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Learning Goals

- 1. Let's talk about "differentiation".
- 2. Let's explore Universal Design for Learning.
- 3. Let's explore Math Language Routines.
- 4. Let's build norms for our mathematical community.

Teach and Learn Virtual Note Page Links

<u>Part 1 | Part 2 | Part 3 | Part 4</u>



Acting on our Beliefs About Students

ALL students...

have brilliant mathematical ideas

have experiences and ideas that are valuable

can make sense of and solve problems

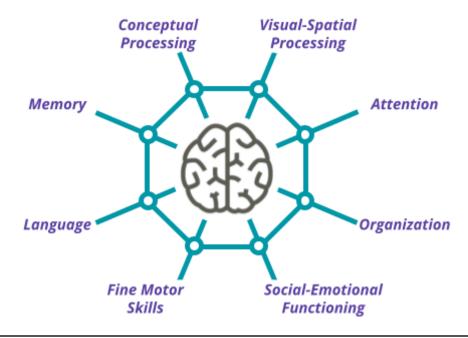
can learn grade-level mathematics

I will act on my belief that students	by
have brilliant mathematical ideas	
have experiences and ideas that are valuable	
can make sense of and solve problems	
can learn grade-level mathematics	



Cognitive Functions and UDL

The following areas of **cognitive functioning** are integral to learning mathematics.



Conceptual Processing →	includes perceptual reasoning, problem solving, and metacognition
Language →	includes auditory and visual language processing and expression
Visual-Spatial Processing →	includes processing visual information and understanding relation in the space of visual mathematical representations and geometric concepts
Organization $ ightarrow$	includes organizational skills, attention, and focus
Memory →	includes working memory and short-term memory
Attention →	includes paying attention to details, maintaining focus, and filtering out extraneous information
Social-Emotional Functioning →	includes interpersonal skills and the cognitive comfort and safety required in order to take risks and make mistakes
Fine Motor Skills →	includes tasks that require small muscle movement and coordination such as manipulating objects (graphing, cutting with scissors, writing)

Brodesky et al. (2002). Accessibility strategies toolkit for mathematics. Education Development Center. http://courses.edtechleaders.org/smi 3/documents/2020/MathAccessStrategies.pdf



Universal Design for Learning

UDL offers students a variety of opportunities for

- **Engagement (the "why" of learning)** to enhance access by recruiting interest, develop effort & persistence, and internalize self-regulation
- Representation (the "what" of learning) to enhance access to perception, develop language & symbols, and internalize comprehension
- Action & Expression (the "how" of learning) to enhance access through physical action, expression, & communication and enhance executive functions

Access for Students with Disabilities

ARE	ARE NOT
suggestions to help students build on, and teachers spot, their strengths	"different tasks" for certain students
accommodations to enhance access to grade-level content	modifications to what students are expected to do or understand
built from the UDL principles; beneficial for all learners	to address unfinished learning from prior grades



Access for Students with Disabilities

Planning for Instruction

- Where is there opportunity to enhance access and challenge already written into the lesson?
- What can I highlight or amplify?
- What questions can I ask to build on students' understanding?
- What tools can I offer to help students make connections?

Group 1 | Group 2 | Group 3 | Group 4

GROUP 1	
One way Universal Design for Learning provides enhanced access to mathematics for all of my students is	I will choose to use an Access for Students with Disabilities instructional strategy when
 Offering a summary at the end of each lesson to review and put important vocabulary for the students to remember Giving models for the students to use 	 Teaching a lesson and placing students in learning groups • • • •

Debbie		
One way Universal Design for Learning provides enhanced access to mathematics for all of my students is		I will choose to use an Access for Students with Disabilities instructional strategy when
• Warm up to	o focus for that day's topic	 Students need a concrete reference to access the concept • • • • • • •



Tiy		
One way Universal Design for Learning provides enhanced access to mathematics for all of my students is		I will choose to use an Access for Students with Disabilities instructional strategy when
•		 The strategies best mirror their accommodations • • • •

Mary		
One way Universal Design for Learning provides enhanced access to mathematics for all of my students is		I will choose to use an Access for Students with Disabilities instructional strategy when
Meet th	em where they are	 They need an anchor to access a standard • •



Mathematical Language Routines (MLR)

MLR1 Stronger and Clearer Each Time

To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.

MLR2 Collect and Display

To capture students' oral words and phrases into a stable, collective reference.

MLR3 Clarify, Critique, Correct

To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.

MLR4 Information Gap

To create a need for students to communicate. This routine allows teachers to facilitate meaningful interactions by giving partners or team members different pieces of necessary information that must be used together to solve a problem or play a game.

MLR5 Co-Craft Questions

To allow students to get inside of a context before feeling pressure to produce answers, and to create space for students to produce the language of mathematical questions themselves.

MLR6 Three Reads

To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to actively make sense of mathematical situations and information (Kelemanik, Lucenta, & Creighton, 2016).

MLR7 Compare and Connect

To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language.

MLR8 Discussion Supports

To support rich discussions about mathematical ideas, representations, contexts, and strategies.



Access for English Learners Design Principles

1. Support sense-making: Scaffold tasks and amplify language so students can make their own meaning.

Students do not need to understand a language completely before they can start making sense of academic content and negotiate meaning in that language. Language learners of all levels can and should engage with grade-level content that is appropriately scaffolded. Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher. In addition, teachers can foster students' sense-making by amplifying rather than simplifying, or watering down, their own use of disciplinary language.

2. Optimize output: Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing.

Linguistic output is the language that students use to communicate their ideas to others. Output can come in various forms, such as oral, written, visual, etc., and refers to all forms of student linguistic expressions except those that include significant back-and-forth negotiation of ideas.

3. Cultivate conversation: Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class).

Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning. They also allow students to hear how other students express their understandings. When students have a reason or purpose to talk and listen to each other, interactive communication is more authentic.

4. Maximize meta-awareness: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.

Language is a tool that not only allows students to communicate their math understanding to others, but also to organize their own experience, ideas, and learning for themselves. **Meta-awareness** is consciously thinking about one's own thought processes or language use. Meta-awareness develops when students and teachers engage in classroom activities or discussions that bring explicit attention to what students need to do to improve communication or reasoning about mathematical concepts.



Access for English Learners

ARE	ARE NOT
suggestions to help students build on, and teachers spot, their strengths	a replacement for English instruction
designed to <i>amplify</i> rather than <i>simplify</i> language	a reduction of the cognitive and language demands of the curriculum
a range of suggestions for teachers to enhance access for English Learners	to address unfinished learning from prior grades
beneficial for all learners	changes to what students are expected to do or understand

Planning for Instruction

- Where is there opportunity to enhance access and challenge already written into the lesson?
- What can I highlight or amplify?
- What questions can I ask to build on students' understanding?
- What tools can I offer to help students make connections?

One way Math Language Routines provide greater access to mathematics for all of my students is...

- Helps students understand what the questions are asking them to do.
- It helps them to make math vocabulary apart of their vocabulary. They will begin to use the correct terms more, the more they hear them.
- Allows the students to focus on what is actually being asked of them
- Collaboration and repetition helps them develop the grade-level vocabulary
- Learn from others



I will choose to use a Math Language Routine when . . .

- When suggested in the teacher materials
- When working with students who might need it
- To increase class participation

• As often as possible

•

•

MLR5 Co-Craft Questions

Grade 8 Unit 7: Exponents and Scientific Notation

Activity 15.3: A Celestial Dance



Learning Goal

Generalize (orally and in writing) a process of adding and subtracting numbers in scientific notation and interpret results in context.

Activity Narrative

In this activity, students add quantities written in scientific notation in order to answer questions in context. To add numbers in scientific notation, students must attend to precision by aligning place value (MP6).

As students work, notice the different strategies used to align place value. One strategy would be to convert all the distances to decimal, align the place values vertically, and then add in the usual way. Another example would be to rewrite all the addends to use the same power of 10 before adding.

- 1. When you add the distances of Mercury, Venus, Earth, and Mars from the Sun, would you reach as far as Jupiter?
- 2. Add all the diameters of all the planets except the Sun. Which is wider, all of these objects side by side, or the Sun? Draw a picture that is close to scale.

object	diameter (km)	distance from the Sun (km)
Sun	1.392×10^6	$0 imes 10^0$
Mercury	4.878×10^3	5.79×10^7
Venus	$1.21 imes 10^4$	$1.08 imes 10^8$
Earth	$1.28 imes 10^4$	$1.47 imes 10^8$
Mars	6.785×10^3	$2.28 imes 10^8$
Jupiter	$1.428 imes 10^5$	$7.79 imes 10^8$



Mathematical Language Routine 5: Co-Craft Questions

Purpose

To allow students to get inside of a context before feeling pressure to produce answers, and to create space for students to produce the language of mathematical questions themselves.

How it happens

Hook: Present a context or a stem for a problem. The hook can also be a picture, video, or list of interesting facts.

Students Write Questions: Students write down possible mathematical questions that could be asked about the situation. (1–2 minutes)

Students Compare Questions: Students compare questions with a partner (1–2 minutes) before sharing questions with the whole class. Select questions to share and discuss with the whole class. Demonstrate (or ask students to demonstrate) identifying specific questions that are aligned to the content goals of the lesson as well as the disciplinary language function. If there are no clear examples, teachers can demonstrate adapting a question or ask students to adapt questions to align with specific content or function goals. (2–3 minutes)

Actual Question(s) Revealed: Reveal the questions students are expected to work on. Alternatively, select from the list of student generated questions.



MLR5 Co-Craft Questions		
MLR Step	Students have access to grade-level mathematics.	Students learn language as they develop mathematical understanding.
Hook	 All students are working at grade level. Multiple entry points. 	•
Students Write Questions	 Plenty of think time and collaboration Lots of collaboration • 	•
Students Compare Questions	 Students are hearing the ideas of their classmates She's recording their questions verbatim 	•
Actual Questions Revealed	•	•

Teacher is curious about and trusts student thinking to drive learning.

- She's recording their questions verbatim
- "Ooh ..."
- "What is your idea?"
- Lack of judgment about what students are saying
- •
- •



Mathematical Community Norms

Mathematical Community		
Doing Math sounds like	Doing Math looks like	
 Using math vocabulary Making mistakes productive struggle Sharing a story/connecting a math concept to a real life experience Real world application Asking questions 	 collaboration Real life application Students are working together to solve a problem Students working in groups to help facilitate their understanding 	
 Respecting each other's thoughts and ideas Agreeing on same thoughts or ideas 		

Group 1 | Group 2 | Group 3 |

Mary

What **norms**, or expectations, help us to do math together as a mathematical community?

In our mathematical community we...

- Feel free to risk-take
- Ask questions
- Respect others value other's opinion
- Wait time given
- Mistakes occur

Debbie

What **norms**, or expectations, help us to do math together as a mathematical community?

In our mathematical community we...

- Respect each other's ideas
- Recognize that everyone's ideas are valuable



- •
- •

Tiy

What norms, or expectations, help us to do math together as a mathematical community?

In our mathematical community we...

- Accept everyone's ideas
- •
- We allow each person time to think and speak
- •

Traci

What norms, or expectations, help us to do math together as a mathematical community?

In our mathematical community we...

- Group collaboration
- Problem solving with partners
- Listen and value others views and opinions
- •

Discussion

What norms, or expectations, help us to do math together as a mathematical community?

In our mathematical community we...

- Watch our air time
- Show our thinking
- Value the process over the answer
- Make mistakes, inspect mistakes, expect mistakes



HANDOUT 8 Learning Targets

1. I can locate resources and opportunities to enhance access and challenge for all students.

- 2. I can make decisions about when and how to use the suggested strategies for English Learners and Students with Disabilities.
- 3. I can recognize the value of co-constructing norms to build a positive and inclusive mathematical community.

HANDOUT 9 My Reflections

	- -,
Enhancing Access to Mathematics	E = Excited What are you excited about?
	W = Worry What are you worried about?••••
	N = Need to Know What do you need to know? • • • • • • • •
	S = Stance or Step What is your current stance? How might you move forward? • • • • • • • •



Are You Ready for More?

We value reflection and consolidation as an important part of learning. To that end, we offer the following set of carefully chosen activities to help deepen your understanding of Enhancing Access to Mathematics and Building Mathematical Community in your classroom.

Suggested Reading

- Access for Students with Disabilities in the course guide (<u>IM 6–8 Math</u> or <u>IM 9–12 Math</u>)
- Learn more about the Universal Design for Learning Guidelines at http://udlguidelines.cast.org
- Access for English Learners in the course guide (<u>IM 6–8 Math</u> or <u>IM 9–12 Math</u>)
- Read more about MLR design at https://imk12.org/MLRdesign
- Read about MLR8 Discussion Supports on <u>Handout 11</u> below and try out some of the sentence frames in your classroom.

Subscribe to the IM Blog

Once you have clicked on a blog post, subscribe to the blog by entering your email address on the right sidebar. Subscribing will ensure you receive email notifications of future blog posts.

Blog Posts on Building Community:

- Building a Mathematical Classroom Community (see plan below on Handout 12)
- Co-Creating Classroom Norms with Students
- Explicit Classroom Norms to Teach Kids How to Learn from Solving Problems

Blog Posts on Enhancing Access and Challenge:

- <u>Tackling Wordy Problems: How the Three Reads Math Language Routine Supports Access</u> for All Learners
- Building Equitable Learning Environments for Each Student
- Leveraging IM 6–12 Math Teacher Materials to Enhance Access to Grade-Level Mathematics
- English Learners and Distance Learning: Enhancing Access
- English Learners and Distance Learning: Clarify, Critique, Correct
- English Learners and Distance Learning: Co-Craft Questions
- English Learners and Distance Learning: Compare and Connect



MLR8 Discussion Supports

Mathematical Language Routine 8: Discussion Supports

Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009). The examples provided can be combined and used together with any of the other routines. They include multi-modal strategies for helping students make sense of complex language, ideas, and classroom communication. The examples can be used to invite and incentivize more student participation, conversation, and meta-awareness of language. Eventually, as teachers continue to model, students should begin using these strategies themselves to prompt each other to engage more deeply in discussions.

How it happens: Unlike the other routines, this support is a collection of strategies and moves that can be combined and used to support discussion during almost any activity.

Examples of whole class strategies:

- Revoice student ideas to model mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gestures, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task. Model detailing steps, describing and justifying reasoning, and questioning strategies.
- Demonstrate uses of disciplinary language functions such as detailing steps, describing and justifying reasoning, and questioning strategies.
- Give students time to make sure that everyone in the group can explain or justify each step
 or part of the problem. Then make sure to vary who is called on to represent the work of
 the group so students get accustomed to preparing each other to fill that role.
- Prompt students to think about different possible audiences for the statement, and about the level of specificity or formality needed for a classmate vs. a mathematician, for example. [Convince Yourself, Convince a Friend, Convince a Skeptic (Mason, Burton, & Stacey, 2010)]



Sentence Frames

Sentence frames can support student language production by providing a structure to communicate about a topic. Helpful sentence frames are open-ended, so as to amplify language production, not constrain it. The table shows examples of generic sentence frames that can support common disciplinary language functions across a variety of content topics.

Describe	It looks like I notice that I wonder if Let's try A quantity that varies is What do you notice? What other details are important?	Critique	That could/couldn't be true because This method works/doesn't work because We can agree that 's idea reminds me of Another strategy would be because Is there another way to say/do ?
Explain	First, I because Then/next, I I noticed so I I tried and what happened was How did you get ? What else could we do?	Compare and contrast	Both and are alike because and are different because One thing that is the same is One thing that is different is How are and different? What do and have in common?
Justify	I know because I predict because If then because Why did you ? How do you know ? Can you give an example?	Represent	represents stands for corresponds to Another way to show is How else could we show this?
Generalize	reminds me of because will always because will never be true because ls it always true that? ls a special case?	Interpret	We are trying to We will need to know We already know It looks like represents Another way to look at it is What does this part of mean? Where does show ?

Chapin, S., O'Connor, C., & Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn, grades K-6* (second edition). Sausalito, CA: Math Solutions Publications.

Mason, J., Burton, L. and Stacey, K. (2010). *Thinking mathematically* (2a. ed.). Harlow: Pearson Education.



Building a Mathematical Classroom Community Plan

Goal for Days 1-3

Let's describe what it looks and sounds like to learn math by doing math.

Day 1: Unit 1, Lesson 1

 Prepare a space, such as a piece of poster paper, titled "Mathematical Community of Learners, Doing Math". Include a two-column table with the headers "Looks like . . ." and "Sounds like . . ."

Mathematical Community of Learners Doing Math			
Looks like	Sounds like		

- Do the warm-up as described in the teacher materials.
- 3. After the warm-up, invite students to take 1–2 minutes of quiet think time to reflect on both individual and group actions while considering the question, "What does it look and sound like to do math together as a mathematical community of learners?"



4. Take 5 minutes to record and display actions using students' exact words under the appropriate header. Students might mention things such as: we talked to each other and to the teacher (sounds like), we had quiet time to think (looks like), we shared our ideas (sounds like), we wrote/drew our ideas on paper (looks like), or referred to a classmates thinking (sounds like).



- 5. Do the rest of the lesson as described, monitoring for the recorded actions as students work.
- 6. After the cool-down, take 5 minutes to revisit the table of actions. Invite students to discuss "Where in the lesson did you get to do each of these?" and "What's missing or could be reworded for clarity?" As a whole group, add or update actions.



Day 2: Unit 1, Lesson 2

1. Before the warm-up, mention to students that they will have an opportunity to revise their "Mathematical Community of Learners" actions during the synthesis of the lesson, so as they work today they can be on the lookout for actions that may be missing from the table.



- 2. Do the warm-up and activities as described in the teacher materials.
- 3. After the cool-down, give students 2–3 minutes to discuss any revisions to the "Doing Math" actions in small groups.
- 4. Share ideas as a whole group and record any revisions.





Day 3: Unit 1, Lesson 3

- 1. Before the warm-up, invite students to keep track of the specific action from their "Doing Math" table they personally experience during the lesson.
- 2. Do the warm-up and activities as described in the teacher materials. As students work, monitor for examples of the "Doing Math" actions.
- 3. After the cool-down, invite students to take 2–3 minutes of quiet think time to individually reflect on the question "Which 'Doing Math' action did you feel was most important in your work today, and why?" and write their response on their cool-down page, on a separate sheet of paper, or in a math journal.
- 4. Collect and read their responses after class. These responses will offer insight into how students feel about their own mathematical work and help you make authentic and personalized connections to the norms they will be creating during days 4–6.



Goal for Days 4-6

Let's build norms that support doing math together within our community of learners.

Day 4: Unit 1, Lesson 4

 Prepare a second space, such as a piece of poster paper, titled "Mathematical Community of Learners, Norms".

Mathematical Community of Learners Norms

In our community we . . .

2. Before the warm-up, describe to students that norms are *community expectations* that help everyone in the room feel safe, comfortable, and productive doing math together. Offer an example, such as "It may help us share our ideas as a whole class if we have the norm 'Listen as others share their ideas." Mention to students that you will pause at two different points of the lesson to identify norms for a mathematical community of learners where everyone does math.



- 2. Do the warm-up as described in the teacher materials.
- 3. After the warm-up, invite students to reflect on both individual and group actions to complete the sentence frame, "In our community we . . ."
- 4. Record and display their responses using students' exact words.
- 5. Resume the lesson as described in the teacher materials.
- 6. After the cool-down, revisit the norms. Ask students to discuss with a partner when a norm was helpful as they did math together. Add or update norms based on their discussion.





Day 5: Unit 1, Lesson 5

- 1. Before the warm-up, invite students to keep track of specific norms they experience during the lesson.
- 1

- 2. Do the entire lesson as described in the teacher materials.
- 3. After the cool-down, invite students to take 2–3 minutes in small groups to discuss any revisions to the norms. Add or update norms based on their discussion.



Day 6: Unit 1, Lesson 6

1. Before the warm-up, invite students to keep track of the most helpful norm they experience during the lesson.



- 2. Do the entire lesson as described in the teacher materials.
- 3. After the cool-down, invite students to take 2–3 minutes of quiet think time to individually reflect on the question "Which one of the norms did you feel was most important in your work today, and why?" and write their response on their cool-down page, on a separate sheet of paper, or in a math journal.



- 4. Collect and read their responses after class.
- Commit and explain to students that as our mathematical community of learners works together over the course of the year, we will continually add to and revise our "Doing Math" actions and "Norms".



Goal for Days 7+

Let's maintain & enhance norms that support doing math together within our community of learners.

Day 7+

Beyond the first 6 days, teachers refer to and revisit these norms weekly to sustain a healthy, positive, and inclusive community of learners. Teachers can:



- highlight helpful norms for students to attend to during an activity launch
- invite students to reflect on the norms during an activity or lesson synthesis by considering which ones are
 - the most challenging for them and why
 - becoming easier to live by and why
- pause and reflect on the norms after teaching a lesson by considering which ones
 - accurately describe the classroom community
 - could use some active support and attention within the classroom community

Consistently returning to these ideas shows students that we value the mathematical community of learners as much as we value the math content.

Adapted from Gray, K., Laib, J., & Caban, S. (2018, July 23). Building a Mathematical Classroom Community [Blog post]. Retrieved from https://illustrativemathematics.blog/2018/07/23/building-a-mathematical-classroom-community/



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Hub for lessons in Desmos Pacing Streamline/which resources to use when