UNIT-III

ELECTROMAGNETIC WAVES AND FIBER OPTICS

SYNOPICS:

1. ELECTROMAGNETICS:

A branch of physics in which electric and magnetic phenomena are studied is called 'Electromagnetic'

2. DIVERGENCE OF A VECTOR:

"Divergence of a vector field at any point is defined as the amount of flux per unit point".

$$\text{Div A} = \nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

3. CURL OF A FIELD:

The curl of a vector field is defined as the maximum line integral of the vector per unit area.

$$\text{Curl A} = \nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

4. STOKES THEOREM:

Stokes theorem states that the line integral of a vector field 'A' around a closed curve is equal to the surface integral of the curl of 'A' taken over the surface 'S' surrounded by the closed curve.

$$\oint_c A.dl = \iint_s curl A.ds = \iint_s (\nabla x\, A) ds$$

5. GAUSS THEOREM:

The gauss theorem states that the surface integral of the normal component of vector 'A' taken over a closed surface 'S' is equal to the volume integral of the divergence of vector 'A' over the volume "V" enclosed by the surface 'S'

$$\iint_S A.ds = \iiint_v div A dv = \iiint_v (\nabla . A) dv$$

6. MAXWELL'S EQUATIONS (DIFFERENTIAL FORM):

1. $\nabla . E = \frac{\rho}{\varepsilon_0}$ (Gauss law for electricity)
2. $\nabla . B = 0$ (Gauss law for magnetism)
3. $\nabla \times E = \frac{-\partial B}{\partial t}$
4. $\nabla \times H = (J + \frac{\partial D}{\partial t})$

7. MAXWELL'S EQUATION IINTEGRAL FORM:

1. $\oint E.ds = \frac{\rho}{\varepsilon_0}$

2. $\oint B.ds = 0$

3. $\oint E.dl = \frac{-\partial \phi B}{\partial t}$

4. $\oint B.dl = \mu_0 (I + \varepsilon_0 \frac{\partial \phi E}{\partial t})$

8. POYNTING THEOREM:

This theorem states that the cross product of electric field vector, E and magnetic field, H at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point.

$$P = E \times H$$

9. ELECTROMAGNETIC WAVE PROPAGATION:

The wave equation in which the wave propagation velocity is $c = \frac{1}{\sqrt{\mu\varepsilon}}$

10. OPTICAL FIBER:

Optical fiber is a thin and transparent guiding medium material which guide the Information carrying light waves

11. CRITICAL ANGLE:

At the core cladding interface the angle of incidence must be greater than the Critical angle defined as $\sin \theta_c = \frac{n_2}{n_1}$

12. ACCEPTANCE ANGLE:

It is defined as the maximum angle that a light ray can have relative to the axis of Fiber and propagate down the fiber

$$\theta_c = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

13. NUMERICAL APETURE:

Power gathering capacity is called numerical aperture it gives $NA=\sqrt{n_1^2 - n_2^2}$

14. TYPES OF OPTICAL FIBER:

Based on the number of modes, they are classified as 1.step index 2.graded index

15. PRINICIPALE OF THE OPTICAL FIBER COMMUNICATION:

A fiber optic system converted an electrical signal to an electrical signal to an light signal the signal is Transmitted through an optical fiber at the end of the optical Fiber. It is reconverted into an electrical signal.

16. APPLICATION:

Fiber optics endoscope is used in medical diagnosis it is used to visualize the inner organ of the body.

PART-A
SHORT QUESTIONS WITH SOLUTIONS

1. **Define divergence of a field?**

Ans: "Divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from that point"

Let 'A' be a vector function differentiable at (x,y,z) in a region of space. Then, divergence of A can be written as

$$\nabla . A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right](iA_x + jA_y + kA_z)$$

$$\text{div A} = \nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

2. **Define curl of a field?**

Ans: "The curl of a vector field is defined as the maximum line integral of the vector per unit area".

If 'A' is function differentiable at (x,y,z) in a space. Then, curl of 'A' can be shown by the cross product of '∇' and A.

$$\text{Curl A} = \nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

3. **Define stokes's theorem?**

Ans: Stokes theorem states that the line integral of a vector field 'A' around a closed curve is equal to the surface integral of the curl of 'A' taken over the surface 'S' surrounded by the closed curve.

$$\oint_c A.dl = \iint_s curlA.ds = \iint_s (\nabla x\, A)ds$$

4. **Define gauss's theorem?**

Ans: The gauss theorem states that the surface integral of the normal component of vector 'A' taken over a closed surface 'S' is equal to the volume integral of the divergence of vector 'A' over the volume "V" enclosed by the surface 'S'

$$\iint_S A.ds = \iiint_v divAdv = \iiint_v (\nabla . A)dv$$

5. **Write the Maxwell's equations in differential?**

Ans:

MAXWELL'S EQUATIONS (DIFFERENTIAL FORM):

1. $\nabla . E = \frac{\rho}{\varepsilon_0}$ (Gauss law for electricity)
2. $\nabla . B = 0$ (Gauss law for magnetism)

3. $\nabla \times E = \frac{-\partial B}{\partial t}$
4. $\nabla \times H = (J + \frac{\partial D}{\partial t})$

6. Write the Maxwell's equations in integral?
Ans: MAXWELL'S EQUATION IINTEGRAL FORM:

1. $\oint E.ds = \frac{\rho}{\varepsilon_0}$

2. $\oint B.ds = 0$

3. $\oint E.dl = \frac{-\partial \phi B}{\partial t}$

4. $\oint B.dl = \mu_0 (I + \varepsilon_0 \frac{\partial \phi E}{\partial t})$

7. States Poynting theorem?
Ans: POYNTING THEOREM:

This theorem states that the cross product of electric field vector, E and magnetic field, H at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point.

$$P = E \times H$$

8. What is an optical fiber and giving its types?
ANS. **<u>Optical fiber:</u>** An optic fiber is a transparent dielectric waveguide of smaller diameter thet transmission Optical data from one point to other point with minimal loss it works on the principle of total internal reflection Types of optical fiber.
The optical fibers are classified into two types they are:
1. Step index fiber 2. Graded index fiber
9. Write the relation between numerical aperture and acceptance angle of an optical fiber.
ANS. <u>Numerical aperture</u>: It is defined as the capability of the fiber together the light.

$$NA = n_1\sqrt{2\Delta} \quad \text{where } \Delta = \frac{n_1 - n_2}{n_1}$$

<u>Acceptance angle</u>: It is defined as the maximum value of that angle of incidence that a light ray process w.r.t the axis in order to propagate down the fiber.
10. What is total internal reflection?
ANS. when light travelling in an optically dense medium hits boundary at a steep angle. The Light is completely reflected this is called total internal reflection.
11 .Mention the application of optical fibers.
ANS. (1) It is also used in sensors and television cables
(2) To carry light from one place to other
(3) Used in medical field to see internal body parts.

12. What is the basic principle of fiber optic communication?

Ans: Total internal reflection is the basic principle of fiber optic communication system.

Principle: When light travels from a denser to a rarer medium, at a particular angle of incidence called critical angle, the ray emerges along the surface of separation. When angle of incidence exceeds the critical angle, the incident ray is reflected in the same medium and this phenomenon is called total internal reflection.

13. What is called mode of propagation in optical fibers?

Ans: Mode of propagation represents the number of possible directions or path of propagation of light through the optical fibers. When single ray of light propagate through a path, then it is called single mode and when many rays propagate through different directions, it is called multimode.

14. What is the role of cladding in optical fiber?

Ans: An optical fiber consists of core which is surrounded by cladding. Here the role of cladding is to make the light to suffer total internal reflection inside the fiber, satisfying the condition that the light should travel from denser medium to rarer medium.

15. Mention any four advantages of fiber optic sensors.

Ans: It has no external interference

(i) It is used in remote sensing.
(ii) Safety of data transfer.
(iii) It is small in size.

16. What are the requirements of light sources used in fiber optic communication?

Ans: 1. The light produced must be as nearly monochromatic as possible.
2. It must modulate the source at high speeds.
3. The light source should have compact size and high efficiency.
4. It should be reliable, durable and inexpensive.
5. It must require very small power for its operation.
6. Spectral line width of the source should be as small as possible.
7. Can operate continuously at room temperature for many years.
8. It should be modulated over a wide range of frequencies.

17. Distinguish between step index fiber and graded index fiber.

Ans:

Step Index Fiber	**Graded Index fiber**
1. The difference in refractive indices between the core and cladding is obtained in a single step and hence called as step index fiber.	1. The difference in refractive indices between the core and cladding gradually increases from centre to interface and hence called graded index fiber.
2. The light propagation in the form of meridional rays.	2. The light propagation in the form of skew rays.
3. The light rays pass through the fiber axis	3. The light rays do not cross the fiber
4. It follows a zigzag path of light propagation	4. It follows a helical path of light propagation
5. It has low bandwidth.	

6. Distortion is more	5. It has high bandwidth. 6. Distortion is very less

18. Distinguish between multimode fiber and single mode fiber

Ans:

Single mode fiber	Multimode fiber
1. In single mode fiber only one mode can be propagated 2.The single mode fiber has a smaller core Diameter and difference in refractive index Of core and cladding is small 3.No dispersion	The fiber in this case allows large number of modes for light to pass through it Here both core and cladding refractive indices difference is large as the core diameter in large Dispersion is more due to degradation of single Due to multimode

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

3.1. ELECTROMAGNETIC WAVES

3.1.1. DIVERGENCE AND CURL OF ELECTRIC AND MAGNETIC FIELD

1. **Derive divergence and curl of a field?**

Ans: **Divergence of a field:**

"Divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from that point"

Let 'A' be a vector function differentiable at (x,y,z) in a region of space. Then, divergence of A can be written as

$$\nabla . A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right](iA_x + jA_y + kA_z)$$

$$\text{div A} = \nabla . A = \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

curl of a field

"The curl of a vector field is defined as the maximum line integral of the vector per unit area".

If 'A' is function differentiable at (x,y,z) in a space. Then, curl of 'A' can be shown by the cross product of '∇' and A.

$$\text{Curl A} = \nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

3.1.2. GAUSS'S THEOREM FOR DIVERGENCE AND STOKES THEOREM FOR CURL

2. **State and explain gauss's theorem for divergence?**

Ans: Gauss's theorem:

Statement:

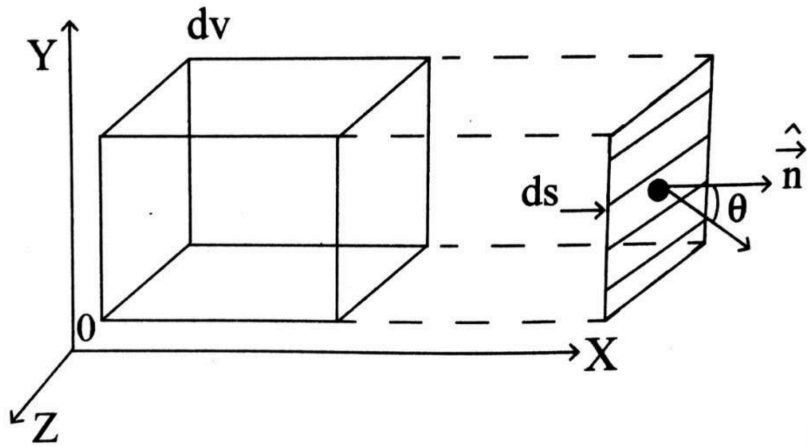
The gauss theorem states that the surface integral of the normal component of vector 'A' taken over a closed surface 'S' is equal to the volume integral of the divergence of vector 'A' over the volume "V' enclosed by the surface 'S'

$$\iint_S A.ds = \iiint_V divA dv = \iiint_V (\nabla . A) dv$$

The relation between volume integrals and surface integrals were proved using this theorem.

Theorem:

Let us consider a surface ‘S’ in a vector field ‘A’ as shown in fig. let ‘V’ be the volume enclosed in the surface and the volume is divided into a large number of cubical volume elements ‘dv’.



We have that ‘divA’ represents the amount of flux diverging per unit volume and hence, the flux diverging from the element of volume ’dv’ will be ‘div A dv’. Hence, the total flux coming from the entire volume can be written as

$$\iiint_V div\, A\, dv \text{---------------------}1$$

Consider a small element of area ‘ds’ with $\hat{n}$ unit vector drawn normal to ‘ds’. Here, outward drawn normal on a surface is positive.

For a field vector ’A’ and outward normal $\hat{n}$ at an angle ‘Θ’ the ‘A’ component along $\hat{n}$ can be written as

$$A \cos cos\, \theta = A.\hat{n}$$

The flux of ‘A’ through the surface element ‘ds’ is given bu

$$(A.\hat{n}) = A.ds$$

The total flux through the entire surface ‘S’ is given by

$$\iint_S A.ds \text{------------------------------}2$$

This must be equal to the total flux diverging from the whole volume ‘V’, enclosed by the surface S. Therefore, from equation 1 and 2 we have

$$\iint_S A.ds = \iiint_V divAdv \text{---------------------} 3$$

Equation 3 can also be written as

$$\iint_S (A.\hat{n})ds = \iiint_V (\nabla.A)dv$$

3. State and explain g stokes theorem for divergence?

Ans: **Stokes theorem:**

Statement:

Stokes theorem states that the line integral of a vector field 'A' around a closed curve is equal to the surface integral of the curl of 'A' taken over the surface 'S' surrounded by the closed curve.

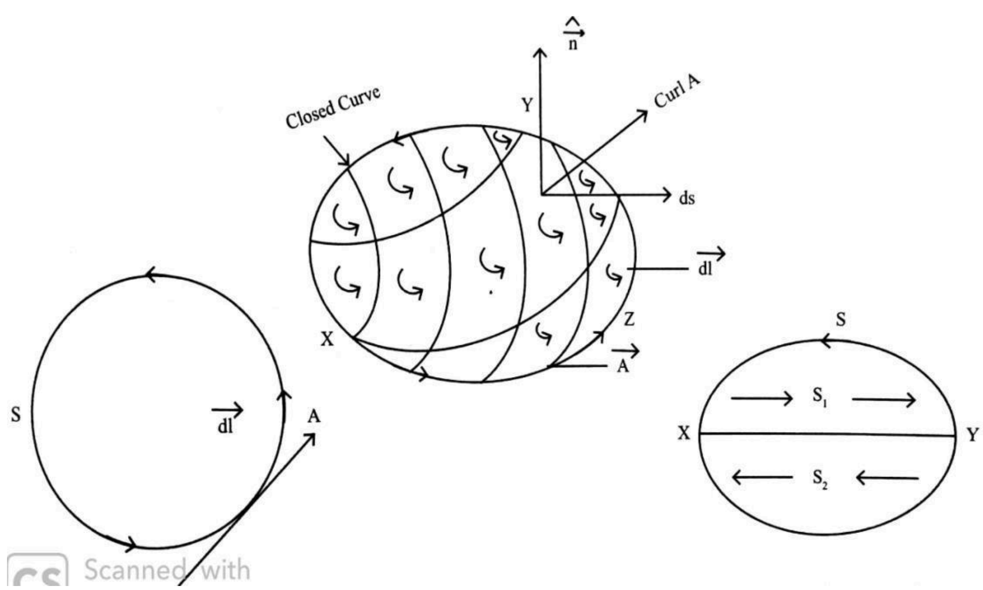
$$\oint_c A.dl = \iint_s curlA.ds = \iint_s (\nabla x\, A)ds$$

Theorem:

Let us consider surface 'S' enclosed in a vector field 'A', as shown in fig. in which a closed curve XYZ represents the boundary of surface 'S'.

The line integral of 'A' around the curve XYZ is

$$\oint_c A.dl$$



Let the entire surface 'S' is divided into large number of square loops. 'ds' be the area enclosed by each small loop, and $\hat{n}$ be unit positive outward normal upon small loop 'ds'. Then vector area of the element is

nds = ds

The curl of a vector field at any point is the maximum line integral of the vector computed per unit area, along the boundary of an infinitesimal area at the point. Then, the line integral of A around the boundary of 'ds' is

curl A.ds

It is applied to all surface element and hence, sum of the line integral of 'A' around thr boundaries of all the area element can be written as

$$\iint_s curlA.ds \text{----------------}1$$

From the fig, we can understand that the line integrals along the common sides of the continuous elements mutually cancel because they are traversed in the opposite direction.

The sides of the elements which lie in the closed curve of the surface contribute to the line integral. The equation (1) gives the sum of the line integrals on the boundary line of the curve.

Therefore,

$$\oint_c A.dl = \iint_s curlA.ds = \iint_s (\nabla x\, A)ds$$

The above expression is known as Stokes theorem.

4. Derive the expression for Maxwell's equations?

Ans: Maxwell's equations

1. $\nabla.E = \frac{\rho}{\varepsilon_0}$ (Gauss law for electricity)
2. $\nabla.B = 0$ (Gauss law for magnetism)
3. $\nabla \times E = \frac{-\partial B}{\partial t}$
4. $\nabla \times H = (J + \frac{\partial D}{\partial t})$

1. Maxwell's first equation, $\nabla.E = \frac{\rho}{\varepsilon_0}$

According to Gauss's theorem, total electric flux passing normally through a closed surface is equal to $\frac{1}{\varepsilon_o}$ time the total charge enclosed with in surface, where ε_o is 8.86 x 10^{-12} C^2 / Nm^2.

Thus,

$$\iint_s \vec{E}.ds = \frac{\rho}{\varepsilon_0}$$

if ρ is volume charge density, then

Therefore, $\iint \vec{E}\,.ds = \frac{1}{\varepsilon_o}\iiint \rho\, dv$

By gauss divergence theorem

$$\iint_s E.ds = \iiint_v div\, E dv$$

div E dv = $\frac{\rho}{\varepsilon_0}$

$$\nabla. E = \frac{\rho}{\varepsilon_0}$$

2. **Maxwell's second equation,** $\nabla. B = 0$

Number of magnetic lines of force entering surface is equal to number of magnetic lines leaving that surface, hence, magnetic flux in that field is zero.

$$\iint_s \vec{B}\,.ds = 0$$

by gauss divergence theorem

$$\iint_s B.ds = \iiint_v div\, B dv$$

$$\iiint_v div\, Bd = 0$$

div B = 0

$$\nabla. B = 0$$

3. **Maxwell's third equation,** ∇ x E = $\frac{-\partial B}{\partial t}$

According to Faraday's law, whenever there is charge in magnetic flux emf is induced which is directly proportional to negative rate of charge of flux.

e = $\frac{-\partial B}{\partial t}$--------1

ø = $\iint_s B.ds$ -----2

e = $\frac{-\partial}{\partial t}\iint_s B.ds$ ----3

e = $\oint_c E.dl$ -----4

Equating equation 3 and 4

$\oint E.dl = \frac{-\partial}{\partial t}\iint B.ds$ ------5 c s

By stokes theorem

$$\oint A.dl = \iint curlE.ds = \iint (\nabla x\ E)ds \text{----}6$$

c s s

Equating equation 5 and 6

$$\iint curlE.ds = \frac{-\partial}{\partial t}\iint B.ds$$

S S

$$\text{Curl E} = \frac{-\partial B}{\partial t}$$

4. Maxwell's fourth equations ∇ x H = (J + $\frac{\partial D}{\partial t}$)

The magnitude of the magnetic field at any point is directly proportional to the strength of the current and inversely proportional to the distance of the point from the straight conductors is called Ampere's law**.**

Consider a wire of current carrying conductor with the current I, since there is an electric field there has to be a magnetic field vector around it. Ampere's circuit law states that "The closed line integral of magnetic field vector is always equal to the total amount of scalar electric field enclosed within the path of any shape" which means the current flowing along the wire (which is a scalar quantity) is equal to the magnetic field vector (which is a vector quantity).

According to Ampere's law

$$\text{Curl B} = \mu_0\left[J + \varepsilon_0 \frac{dE}{dt}\right]$$

1. **Write physical significance of Maxwell's equations?**

Ans: Physical significance of Maxwell's equations:

1. Maxwell's first equation $\iint \vec{E}.ds = \frac{\rho}{\varepsilon_0}$ represents the Gauss's law in electrostatics for the statics charges, which states that the electric flux through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the total charges enclosed by the surface.
2. Maxwell's second equation $\oint B.ds = 0$ expresses Gauss's law in magnetostatics, which states that the net magnetic flux through any closed surface is zero.
3. Maxwell's third equation is the Faraday's law of electromagnetic induction, i.e. ($\oint E.dl = \frac{-\partial \phi B}{\partial t}$). it states that the induced electromotive force around any closed surface is equal to the negative time rate of change of the magnetic flux through the path enclosing the surface.
4. Maxwell's fourth equation ∇ x H = (J + $\frac{\partial D}{\partial t}$) represents the generalized form of Ampere's law as extended by Maxwell to account for the time varying magnetic fields.

6. State and explain poynting theorem?

Ans: Poynting Theorem.

Statement:

This theorem states that the cross product of electric field vector, E and magnetic field, H at any Point is a measure of the rate of flow of electromagnetic energy per unit area at that point.

$$P = E \times H$$

Let us consider an electromagnetic wave travelling along the x-axis with the magnetic and electric fields H and E confined in the transmission planes along the z and y axis, respectively. It propagates along the direction of propagation vector (E x H).

Taking divergence of above equation, we have

$$\nabla.(E \times H) = H.(\nabla \times E) - E.(\nabla \times H)$$

$$= -H.\frac{\partial B}{\partial t} - E.\frac{\partial D}{\partial t} \qquad \left[\therefore . \nabla \times E = \frac{-\partial B}{\partial t} \text{ and } \nabla \times H = \frac{\partial D}{\partial t}\right]$$

$$= -\left[E.\frac{\partial D}{\partial t} + H.\frac{\partial B}{\partial t}\right] \text{-------1}$$

We know that,

$$D = \varepsilon_0 E$$

$$B = \mu_0 H$$

Substitute the above expressions in equations (1) we have

$$= -\left[\varepsilon_0 . E\frac{\partial E}{\partial t} + \mu_0 H.\frac{\partial H}{\partial t}\right]$$

$$= -\left[\frac{1}{2}\varepsilon_0 2E\frac{\partial E}{\partial t} + \frac{1}{2}\mu_0 2H\frac{\partial H}{\partial t}\right]$$

$$= -\left[\frac{1}{2}\varepsilon_0 \frac{\partial (E)^2}{\partial t} + \frac{1}{2}\mu_0 \frac{\partial (H)^2}{\partial t}\right]$$

$$= \frac{-\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) \text{----------2}$$

Since surface ‘S’ bounds a volume ‘V’, we integrate equations (3) over the volume ‘V’, we have

$$\int_V \nabla.(E \times H) = \frac{-\partial}{\partial t}\int_V \left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) dv \text{----------3}$$

Applying the divergence theorem to equation (3) we get

$$\int (E \times H).ds = \frac{-\partial}{\partial t}\int \left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) dv \text{------4}$$

s v

The right hand side of equation (4) is the sum of energies of electric and magnetic fields. Hence, it represents the amount of energy transferred over the volume 'V' in one second i.e. rate of energy over the volume V.

The vector P = E x H in equation (4) represents the amount of filed energy passing through a unit area of surface in time, normal to the direction of flow of energy. This statement is called 'Poynting 'Theorem' and vector 'P' known as 'Poynting vector'.

7. Explain the electromagnetic wave in free space?

(or)

Explain the expression for non- conducting medium?

Ans: Non- conducting medium:

Let us assume the medium to be perfectly non- conducting, homogeneous and isotropic, having permeability and permittivity of the medium as μ and ε

All Maxwell's equations will remain unchanged, except the replacement of 'ε_0' and 'μ_0' with 'ε and 'μ' respectively.

If the field vectors of the electromagnetic wave the same magnitude at all points of any plane perpendicular to the direction of wave propagation, such as electromagnetic wave is called a plane wave. The time variation of the field vectors of the wave which obeys a harmonic law with a certain constant frequency is called monochromatic Waves.

From this Maxwell's equations can be written as

$$\nabla \times B = \mu\,\varepsilon \frac{\partial E}{\partial t} \text{ -----------------------1}$$

And $$\nabla \times E = -\frac{\partial B}{\partial t} \text{ ----------------------------2}$$

Differentiate equation (1) w.r.t. 't' we get

$$\frac{\partial}{\partial t}(\nabla \times B) = \frac{\partial}{\partial t}\left(\mu\,\varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial B}{\partial t} = \mu\,\varepsilon \frac{\partial^2 E}{\partial t^2}$$

Substituting for '$\frac{\partial E}{\partial t}$ 'form equation (2) we get,

$$\nabla \times (-\nabla \times E) = \mu\,\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$-\nabla \times (\nabla \times E) = \mu\,\varepsilon \frac{\partial^2 E}{\partial t^2} \text{ -------------3}$$

We have from the property of the operator '∇'

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E \text{--------4}$$

Substituting equation (4) in left hand part of equation (3), we have

$$\nabla \; (\nabla .\; E) + \nabla^2 E = \mu\, \varepsilon \frac{\partial^2 E}{\partial t^2} \text{ ---------------5}$$

Since there are no free charges, we can write $\nabla.\; E = 0$ in equation (5)

$$\therefore \; \nabla^2 E \; - \mu\, \varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \text{ --------------6}$$

similarly, for vector filed 'B' we have

$$\nabla^2 B - \mu\, \varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \text{ ----------------7}$$

By comparing equation (6) and (7) with simple harmonic motion of type $\frac{d^2 y}{dt^2} - \mu y = 0$ this velocity of wave propagation is $\frac{1}{\sqrt{\mu}}$.

Therefore, filed vector satisfy the wave equation in which the wave propagation velocity is

$$cl = c = \frac{1}{\sqrt{\mu\varepsilon}}.$$

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

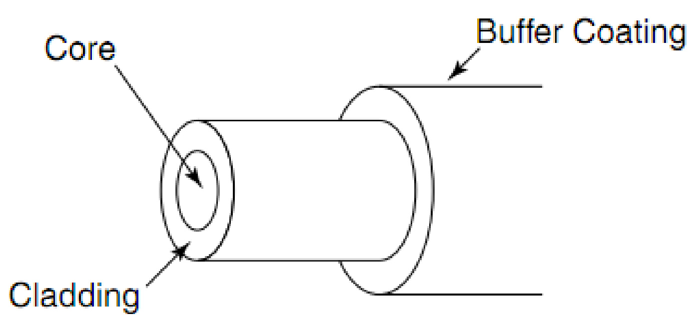
3.2 OPTICAL FIBER

3.2.1. Introduction to optical fibers – Total internal reflection construction of optical fibers, Critical angle of propagation.

1. Define optical fiber? Describe the construction and working principle of optical fiber.

ANS. Optical fiber: An optical fiber is a transparent dielectric wave guide of smaller diameter That transmits optical data from one point to other with minimal loss It works on the principle Of total internal reflection

Construction: The construction of an optical fiber is shown in fig below



Optical fiber construction

Optical fiber consists of a inner cylinder core a second cylindrical shell cladding and outer Cylindrical shell buffer or jacket

Cladding: The cladding is a dielectric medium of refractive index n_2 *which* surrounds the core the refractive index n_1 the cladding is less than refractive index of core. The thickness of the cladding is 125 to 150μm

<u>Core:</u> core is inner cylindrical shell protected by cladding the refractive index of core is greater than the refractive index of cladding (i.e. $n_4 >> n_2$ The thickness of core is 80µm

<u>Buffer (Or) Jacket</u>; The core surround3ed by classing are encapsulated in an elastic abrasion-
Resistant plastic material called buffer coating

<u>Working principle</u>: Optical fiber works on the principle of total reflection Consider an ray of light incident at the boundary of two medium one part of a ray gets reflected and the other parts gets refracted. According to snell's law when angle of incidence $\emptyset_1$ *reaches s crtical angle* $\emptyset_c$ then the angle of Refraction reaches 90^0 and emerges parallel to the medium interface when $\emptyset_1$ exceeds $\emptyset_c$ the Incident light rays gets reflects in the same direction. This phenomena is called as total internal reflection

According to snells law

$$n_1 \, sin\emptyset_1 = \, n_2$$

$$sin\emptyset_1 \, \frac{n_2}{n_1} \, sin\emptyset_2$$

when $\emptyset_1 = \emptyset_c \& \emptyset_2$

$$\sin \, \emptyset_c = \frac{n_2}{n_1} \sin 90^0$$

sin

$$\emptyset_c = sin^{-1} \frac{n_2}{n_1}$$

Where $\emptyset_1$ = Incidence ray angle

$\emptyset_2$ = Refracted ray angle

$\emptyset_c$ = critical angle

$n_{1=}$Refractive index of core

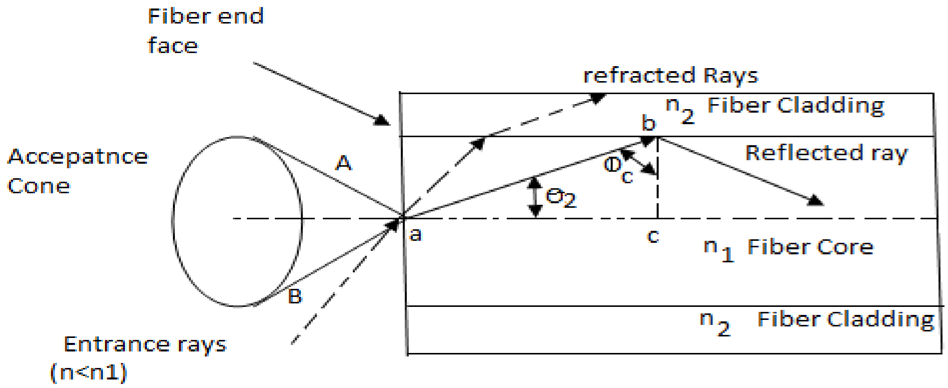
n_2 = Refractive index of cladding

3.2.2 Acceptance angle – Numerical aperture

2. Obtain the expression foe numerical aperture and acceptance angle of an optical angle.

Let us consider a cylindrical fiber it consists of are refractive index n_1 and let n_0 be the refractive index of the air medium in which the optical fiber is placed.

The incident ray travel along AO and enters the centre at an angle I to the fiber axis



The ray is refracted along OB at angle θ in the core it further proceeds to fall at critical angle Of incidence $\emptyset_c$ = 90-θ on the interface between core and cladding at this angle the ray just moves a long BC

Any way which enters into the core at angle of incidence less than '1' will have refractive angle less than θ

Hence the angle of incidence ($\emptyset = 90 - \theta$) at the interface of core and cladding will be more than the critical angle. Hence the ray is totally internally reflected ray.

Mathematical relation:

(i) Applying Snell's law at point of entry (AO) we have

$$n_0 \sin i = n_1 \sin\theta$$

$$\text{SiniI} = \frac{n_1}{n_2}\sin\theta$$

$$\text{Sin i} = \sqrt{1 - cos^2\theta} \;\; -1 \text{ ----------------------- } (1)$$

(ii) Applying snell's law at point B

$$n_1 \sin\emptyset = n_2 \sin 90^0$$

$$\text{Sin}\,\emptyset = \frac{n_2}{m_1}$$

$$\text{Sin (90-0)} = \frac{n_2}{n_1}$$

$$\text{Cos}\,\theta = \frac{n_2}{n_1} \text{ ----------------------------- } (2)$$

Substitute equ (2) in equ (1) we get

$$\text{Sin i} = \frac{n_2}{n_1}\;\{\sqrt{1 - (\frac{n_2}{n_1})^2}$$

$$= \frac{n_1}{n_2} \times \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

$$i = sin^{-1}(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the refractive index of air $n_0 = 1$ then the maximum value of sini is given as

$$\text{Sin } i_{max} = \sqrt{n_1^2 - n_2^2}$$

Numerical aperture: It is defined as the sine of the acceptance and the fiber is

$$\text{NA} = = \sqrt{n_1^2 - n_2^2}$$

Acceptance angle: The maximum angle at which the light can suffer total internal reflection is called as acceptance angle

$$\text{Sin}\theta_a = \text{NA}$$

$$\text{Sin}\theta_a = \sqrt{n_1^2 - n_2^2}$$

=

$$\theta_a = sin^{-1} \sqrt{n_1^2 - n_2^2}$$

3. Classify the optical fibers on the basis of materials modes of propagation and refractive Index difference

Or

Explain different types of optical fiber with propagation of light in step index and graded index fiber

ANS. Types of optical fiber

The optical fibers are classified into two types they are

(i)step index fiber and

(ii) graded index fiber

(i) Step index fiber: The refractive index of air cladding and core varies by step by step and hence it is called as step index fiber

In step index fiver we have both single mode and multimode fiber as shown in fig(a) ,Fig(b) respectively.

In both the fibers the variation in refractive indices will be in step by step since mode fiber has less dispersion than multimode the single mode step index fiber also low intermodal dispersion compared to multimode step index fiber.

Fig (a) single mode-step index

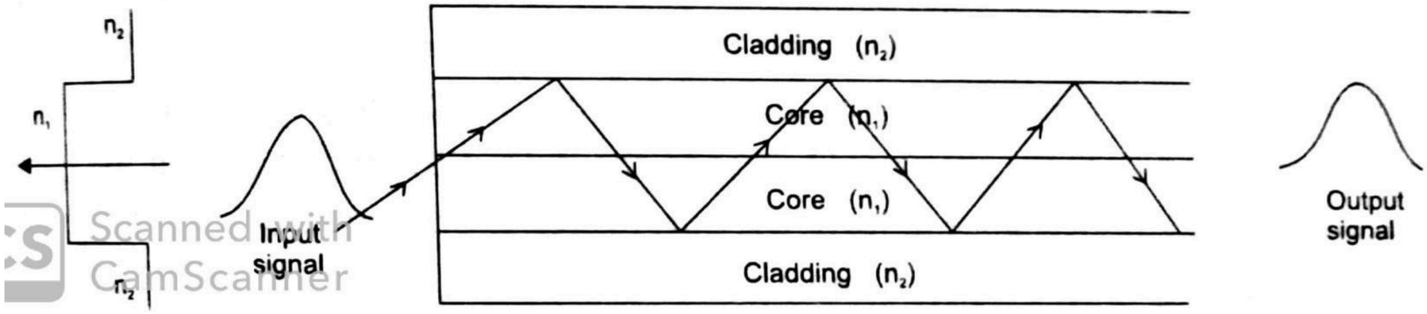
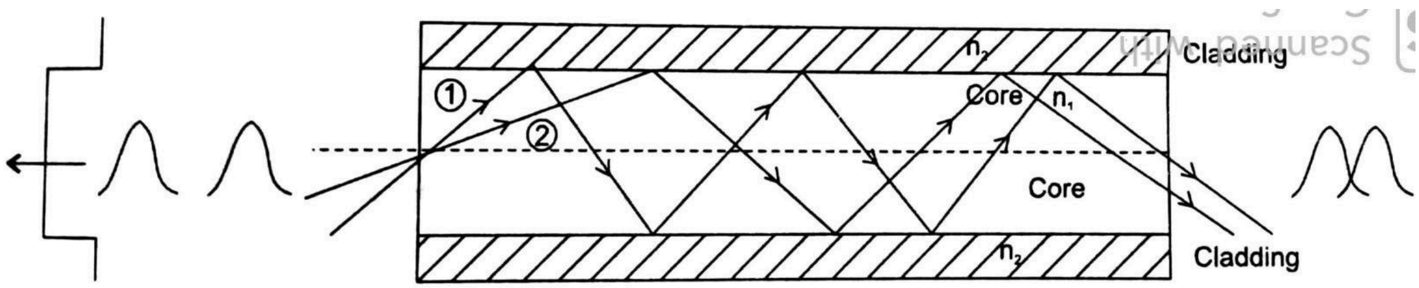


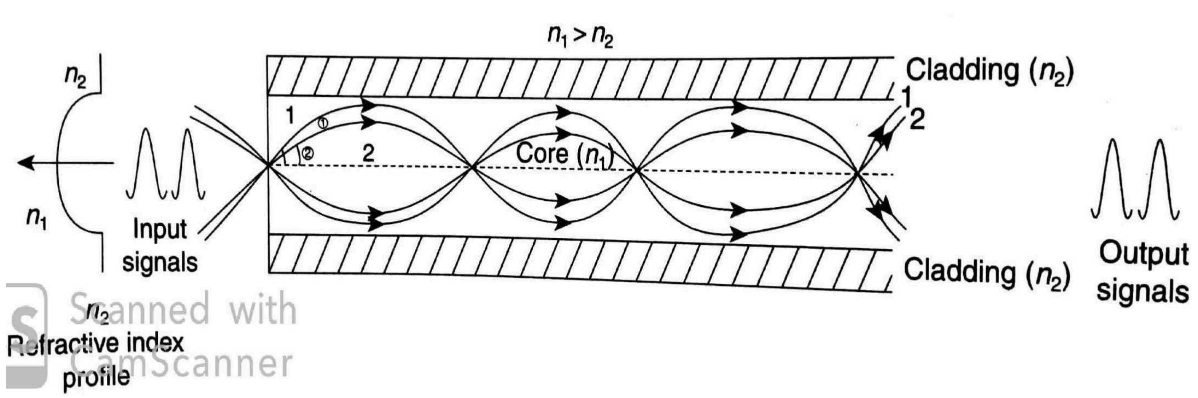
Fig (b) multi mode-step index



(ii)Graded index fiber:

The refractive index of the core radially from the axis of the fiber. The refractive index of the core is maximum along the fiber axis and it gradually decreases. Thus it is called as graded index fiber.

In general the ground index fibers will be of multimode system the multimode graded index fiber has very less inter molded dispersion compared to multimode step index fiber A typical Multimode graded index fiber is a shown in fig (a)



4. Distinguish between multimode fiber and single mode fiber

ANS.

Single mode fiber	Multimode fiber
1. In single mode fiber only one mode can be propagated	The fiber in this case allows large number of modes for light to pass through it
2.The single mode fiber has a smaller core Diameter and difference in refractive index Of core and cladding is small	Here both core and cladding refractive indices difference is large as the core diameter in large
3.No dispersion	Dispersion is more due to degradation of single Due to multimode
4.Information can be carried to longer distances	Information can be carried to shorter distances only
5.launching of light and connecting the fibers are difficult	Launching of light and also correcting two fibers is easy
6.intsallation is difficult it is more costly	Fabrication in easy and the installation cost is Low

3.2.3 Propagation of electromagnetic wave through optical fiber importance of v number

5 .Explain the importance of v number through an optical fiber?

ANS. **<u>v – number (or) normalized frequency parameters</u>**

The normalized frequency parameter or v-number gives the upper limit of the number of modes that can be transmitted in a multimode optical fiber It depends on the core diameter the Numerical aperture and the wave length is given by

$$V=\frac{2\pi}{\lambda}a\sqrt{n_1^2 - n_2^2}$$

$$V=\frac{2\pi}{\lambda}(NA)$$

$$V== \frac{2\pi a n_1}{\lambda} \quad (\sqrt{2\Delta})s$$

Where a= radius of the core

λ=wave length of ray

N.A = numerical aperture

n_1 *and* n_2 = the refractive indices of the core cladding

Fiber with a v-parameter of less than 1.1585 only supports the fundamental mode and is Therefore a single mode fiber the fundamentals mode and is therefore a single mode fiber Whereas fiber with a higher v- parameter has multiple mode the value of the normalized frequency parameter (v) relates core size with mode propagation .The number of modes in an optical fiber distinguishes multimode optical fiber from single mode optical fiber

Graded index fiber

Graded index fiber does not have a constant refractive index in the core due to this property

They are also called in homogeneous core fiber

For guide modes we know that v- number gives as

$$V=\frac{2\pi}{\lambda} a\sqrt{n_1^2 - n_2^2}$$

$$V=\frac{2\pi a}{\lambda}(NA)$$

$$V== \frac{2\pi a n_1}{\lambda} \quad (\sqrt{2\Delta})$$

Where Δ = relative refractive index

$$\Delta = (\frac{n_1 - n_2}{n_1})$$

Total number of guided mode in

$$M_g = \frac{\propto}{\alpha+2} \times (n_1 \times \frac{2\pi}{\lambda}^2).\ \Delta$$

But

$$n_1 \times \frac{2\pi}{\lambda}^2 .\ a.\ \sqrt{2}\ .\ \Delta = v$$

$$(n_1 \times \frac{2\pi}{\lambda}^2).\ \Delta = v^2/2$$

$$M_g \ =\frac{\propto}{\alpha+2} v^2/2$$

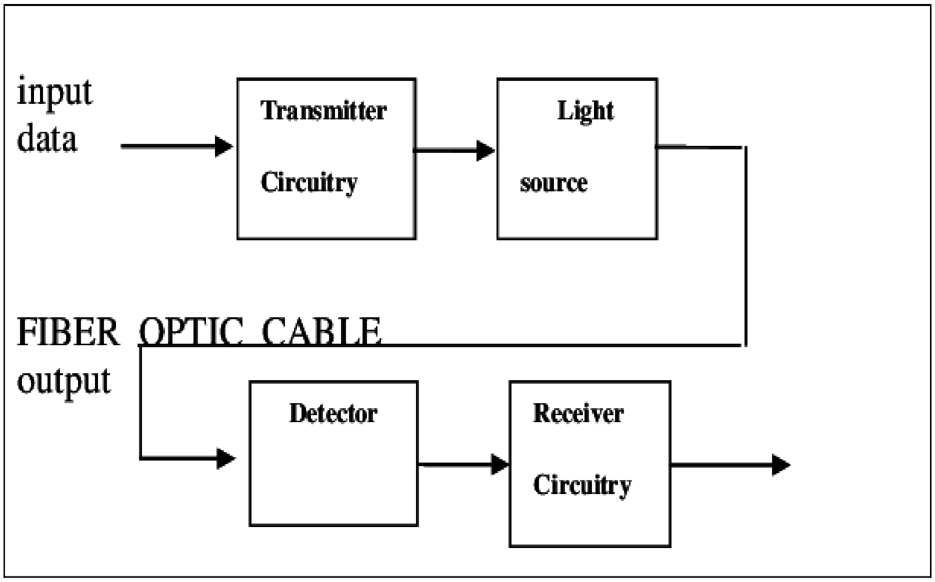
For a parabolic refractive index profile core fiber (∝=2)

$$m_g = v^2/4$$

3.2.4 Block diagram if fiber optic communication system medical application

6. Draw the block diagram of an optical fiber communication system and explain the Function of each block

ANS. Optical fiber communication system is a system used to transmit information In the form of optical signal through a wave guide. In this electrical signal is converted Into optical signal at the transmitting end and at the receiving end, optical signal is Converted into electrical signal



The communication system consists of

1.Information source

2.Transmitter

3.Tranmsision medium

4.Receiver

1**. Information source**

In this the original signal at the signal source is along signal is converted into electrical signals and then passed through the transmitter

2. **Transmitter**

The transmitter is a modulation device which consists of a Driver and light source The analog electrical signals that are received by the driver are converted into digital pulses. Therefore pulses are in turn converted into optical pulses using LED or LASER

3. **Transmission medium**

In this the optical signals is transmitted to the other end by following the principle of total Internal reflection

4. **Receiver**

Receiver is a demodulator device which contains photo detector amplifier and signal restorer the optical pulses received from the optical fiber are converted into an electrical pulses by photo detector the pulses are than amplified and converted into the required signal form.

7. Write applications of optical fiber in medical?

ANS. Fiber optics applications in medical

1. Optical fiber plays a crucial role in traditional application such as x-ray imaging light therapy surgical microscopy etc
2. With the help of optical **fiber** wide range of application like illumination, image and laser signal delivery can be designed
3. Fiber scope in endoscopy is one of the widely used optical techniques to view the internal parts of the disease affected body. In these optical fibers plays a major role in visualization of internal of human body but also in the selective cauterization of tissues, using laser beam.
4. Optical fibers are used in photodynamic therapy for cancer
5. They are used in the treatment of lung disorders.
6. They are used in the treatment of bleeding ulcers.
7. They are used in arthroscopic surgery for damaged cartilage, ligaments and tenors in major joints such as knees and shoulders.
8. They are used in the investigation of heart. Respiratory system and pancreas.

SOLVED PROBLEMS

1. **An optical fiber has a core material of refractive index 1.55 and the cladding material of refractive index 1.50vand light is launched into it in air. Calculate its numerical aperture.**

Solutions: Given,

Core $n_1 = 1.55$,   cladding $n_2 = 1.50$

w.k.t. numerical aperture of the optical fiber

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{(1.55)^2 - (1.50)^2}$$

$$NA = 0.39$$

2. **Calculate the angle of acceptance of given optical fiber if the refractive index of the core and cladding are 1.563 and 1.498.**

Solutions: Given,

$n_1 = 1.563$ and $n_2 = 1.498$

We know that

$$\text{Acceptance angle } \theta_a = sin^{-1}\sqrt{n_1^2 - n_2^2}$$

$$= sin^{-1}\sqrt{(1.563)^2 - (1.498)^2}$$

$$= sin^{-1}\sqrt{0.1989}$$

$$= sin^{-1}(0.446)$$

$$\theta_a = 26^0 29^1$$

3. **Calculate the fractional index change for a given optical fiber if the refractive index of the core and cladding are 1.563 and 1.498.**

Solutions: Given,

$n_1 = 1.563$ and $n_2 = 1.498$

∴ The fractional index change,

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$= \frac{1.563 - 1.498}{1.563}$$

$$= \frac{0.065}{1.563}$$

$$\Delta = 0.0416$$

4. **An optical fiber of 1mW is guided into the optical fiber of length 100m. If the output power at the other end is 0.3 mW. Calculate the fiber attenuation.**

Solutions: Given,

Input power $P_{in} = 1mW$

Output power $P_{out} = 0.3mW$

Length of the fiber L = 100m = 0.1 km

∴ We know that fiber attenuation

$$\alpha = \frac{10}{L} log\left(\frac{P_{in}}{P_{out}}\right) \text{ dB/km}$$

$$= \frac{10}{L} log\left(\frac{1}{0.3}\right) \text{ dB/km}$$

$$\alpha = 52.29 \text{ dB / km}$$