right)={x}^{n}[/latex] reveal symmetry of one kind or another. First, in the even-powered power functions, we see that even functions of the form [latex]f\left(x\right)={x}^{n}\text{, }n\text{ even,}[/latex] are symmetric about the *y*-axis. In the odd-powered power functions, we see that odd functions of the form [latex]f\left(x\right)={x}^{n}\text{, }n\text{ odd,}[/latex] are symmetric about the origin.

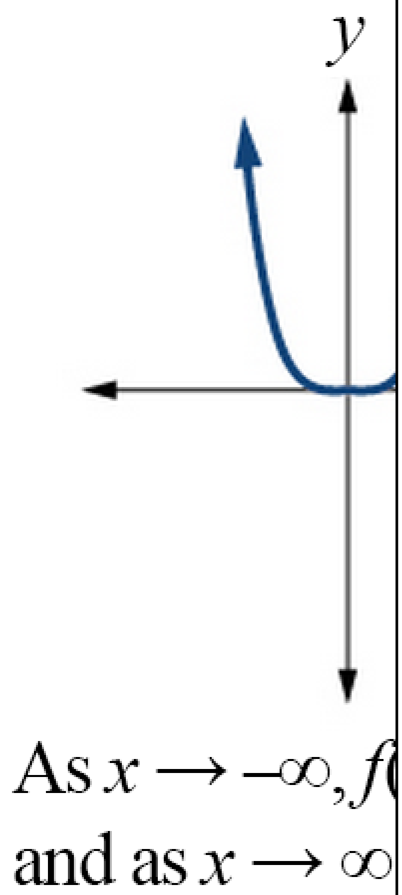
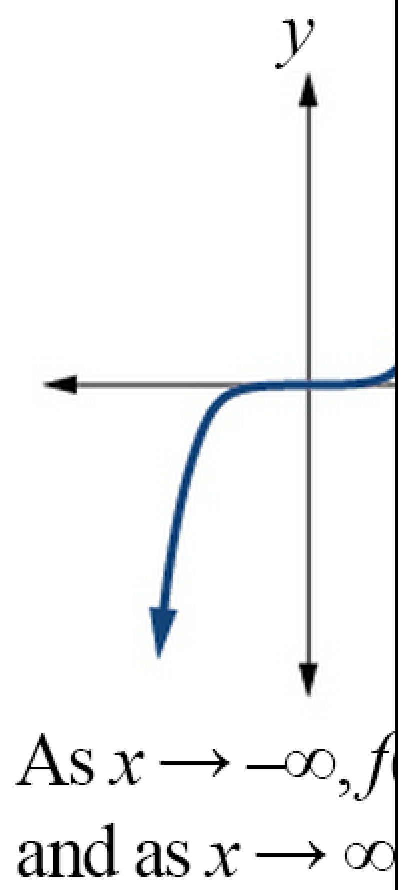
For these odd power functions, as *x* approaches negative infinity, [latex]f\left(x\right)[/latex] decreases without bound. As *x* approaches positive infinity, [latex]f\left(x\right)[/latex] increases without bound. In symbolic form we write

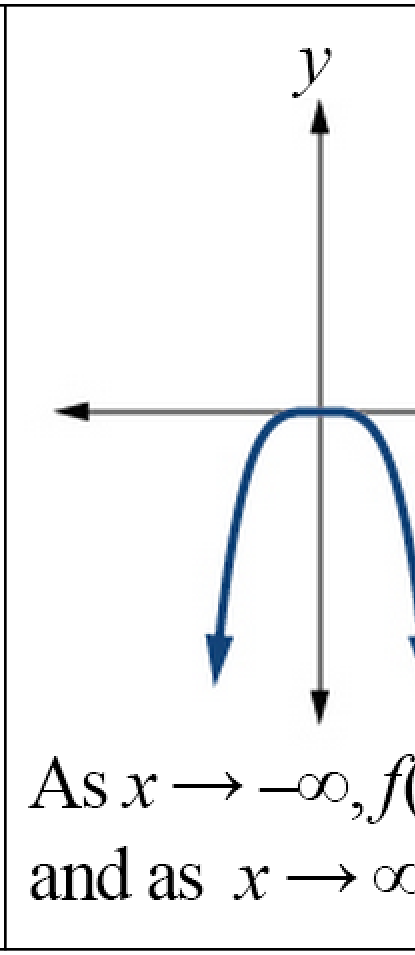
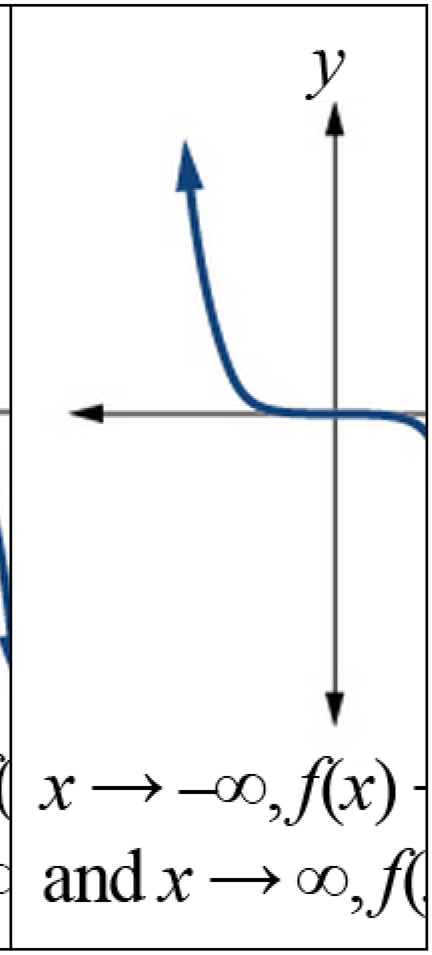
[latex]\begin{array}{c}\text{as } x\to -\infty , f\left(x\right)\to -\infty \\ \text{as } x\to \infty , f\left(x\right)\to \infty \end{array}[/latex]

The behavior of the graph of a function as the input values get very small ([latex]x\to -\infty[/latex]) and get very large ([latex]x\to \infty[/latex]) is referred to as the **end behavior** of the function. We can use words or symbols to describe end behavior.

The table below shows the end behavior of power functions of the form [latex]f\left(x\right)=a{x}^{n}[/latex] where [latex]n[/latex] is a non-negative integer depending on the power and the constant.

	Even power	Odd power

Positive constant ***a* > 0**		

Negative constant ***a* < 0**		

How To: Given a power function [latex]f\left(x\right)=a{x}^{n}[/latex] where [latex]n[/latex] is a non-negative integer, identify the end behavior.

1. Determine whether the power is even or odd.
2. Determine whether the constant is positive or negative.
3. Use the above graphs to identify the end behavior.

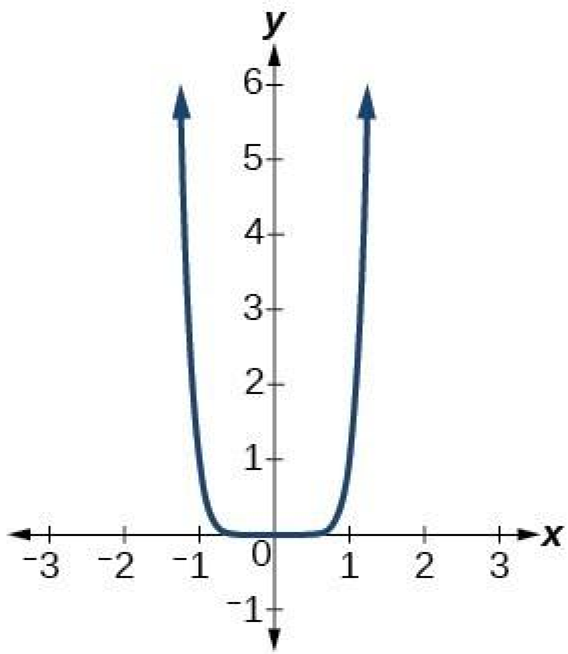
Example: Identifying the End Behavior of a Power Function

Describe the end behavior of the graph of [latex]f\left(x\right)={x}^{8}[/latex].

Show Solution

The coefficient is 1 (positive) and the exponent of the power function is 8 (an even number). As

x (input) approaches infinity, [latex]f\left(x\right)[/latex] (output) increases without bound. We write as [latex]x\to \infty , f\left(x\right)\to \infty [/latex]. As x approaches negative infinity, the output increases without bound. In symbolic form, as [latex]x\to -\infty , f\left(x\right)\to \infty[/latex]. We can graphically represent the function.

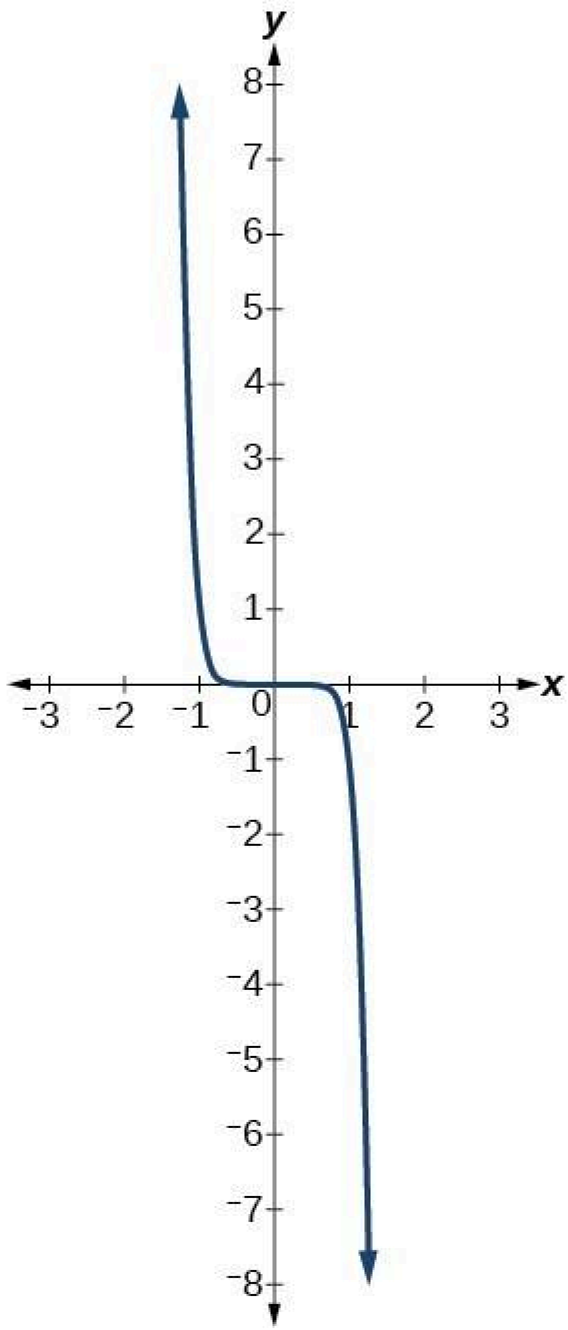


Example: Identifying the End Behavior of a Power Function

Describe the end behavior of the graph of [latex]f\left(x\right)=-{x}^{9}[/latex].

Show Solution

The exponent of the power function is 9 (an odd number). Because the coefficient is –1 (negative), the graph is the reflection about the x-axis of the graph of [latex]f\left(x\right)={x}^{9}[/latex]. The graph shows that as x approaches infinity, the output decreases without bound. As x approaches negative infinity, the output increases without bound. In symbolic form, we would write as [latex]x\to -\infty , f\left(x\right)\to \infty[/latex] and as [latex]x\to \infty , f\left(x\right)\to -\infty[/latex].



Try It

Describe in words and symbols the end behavior of [latex]f\left(x\right)=-5{x}^{4}[/latex].

Show Solution

As *x* approaches positive or negative infinity, [latex]f\left(x\right)[/latex] decreases without bound: as [latex]x\to \pm \infty , f\left(x\right)\to -\infty[/latex] because of the negative coefficient.

[See this interactive in the course material.]

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