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M.Sc. (Mathematics) (Semester – 3rd)
MATHEMATICAL METHODS
Subject Code: MMAT1-314
Paper ID: [19220514]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It carries 16 marks. It consists of 4 questions of 4 marks each.
2. Section B consist of 4 questions of 8 marks each. The student has to attempt any 3 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(4 marks each)**

Q1. Convert the following differential equation into integral equation

$$y(x) = \int_0^x (x - t)y(t)dt + 3sinx$$

Q2. Solve the I.E $y(x) = x + 2\int_0^x \cos cos\,(x - t)y(t)dt$.

Q3. Find the curves on which the functional $\int_0^1 \left[\left(y^1\right)^2 + 12xy\right]dx$ with $y(0) = 0$ and $y(1)1$ can be Extremised.

Q4. Show that the geodesics on a plane are straight lines.

Section – B **(8 marks each)**

Q5. Find the Eigen values and Eigen function of $y(x) = \lambda\int_0^{2\pi} \sin sin\,(x + t)y(t)dt$.

Q6. Prove that a necessary condition for $I = \int_{x_1}^{x_2} f(x, y, y', y'')dx$ to be extremum is

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$$

Q7. Find a function $y(x)$ such that $\int_0^\pi y^2dx = 1$ which makes $\int_0^\pi \left(\frac{d^2y}{dx^2}\right)^2 dx$ a minimum of $y(0) = 0 = y(\pi),\ y''(0) = 0 = y''(\pi)$

Q8. Solve the integral equation $\int_0^x \frac{y(t)}{(x-t)^{1/3}}dt = x(x + 1)$

Section – C **(10 marks each)**

Q9. Prove that the sphere is the solid figure of revolution which for a given surface area has maximum value.

Q10. Find the resolvent kernel of the Volterra integral equation with kernel $K(x,t) = 1$.

Q11. (a) Prove that the Eigen values of a symmetric kernel are real.

(b) Prove that the Eigen function of a symmetric kernel corresponding to different Eigen values are orthogonal.