STATWAY® PATHWAY MODULE 6 v4.1

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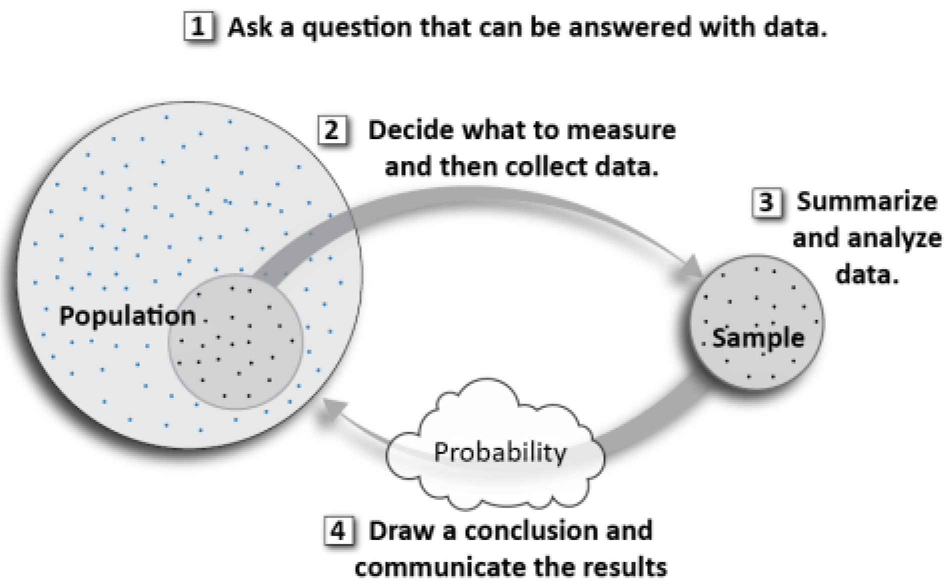
Preparation 6.1

THE BIG PICTURE

We have described statistical analysis as a process having four steps. These are represented below.



In Module 6, we turn our attention to the fourth step: using sample data to draw inferences about a population. Module 6 focuses on categorical variables. With categorical variables, individuals in the population fall into some category. We summarize categorical data in a sample by calculating a sample proportion.

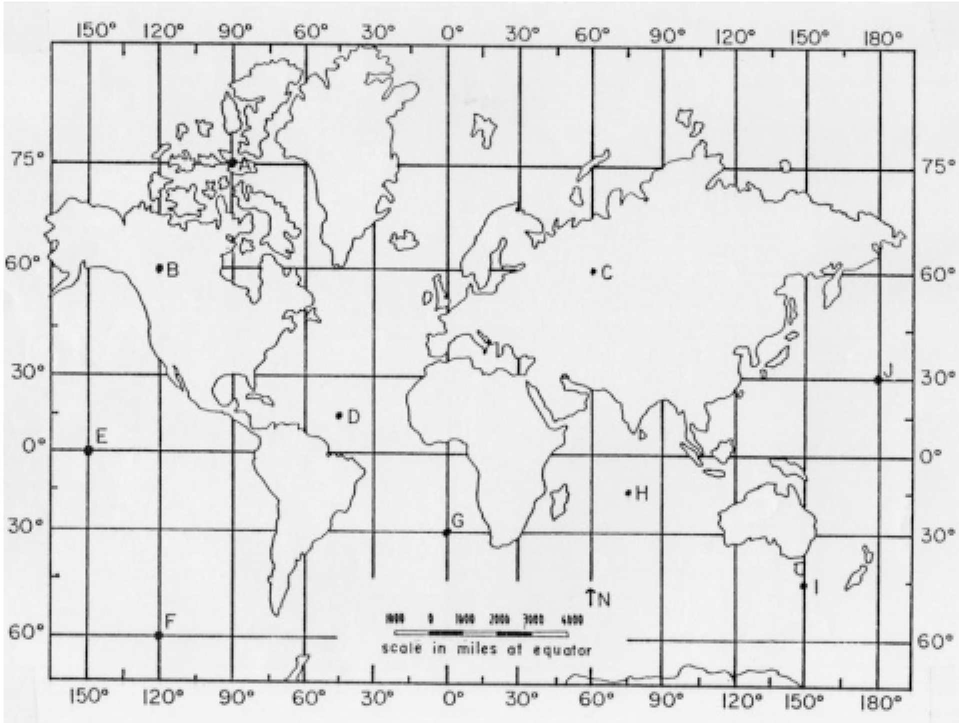
We will use sample proportions to draw conclusions about **population proportions**. A population proportion is the proportion for the entire population.

When we use sample data to draw a conclusion about a population, we say we are making a **statistical inference** about the population. In this module we learn about two main types of inference: (1) using sample data to estimate a population proportion (Unit 6.2) and (2) using sample data to test a claim about a population proportion (Units 6.3, 6.4, and 6.5).

Before we learn about the two types of inference, we need to explore how proportions vary as we take different samples.

INTRODUCTION

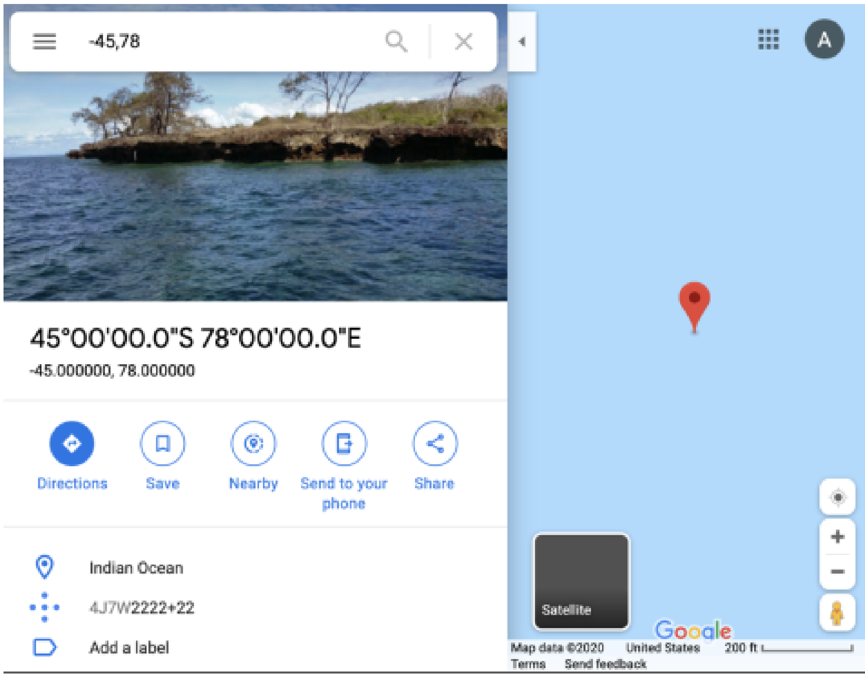
Suppose we want to know the proportion of the Earth's surface which is covered in water. If we collected all points on the surface of the Earth, what proportion of them would be on water? We can use the latitude-longitude coordinate system to investigate this question. Every point on the surface of the Earth can be described by a latitude-longitude coordinate. See https://carnegiemathpathways.org/go/britlat for a detailed description.



Latitudes are horizontal lines that are parallel to the equator. They take values between -90 degrees and 90 degrees. Latitudes below the equator (0 degree horizontal line) are considered negative. Longitudes are vertical lines, perpendicular to latitudes, and take values between -180 degrees and 180 degrees. Longitudes to the left of the prime meridian (0 degree vertical line) are considered negative. Each point on the surface of the Earth has a latitude and longitude pair.

Plotting a Random Point on the Surface of the Earth

- Randomly choose a point on Earth. Randomly pick a latitude by selecting a number between -90 and 90. Randomly pick a longitude by selecting a number between -180 and 180. For example, one possible combination is Latitude = -45 and Longitude = 78.
- Open up Google Maps (https://www.google.com/maps/).
- Enter the latitude and longitude of your point into the *Search Google Maps* input box to plot your point in Google Maps. For example, if the coordinates are -45, 78, enter in these two values separated by a comma. Below is a screenshot of this example. The description in the left panel describes the location of the point. Click the "–" (Zoom Out) button to zoom out and get a better visual of this location on the surface of the Earth. The point Latitude = -45 and Longitude = 78 is a point on the Indian Ocean.



1 What is the latitude and longitude of your point? Write your answers as decimals.

Latitude =

Longitude =

Be sure to note whether your point was over water or not.

Parameters & Statistics

For each point on Earth, the variable is whether or not the point is over water. When we consider a single point, if it is over water we call it a **success**. If it is over land, we call it a **failure**. Note that success doesn't mean good and failure doesn't mean bad. A success is just the outcome we are interested in, and failure is something else. The proportion of *all* points that are over water is the total number of points that are over water divided by the total number of points. This *population proportion* is an example of a **parameter**. A population proportion is denoted by the symbol *p*.

Language Tip
Parameters **are numerical summaries of populations.** ***Statistics*** **are numerical summaries of samples.**

We calculate the proportion of points that are over water in a sample by dividing the number of points that are over water in the sample by the number of points in the sample (the sample size). The proportion of points on Earth over water in a sample is an example of a **statistic**, and is denoted by the symbol $\hat{p}$, pronounced *p*-hat.

$$\hat{p} = \frac{number\ of\ successes\ in\ the\ sample}{sample\ size}$$

The table below summarizes these ideas.

Population	**Sample**
Collection of all points on Earth	A random sample of 20 points on Earth

Parameter	**Statistic**
The proportion of points that are over water (p)	Proportion of points that are over water in the sample ($\widehat{p}$)

It is important to recognize that there are many samples of points on Earth, each with their own proportion of points over water, but *there is only one population proportion*! Sample proportions vary from sample to sample, but the population proportion is a single number.

In this activity, we will examine multiple samples of 20 points on Earth. We will calculate a sample proportion $\widehat{p}$ for each sample. These proportions are a small part of the collection of *all* sample proportions. The collection of all sample proportions forms a distribution of values called the **sampling distribution of sample proportions**.

Finding & Calculating a Sample Proportion

The map below contains 20 random points. The individual points are labeled 1 through 20. You are going to calculate the sample proportion of points over water. If you are unsure about a particular point, make a judgment call (or check on Google Maps) to determine whether the point is over water or over land.

2 Record the proportion of points over water in your sample. This is your sample proportion. Write the sample proportion as a decimal.

3 Your sample proportion is an estimate of the population proportion - the proportion of all points that are over water. Do you believe that your sample proportion will be the same as the population proportion? Choose an answer below.

(i) Yes, since the sample proportion is from a random sample.

(ii) Yes, since the sample proportion is between 0 and 1.

(iii) No, since the sample size is less than 100 points.

(iv) No, since proportions from sample to sample vary, so the proportion from one sample is not likely to be the same as the population proportion.

6.1: Sampling Distributions of Sample Proportions

LEARNING GOALS

By the end of this collaboration, you should understand that:

- Parameters are numerical values calculated from populations; statistics are numerical values calculated from samples.
- Statistics vary from sample to sample but the population parameter is fixed.
- Statistics vary according to a pattern called a sampling distribution, which has a center, shape, and spread.
- Sample size affects the variability of the sampling distributions.
- The distribution of sample proportions is approximately normal under certain conditions.
- When a distribution of sample proportions is approximately normal, the normal distribution can be used to find probabilities related to given sample proportions.

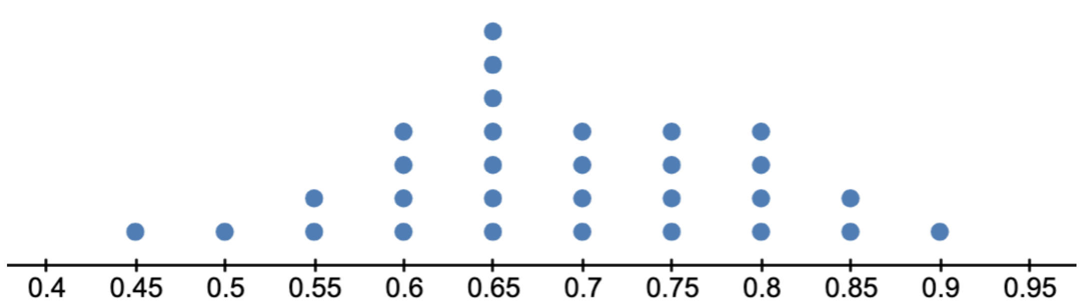
By the end of this collaboration, you should be able to:

- Describe the characteristics of distributions of sample proportions in terms of shape, center, and spread.
- Determine the mean and standard error of a sampling distribution of sample proportions.
- Use the criteria for approximate normality to determine when the normal distribution may be used to approximate a sampling distribution of sample proportions.
- Use the normal distribution and technology or tables to determine the probability that a sample proportion is within a specific range of values.

INTRODUCTION

Sampling Distributions of Sample Proportions

In Preparation 6.1, you computed a sample proportion — the proportion of points over water — from a random sample of 20 points on the surface of the Earth. The dotplot below shows the results of 30 samples. Each dot represents a sample proportion from a random sample of 20 points on Earth (just as each of you did in Preparation 6.1). Use the dotplot to answer the following questions.



1 Did each random sample of 20 points yield the same sample proportion?

2 Each dot represents a sample proportion of points on Earth over water. Using the dotplot, what is the best estimate of the population proportion of *all* points on Earth over water?

3 Think about the distribution of 30 sample proportions in the dotplot above.

A Describe the shape of the distribution of sample proportions.

B Estimate the mean of the sample proportions in the dotplot.

C In Question 2 you estimated the proportion of all points on Earth over water. In Question 3B you estimated the mean of all sample proportions. How are these values related to one another?

YOU NEED TO KNOW

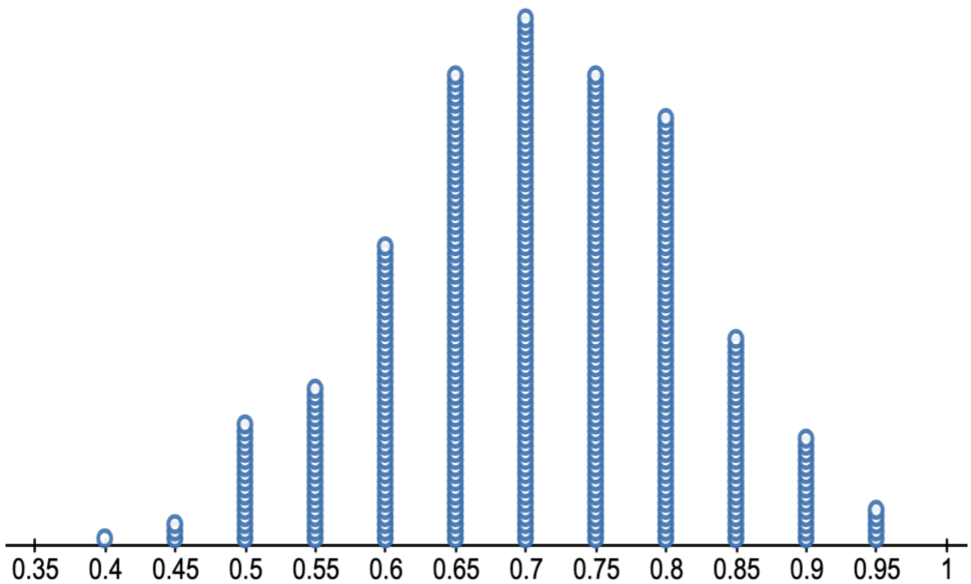
The dotplot above is part of the *sampling distribution of sample proportions*. A **sampling distribution of sample proportions** is the distribution of *all* possible sample proportions from samples of a given size.

Language Tip

A *sampling distribution* is a description of all possible values of a statistic from random samples of a given size.

NEXT STEPS
Simulating a Distribution of Sample Proportions

In the next part of this activity, we use a computer to further simulate part of the sampling distribution of sample proportions of points on Earth over water. Counting out points on a map is time consuming and inefficient, but technology can be used to better simulate a distribution of sample proportions. The dotplot below displays 400 sample proportions. Each sample proportion is based on a random sample of 20 points on Earth.



4 Visually estimate the mean of the distribution of 400 sample proportions.

5 Describe the distribution of 400 sample proportions in terms of center, shape, and spread.

The Central Limit Theorem for Sample Proportions

It turns out that sampling distributions of sample proportions become more normal as the sample size increases. A *sampling distribution of sample proportions* is the distribution of *all* possible sample proportions from samples of a given size. If the sample size is large enough, this distribution is approximately normal. You will explore this idea by examining how distributions of 1000 sample proportions change as the sample size increases.

Scientists estimate that 71% of the surface of the Earth is covered in water. So, the population proportion of all points on Earth over water is 0.71. We will now use a simulation to create random samples and compute sample proportions from a population where the population proportion is 0.71.

- Open the Statkey simulator by going to https://carnegiemathpathways.org/go/statkeysampling
- Click on Edit Proportion and set the proportion to 0.71
- Click on the Sample Size (n =) and set the sample size to 10.
- Click Generate 1000 Samples to generate and plot 1000 sample proportions in the dotplot. The number of sample proportions will be shown at the top right of the viewing window. Each dot in the dotplot represents a sample proportion.
- The mean and standard deviation (std. error) of the sample proportions are displayed in the upper right corner of the main viewing window. Because sample proportions are estimates of the population proportion, deviations are actually errors, so now we refer to the standard deviation of any statistic as its standard error.

6 A What is the mean, standard deviation, and shape of the distribution of sample proportions when the sample size is **10**?

Mean of sample proportions =

Standard Deviation (denoted std. error on the applet) of sample proportions =

Shape of distribution of sample proportions =

B What is the mean, standard deviation, and shape of the distribution of sample proportions when the sample size is 25? Go back to Statkey. Click on the Sample Size (n =) and set the sample size to **25**. Then click on "Generate 1000 samples" to generate and plot the sample proportions.

Mean of sample proportions =

Standard Deviation (denoted std. error on the applet) of sample proportions =

Shape of distribution of sample proportions =

C What is the mean, standard deviation, and shape of the distribution of sample proportions when the sample size is **50**? Go back to Statkey. Click on the Sample Size (n =) and set the sample size to 50. Then click on "Generate 1000 samples" to generate and plot the sample proportions.

Mean of sample proportions =

Standard Deviation (denoted std. error on the applet) of sample proportions =

Shape of distribution of sample proportions =

7 How does the mean of a distribution of sample proportions relate to the population proportion?

8 How does the distribution of sample proportions change as the sample size increases?

(i) As the sample size increases, the population proportion changes.

(ii) As the sample size increases, the standard deviation of the sample proportions increases.

(iii) As the sample size increases, the shape of the distribution of sample proportions becomes more skewed.

(iv) As the sample size increases, the standard deviation of the sample proportions decreases.

Mean and Standard Error of Sampling Distributions

We denote the **mean** of sample proportions as $\mu_{\hat{p}}$. We have seen that this mean is equal to the population proportion (p).

Mean of sample proportions: $\mu_{\hat{p}} = p$

A sample proportion is an estimate of the population proportion. When a sample proportion deviates from the population proportion, the deviation is an error in the estimate. Because of this, the standard deviation of sample proportions is called the **standard error** of sample proportions. We denote the standard error of sample proportions as $\sigma_{\hat{p}}$.

The formula for the standard error is:

Standard error of sample proportions: $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

9 71% of points on the surface of Earth are on water. So, the population proportion of points on Earth covered over water is 0.71.

Use the formulas above to compute the mean and standard error of all sample proportions of points on Earth over water when the sample size is 25. Round the standard error to three decimal places.

Mean of sample proportions =

Standard error of sample proportions =

Criteria for Approximate Normality

Statisticians have learned that sampling distributions of sample proportions are approximately normal whenever $np \geq 10$ and $n(1 - p) \geq 10$. Since p is the proportion of successes, and n is the sample size, np is the expected, or mean, number of successes in a sample of size n. That is, on average, samples of size n will have np successes. Similarly, $1 - p$ is the proportion of failures, and $n(1 - p)$ is the expected number of failures. Thus the sampling distribution of sample proportions is approximately normal if it meets the *Criteria for Approximate Normality*, which requires there are at least 10 expected successes and 10 expected failures in the sample.

As an example, suppose we sample 50 points on Earth and assume that 71% of all points on Earth are over water. The expected number of successes in a sample is $np = 35.5$ and the expected number of failures is $n(1 - p) = 50(0.29) = 14.5$.

This means that, on average, samples of 50 points on Earth contain 35.5 points over water and 14.5 points over land. Of course, the number of points over water will vary in individual samples. But since both of the numbers are greater than 10, the normal distribution is a good approximation for the sampling distribution of sample proportions of points on Earth over water in samples of size 50.

YOU NEED TO KNOW

The Central Limit Theorem for Sample Proportions

A sampling distribution of sample proportions from a population with proportion p is approximately normal if $np \geq 10$ and $n(1 - p) \geq 10$.
The **mean** ($\mu_{\hat{p}}$) and **standard error** ($\sigma_{\hat{p}}$) of a sampling distribution of sample proportions are:

$$\textit{Mean} = \mu_{\hat{p}} = p$$

$$\textit{Standard error} = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

where p is the population proportion and n is the sample size.

TRY THESE

Representing a Sampling Distribution with a Normal Distribution

When a sampling distribution of sample proportions satisfies the normality criteria we can use the normal distribution properties to find probabilities corresponding to sample proportions.

The Gallup organization conducts surveys in countries throughout the world to obtain categorical and quantitative data on people and their views about important issues. Gallup surveyed people in the United States in March 2019 to obtain information on U.S. adults' views regarding global warming. They found that 51% of U.S. adults are "concerned believers" who take global warming seriously and believe it poses a serious threat within their lifetime.

10 For this activity let's assume that Gallup's result (0.51) is the proportion of all U.S. adults who take global warming seriously. Suppose we sample 120 U.S. adults and determine the proportion who take global warming seriously. We can apply the Central Limit Theorem to describe the sample proportions that are likely to occur given the sample size and assumed population proportion.

A What is the sample size (n) and population proportion (p)?

Sample size, n = ___________

Population proportion, p = ___________

B Let's explore if the normality criteria are met. First, find the following values. Round answers to one decimal place.

np = ___________

$n(1 - p)$ = ___________

C Are the normality criteria met?

D Find the mean and compute the standard error of the sampling distribution of sample proportions (use three decimal places for the standard error).

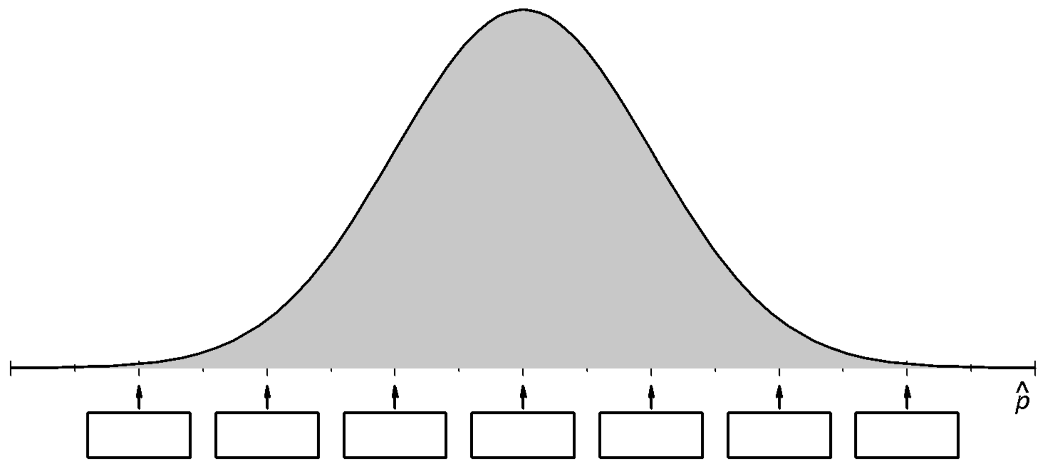
Mean = $\mu_{\hat{p}} = p$ = ___________

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ = ___________

E Which statement below best describes the standard error of the sampling distribution of sample proportions?

(i) Sample proportions vary, and the difference between the lowest possible sample proportion and highest possible sample proportion is 0.046.
(ii) 0.046 is the typical distance which sample proportions are from the population proportion.
(iii) All sample proportions are within 0.044 from the population proportion.
(iv) No sample proportions equal the population proportion.

F The boxes under the normal distribution below are one standard error apart, with the center box at the mean. Use the mean and standard error above to enter the correct values into the boxes.



G Suppose that in a random sample of 120 U.S. adults 48 respond that they take global warming seriously. Compute the sample proportion, $\widehat{p}$. Write your answer as a decimal rounded to two decimal places.

H What is the *Z*-score of the sample proportion from Question 10G? Use the mean and standard error from Question 10D.

I Does this *Z*-score indicate that this sample proportion is unusual?

J Use technology or a normal distribution table to find the probability of observing a sample proportion that is less than or equal to the value from Question 10G. Round your answer to four places after the decimal.

LET'S SUMMARIZE

Please consider the following key points:

- The Central Limit Theorem for sample proportions states that a sampling distribution of sample proportions is approximately normal if the sample size is large enough. Our criteria for determining this are: $np \geq 10$ and $n(1 - p) \geq 10$.
- When the criteria for approximate normality are satisfied, the normal distribution may be used to determine probabilities about sample proportions.
- The mean and standard error (or standard deviation) for the sampling distribution of sample proportions are given by:

$$\textit{Mean} = \mu_{\hat{p}} = p$$

$$\textit{Standard error} = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Statway Cohort Contract

By signing below, I agree to fulfill the following requirements for participation in this Statway course, and acknowledge that I understand the requirements for continued enrollment.

Commitments to the class:

- I commit to successfully completing this Statway course with the members of my cohort.
- I commit to supporting all of my cohort members to understand statistics
- I commit to supporting all of my cohort members to complete the Statway course.
- I will come to class every day prepared to participate in all classroom activities.
- I will contribute to creating a productive classroom atmosphere that supports everyone learning.
- I will keep an open mind and a positive attitude, and will be willing to try out new learning strategies and study skills.
- I will ____
  ____

Commitments to my group:

- We commit to coming to class and working with our group everyday
- We commit to____
  ____
- We commit to____
  ____

Printed Name:______________________Signature:_________________________Date:________

Printed Name:______________________Witness: __________________________Date: _______

Exercise 6.1

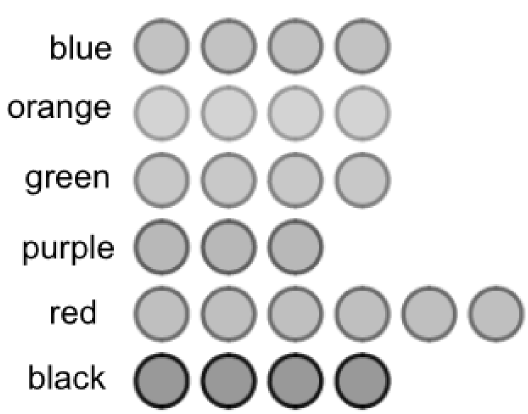
A recent study reported that 40% of community college students in the U.S. receive federal loans to pay for college tuition and other college expenses. The study is based on data from a national survey of 1350 students. The sample was representative of all community college students, so researchers use this study to describe the characteristics of all community college students.

1 In this study, what is the sample and sample statistic?

2 What is the population and the population parameter that the sample statistic is estimating?

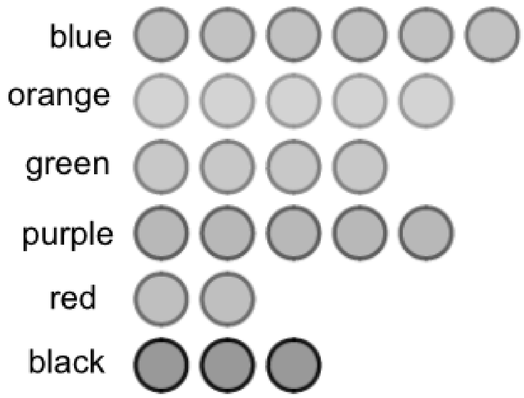
Note: If doing this exercise online, follow the instructions given online for **Questions 3-8**.

3 The image below displays the results when 25 M&M's are randomly sampled from a large population of M&M's. You can see the number of each color of M&M in the sample. Write the number of Blue M&M's in the sample.



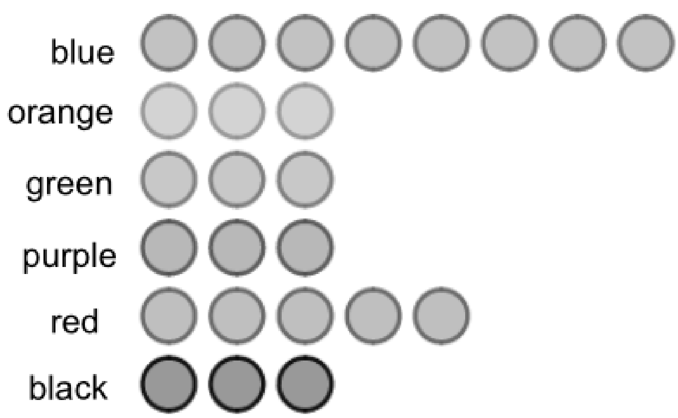
4 What is the sample proportion of blue M&M's? Write the sample proportion as a decimal.

5 Below is an image of a second random sample of 25 M&M's. Write the number of Blue M&M's in the sample.



6 What is the sample proportion of blue M&M's in the second sample? Write the sample proportion as a decimal.

7 Below is an image of a third random sample of 25 M&M's. Write the number of Blue M&M's in the sample.



8 What is the sample proportion of blue M&M's in the third sample? Write the sample proportion as a decimal.

9 Are the sample proportions which you found above statistics or parameters?

(i) Parameters - Each proportion describes the population.

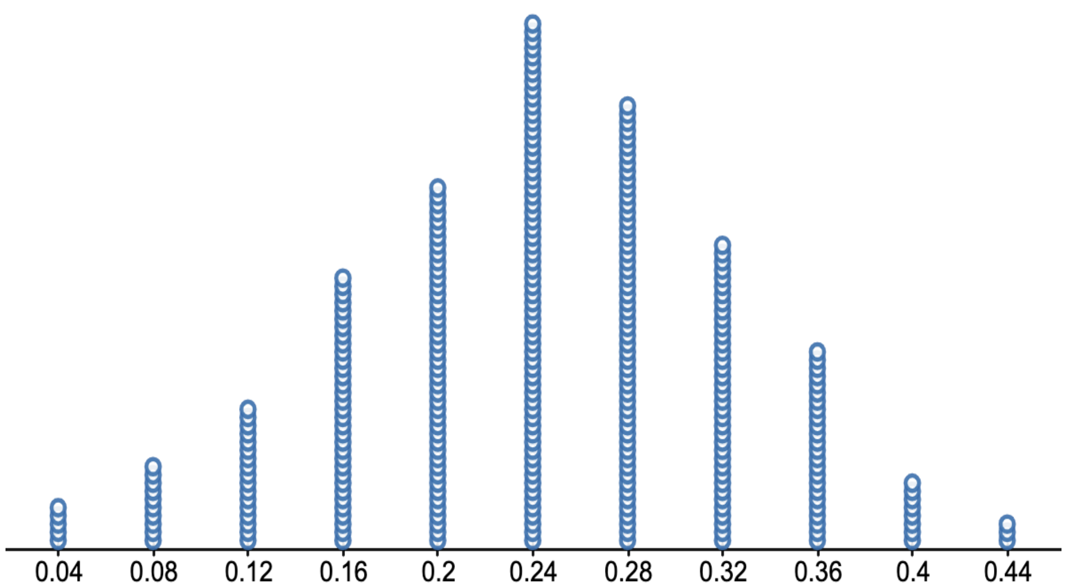
(ii) Parameters - Each proportion describes a sample.

(iii) Statistics - Each proportion describes the population.

(iv) Statistics - Each proportion describes a sample.

10 Based on your samples, what is your best estimate for the population proportion of blue M&M's? This is the proportion of blue M&M's in all M&M's.

11 Let's assume that 24% of all M&M's are blue. This means the population proportion of M&M's that are blue is $p = 0.24$. Suppose random samples of 25 M&M's are obtained and the proportion of blue M&M's are found. The image below displays a simulated distribution of 300 sample proportions of M&M's that are blue, for samples of size $n = 25$.



Estimate the mean of the sample proportions in the dotplot.

12 The standard deviation of the 300 sample proportions in the dotplot is 0.082. 0.082 is an estimate of the standard error of all sample proportions from random samples of size 25. What statement below best interprets this value?

(i) If we select two sample proportions, they will vary by 0.082.
(ii) Sample proportions, from random samples of size 25, vary by at most 0.082.
(iii) Sample proportions, from random samples of size 25, have a typical deviation from the population proportion of 0.082.
(iv) Sample proportions vary.

13 The mean of a sampling distribution of sample proportions is $\mu_{\widehat{p}} = p$. Determine the mean of the sampling distribution in this situation.

14 The standard error of a sampling distribution of sample proportions is $\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$. Determine the standard error of the sampling distribution in this situation. Round the standard error to three decimal places.

15 Let's make a prediction. What will happen to the *mean* and *standard error* of the distribution of sample proportions if we increase the sample size?

How does the **mean** of the distribution of sample proportions change as the sample size increases?

16 How does the **standard error** of the distribution of sample proportions change as the sample size increases?

17 We can use the Central Limit Theorem formulas to examine what happens as we increase the sample size from 25 to 100.

Compute the standard error, $\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$, of the sample proportions of blue M&M's in the simulated distribution of sample proportions when the sample size is n = 100. Round the standard error to three decimal places.

18 Was your prediction correct? What effect did increasing the sample size have on the variability of sample proportions in the distribution of sample proportions?

Gallup reported that 67% of adults in Latin America believe that global warming is a serious threat to themselves and their family.[1] Let's assume this value represents the population proportion. Now imagine that a researcher surveys a random sample of 150 adults in Latin America, asking about their beliefs on global warming. The researcher wants to know whether people's opinions have changed. Let's use the Central Limit Theorem to think about the results that can occur.

19 If samples of size 150 are taken, find the mean ($\mu_{\widehat{p}}$) of the resulting sampling distribution of sample proportions ($\widehat{p}$). Assume the population proportion is p = 0.67.

20 If samples of size 150 are taken, find the standard error ($\sigma_{\widehat{p}}$) of the resulting sampling distribution of sample proportions ($\widehat{p}$). Round the standard error to three decimal places. Assume the population proportion is p = 0.67.

[1] http://www.gallup.com/poll/147203/fewer-americans-europeans-view-global-warming-threat.aspx

21 A sampling distribution is a description of the possible values of a statistic. What does *this* sampling distribution represent?

(i) The proportion of all adults in Latin America who believe global warming is a serious threat.
(ii) The proportion of one random sample of 150 Latin American adults who believe global warming is a serious threat.
(iii) The distribution of sample proportions from all random samples of 150 Latin American adults who believe global warming is a serious threat.
(iv) The distribution of sample proportions from all random samples of any size.

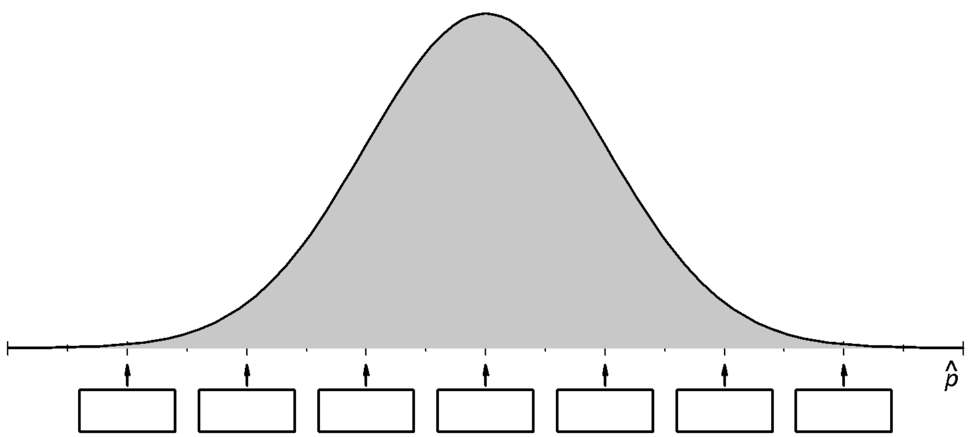
22 Find the values of np and $n(1 - p)$.

np = __________

$n(1 - p)$ = __________

23 Is the sampling distribution of sample proportions approximately normal? How do you know?

24 The boxes under the normal distribution below are one standard error apart, with the center box at the mean. Use the mean and standard error from Questions 19 and 20 to enter the correct values. Round to three decimal places.



25 Use the Empirical Rule to find the interval centered at p that contains approximately 95% of all sample proportions.

26 What sample proportions would you consider unusual?

27 In a random sample of 150 adults in Latin America, 113 say global warming is a serious threat. Find the sample proportion ($\widehat{p}$). Round the value to three decimal places.

28 Find the *Z*-score for this sample proportion. Round the value to two decimal places.

29 Use technology or tables to find the probability that in a random sample of 150 adults in Latin America, 113 or more will say global warming is a serious threat. Round the value to three decimal places.

6.1 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Describe the characteristics of distributions of sample proportions in terms of shape, center, and spread.	
Determine the mean and standard error of a sampling distribution of sample proportions.	
Use the criteria for approximate normality to determine when the normal distribution may be used to approximate a sampling distribution of sample proportions.	
Use the normal distribution and technology or tables to determine the probability that a sample proportion is within a specific range of values.	

6.1-S: Parameters & Statistics

LEARNING GOALS

By the end of this lesson, you should understand that:

- Parameters are numerical measures of population characteristics; statistics are numerical measures calculated from samples.
- A distribution of sample proportions is approximately normal under certain conditions.
- When a distribution of sample proportions is approximately normal, the normal distribution can be used to find probabilities related to given sample proportions.

By the end of this lesson, you should be able to:

- Identify sample statistics and the parameter of interest from a description of a statistical study.
- Determine the mean and standard error of a sampling distribution of sample proportions.
- Find and interpret a Z-score of a sample proportion based on its sampling distribution.
- Use the mean and standard error of a distribution of sample proportions to determine intervals containing 95% of sample proportions in a distribution.

INTRODUCTION

We learned previously that a *statistic* is a characteristic of a sample and that a parameter is a characteristic of a population. When reading a study, we must determine whether numerical results within the study describe samples or populations. Additionally, when a sample statistic is provided, we must be able to determine the population parameter of interest which the statistic is estimating.

Read the following study then answer the questions about it.

> Excessive alcohol drinking can lead to serious health concerns and significant academic and social challenges. The National Survey on Drug Use and Health (NSDUH) is an annual survey sponsored by SAMHSA within the U.S. Department of Health and Human Services (HHS). Results in the 2020 NSDUH indicate the following alcohol consumption characteristics for adults aged 18-24:
>
> - 51.5% drank alcohol in the past month, and of these
> - 31.4% engaged in binge drinking and
> - 8.6% engaged in heavy alcohol use[2].
>
> SAMHSA is an agency within the U.S. Department of Health and Humans Services whose mission consists of leading public health efforts to advance the behavioral health of people throughout the United States. The study is based on data collected from 16,875 adults aged 18 - 24 across the U.S. in 2020.

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[2] https://www.samhsa.gov/data/sites/default/files/reports/rpt35325/NSDUHFFRPDFWHTMLFiles2020/2020NSDUHFFR1PDFW102121.pdf

1 For the above study, answer the following:

A Describe the sample and the statistic(s).

B Describe the population parameter(s) of interest.

C Make a reasonable inference based on the given information.

Read the following study and then answer the questions about it.

> The Pew Research Center conducted a national study on U.S. adults' experiences with online harassment. The study found that among U.S. adults aged 18 to 29, 48% say they have personally experienced more severe online harassment, and 16% say they have experienced less severe online harassment. The study found that U.S. adults aged 50 to 64 are less likely to experience online harassment. In this age group, 16% say they have personally experienced more severe online harassment and 14% say they say they have experienced less severe online harassment. This study was conducted between September 8-13, 2020 and gathered data from 983 U.S. adults aged 18 to 29 and 3,022 U.S. adults aged 50 to 64.[3]

2 For the above study, answer the following

A Describe the sample and the statistic(s).

[3] https://www.pewresearch.org/internet/2021/01/13/the-state-of-online-harassment/

B Describe the population parameter(s) of interest.

C Make a reasonable inference based on the given information.

NEXT STEPS

In a previous lesson we learned about the *Central Limit Theorem for Sample Proportions*. This theorem enables us to predict the center and spread of a distribution of sample proportions from random samples of a certain size.

The distribution would be centered at the population proportion, *p*. If the sample size is sufficiently large, we can model sample proportions with a normal distribution and compute probabilities associated with specific sample proportions. The sample is considered sufficiently large if we can expect at least ten successes and at least ten failures in the sample of size *n*. This can be determined with two calculations:

$np \geq 10$ (at least 10 successes) and $n(1 - p) \geq 10$ (at least 10 failures)

These techniques serve as the foundation to the statistical inference on population proportions which we will perform later in Module 6.

3 Let's return to the Pew Research Center study on online harassment that was referenced above. The majority of people who have been harassed online recently say the most recent experience occurred on social media. The study found that approximately 8 in 10 U.S. adults believe that social media companies are doing a "poor" or "only fair" job addressing online harassment.

For this activity, let's assume that the Pew result (80%) is the proportion of all U.S. adults who believe that social media companies are doing a "poor" or "only fair" job at addressing online harassment. Suppose random samples of 100 U.S. adults are selected and sample proportions are obtained. We can apply the Central Limit Theorem to predict how these sample proportions will vary from the population proportion.

A Based on the assumed population proportion and given sample size, are the normality criteria met?

np = ___________

B Based on the assumed population proportion and given sample size, let's see if the normality criteria are met.

$n(1 - p)$ = ___________

C Are the normality criteria met?

D Find the mean of the sampling distribution of sample proportions.
Mean = $\mu_{\hat{p}} = p$ = ___________

E Compute the standard error of the sampling distribution of sample proportions. Round to 2 decimal places.

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ = _____________

F Use the Empirical Rule to determine the interval that contains 95% of the sample proportions in the sampling distribution.

4 Suppose we obtain a random sample of 100 U.S. adults in which 75 believe that social media companies are doing a "poor" or "only fair" job at addressing online harassment.

A What is the sample proportion ($\hat{p}$)?

B Find the *Z*-score of the sample proportion from Question 4A. Use the mean and standard error of the sampling distribution. Round to two decimal places.

C Is this value unusual? Explain your answer.

D Use technology or a normal distribution table to find the probability of observing a sample proportion that is less than or equal to the value from Question 4A. Round to three decimal places.

E Based on the probability you found in Question 4D, should the sample proportion from Question 4A be considered unusual?

5 Suppose we obtain a different random sample of 100 U.S. adults in which 90 believe that social media companies are doing a "poor" or "only fair" job at addressing online harassment.

A What is the sample proportion ($\widehat{p}$)?

B Find the *Z*-score of the sample proportion from Question 5A. Use the mean and standard error of the sampling distribution. Round to two decimal places.

C Is this value unusual? Explain your answer.

D Use technology or a normal distribution table to find the probability of observing a sample proportion that is greater than or equal to the value from Question 5A.

E Based on the probability you found in Question 5D, should the sample proportion from Question 5A be considered unusual?

6 The previous question showed that If 80% of all U.S. adults believe that social media companies are doing a “poor” or “only fair” job at addressing online harassment, it would be very unlikely for us to obtain a random sample of 100 U.S. adults in which 90% or more held this view. Suppose we instead sampled 500 U.S. adults. Would we be less likely or more likely to obtain a random sample in which 90% or less held this view? Explain.

LET’S SUMMARIZE

In this lesson, you examined parameters and statistics from statistical studies and applied the Central Limit Theorem for Sample Proportions to model sample proportions and characterize sample proportions as being unusual.

You should now understand that:

- Parameters are numerical measures of population characteristics; statistics are numerical measures calculated from samples.
- A distribution of sample proportions is approximately normal under certain conditions.
- When a distribution of sample proportions is approximately normal, the normal distribution can be used to find probabilities related to given sample proportions.

Practice Problems 6.1-S

1 Explain the difference between a sample statistic and a population parameter. Provide an example of each.

In a large city in the Midwestern United States, 55% of all registered voters support the democratic candidate for Congress. Suppose random samples of 200 voters are selected and sample proportions are obtained. Apply the Central Limit Theorem to predict how these sample proportions will vary from the population proportion.

Based on the assumed population proportion and given sample size, let's see if the normality criteria are met.

2 np = ___________

3 $n(1-p)$ = ___________

4 Are the normality criteria met? Explain.

5 Find the mean of the sampling distribution of sample proportions.

Mean = $\mu_{\hat{p}} = p$ = ___________

6 Compute the standard error of the sampling distribution of sample proportions. Round to 3 decimal places.

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ = _____________

7 Given the above mean and standard error), would a sample proportion with a mean of 0.47 be considered unusual?

8 Find the *Z*-score of this sample proportion. Use the mean and standard error of the sampling distribution. Round to three decimal places.

9 Use technology or a normal distribution table to find the probability of observing a sample proportion that is less than or equal to the sample proportion. Round to 3 decimal places.

P-value = _______________

10 A January 2018 Gallup poll found that 62% of American adults feel that the federal government does not provide enough support for older Americans[4]. Given this assumed population proportion, would it be unlikely to find a random sample of 800 American adults in which less than 60% of the sample held this view? Explain.

[4] http://www.people-press.org/2018/01/30/majorities-say-government-does-too-little-for-older-people-the-poor-and-the-middle-class/

6.2: Confidence Intervals for a Population Proportion

LEARNING GOALS

By the end of this collaboration, you should understand that:

- A population parameter can be estimated by a point estimate and an interval estimate.
- A point estimate is a single value and an interval estimate is a range of values.
- A confidence interval is based on a single sample and indicates a range of values that may contain the population parameter, but only at a certain level of confidence.
- The confidence level is the percentage of all theoretical confidence intervals (from samples of size n) that contain the true parameter.
- Increasing the sample size or decreasing the confidence level will lower the margin of error and narrow the confidence interval. The best way to lower the margin of error is to increase the sample size.

By the end of this collaboration, you should be able to:

- Calculate a point estimate.
- Determine the number of standard errors (i.e. the *critical value*, Z_c) that corresponds to a given level of confidence.
- Compute the margin of error in a sample proportion, corresponding to a given level of confidence.
- Compute and interpret a confidence interval for a population proportion.
- Use a confidence interval to reason about a population proportion.

INTRODUCTION

NBC News and the Wall Street Journal conduct phone surveys to gather the opinions of adults in the United States on a variety of political issues. In an October 2017 NBC News/Wall Street Journal poll researchers asked a random sample of registered voters in the U.S. if they approved of the job Donald Trump is doing as President.[5] **Thirty-eight percent (38%)** of the respondents indicated that they approved of President Trump's job performance. In the article summarizing the results of the survey, NBC News/Wall Street Journal noted that the **margin of error** for the survey was ±3.3 percentage points.

[5] https://www.nbcnews.com/politics/donald-trump/trump-s-approval-rating-drops-lowest-level-yet-new-nbc-n815321

TRY THESE 1
Estimating President Trump's Approval Rating

1 Discuss the following questions in your group. Write a group response.

A Is the 38% a parameter or a statistic? Give a reason for your answer.

B Do you think the population proportion of U.S. adults who approved of President Trump's job performance in October 2017 was exactly 38%? Give a reason for your answer.

C What do you think the margin of error of ±3.3 percentage points tells us about the population proportion?

The sample proportion, $\widehat{p}$, is a single estimate of the population proportion, often referred to as a **point estimate**. An **interval estimate** is a range of values used to estimate a population proportion. Interval estimates are centered at a point estimate. The interval is formed as

Point Estimate ± Margin of Error

2 Using the information about the poll above, fill in the following tables. Give your answers in decimal form.

A Identify the point estimate and margin of error. Write both values as decimals.

Point Estimate, $\widehat{p}$	**Margin of Error, E**

B The point estimate and margin of error can be used to construct the endpoints of the interval estimate. Give the lower and upper limit of the interval below.

Lower Limit = Point Estimate − Margin of Error ($\widehat{p} - E$)	**Upper Limit = Point Estimate + Margin of Error** ($\widehat{p} + E$)

Interval estimates like the ones in Question 2B give a range of values that *usually* contain the population proportion, which in our example is the proportion of all registered voters in the U.S. who approve of President Trump's job performance.

3 Do you think it's reasonable to conclude that the proportion of all U.S. adults who approved of President Trump's job performance in October 2017 was as low as 35%? Explain.

4 Do you think it's reasonable to conclude that the proportion of all U.S. adults who approved of President Trump's job performance in October 2017 was as high as 45%? Explain.

Confidence Intervals for the Population Proportion

We call the interval estimate for a population parameter a **confidence interval**. To construct a confidence interval for a population proportion we need two things:

- A *point estimate*
- A *margin of error*

Point estimates, such as sample proportions and sample means, are easy to compute. We have already practiced computing sample proportions. To compute a margin of error, we have to think about the sampling distribution of all sample proportions from a given sample size.

NEXT STEPS

To learn about margins of error, let's think about sampling distributions of sample proportions. We learned that the mean of the sampling distribution of sample proportions, $\mu_{\widehat{p}}$, is the population proportion.

$$Mean: \mu_{\widehat{p}} = p$$

The standard deviation of the sampling distribution of sample proportions, $\sigma_{\widehat{p}}$, is called the standard error, because deviations in estimates are errors. The formula is:

$$Standard\ error: \sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Recall that the mean of a sampling distribution of sample proportions details the center of the distribution of sample proportions, and is the population proportion. The standard error describes the typical variability of sample proportions from the population proportion.

When *p* Is Unknown

When the population proportion (p) is unknown, we estimate it using a sample proportion ($\widehat{p}$). We can then *estimate* the standard error using a sample proportion ($\widehat{p}$) in place of p.

$$\textbf{\textit{Estimated standard error}} = \sqrt{\frac{\widehat{p}\,(1-\widehat{p}\,)}{n}}$$

When p is unknown, we have another problem. We have been saying that the sampling distribution of sample proportions is approximately normal when np and $n(1 – p)$ (the expected number of successes and failures) are both at least 10. Unfortunately, if p is unknown, we cannot determine if these criteria are satisfied, so we use the sample proportion, $\hat{p}$, as an estimate of p instead.

YOU NEED TO KNOW

In the context of a confidence interval for an unknown population proportion (p), the **criteria for approximate normality** are satisfied if $n\widehat{p}$ and $n(1-\widehat{p})$ are at least 10. These are the actual (not expected) number of successes and failures, so we only need to check for at least 10 successes and failures in our random sample.

TRY THESE 2

5 In a previous unit, we learned that 24% of M&M's are blue, but let's pretend we **don't** know that. We take a random sample of M&M's and construct a confidence interval to **estimate** the proportion of M&M's that are blue. Suppose we randomly select n = 50 M&M's and 16 of them are blue.

A Are the criteria for approximate normality of the sampling distribution met?

B What is the sample proportion of M&M's that are blue? Round the value to two decimal places.

$\widehat{p}$ = ______________

C Compute the *estimated* standard error for this sample proportion. Round the value to three decimal places.

Estimated standard error = ______________

D Interpret the estimated standard error for this sample proportion.

To compute a *margin* of error we determine the number of standard errors from the mean needed to capture a given percentage of sample proportions. This number of standard errors is called a **critical value**. A number of standard errors (or deviations) from the mean is, by definition, a *Z-score*. Critical values are denoted Z_c, and can be found by examining the standard normal distribution.

Z_c is the number of standard errors in the margin of error. From the Empirical Rule, we know that about 95% of sample proportions are within two standard errors of the mean. The Empirical Rule uses rounded values. It is more accurate to say about 95% of sample proportions are within **1.96** standard errors of the mean. **So, the critical value for the 95% confidence level is Z_c = 1.96**. 95% of sample proportions are within 1.96 standard errors of the population proportion. If we randomly choose a sample proportion, we are 95% confident that it is within 1.96 standard errors from the population proportion.

Margin of Error

To compute a margin of error for a given level of confidence, we find the corresponding critical value, Z_c. Remember, Z_c is the number of standard errors in the margin of error. So the margin of error formula is

$$E = Z_c \cdot Estimated\ Standard\ Error$$

$$= Z_c\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

The **margin of error** is the approximate distance that a certain percentage of sample proportions are from the population proportion. Once the level of confidence and margin of error are known, we can compute a confidence interval.

When we interpret a given confidence interval, we can say that the **level of confidence** is the proportion of similarly constructed intervals that contain the population proportion, or that it describes how confident we are that the population proportion is in the interval.

E Compute and interpret the margin of error for 95% of sample proportions. Round the value to two decimal places.

E = _______________

F We can express a confidence interval for a population proportion as $\widehat{p} \pm E$. Use this form. Write in the sample proportion and margin of error.

_______________ ± _______________

G Sometimes we write a confidence interval using *interval notation*, $(\widehat{p} - E, \widehat{p} + E)$. This notation represents all numbers between the lower and upper limits of the interval. Give your interval using this notation.

$(\widehat{p} - E, \widehat{p} + E)$ = (_______________,_______________)

Why Do We Center the Interval at the Sample Proportion?

When the confidence level is 95%, we know that about 95% of sample proportions will be within 1.96 standard errors of the population proportion. This means that for 95% of sample proportions, the distance to the population proportion will be less than this margin of error. So 95% of the time, the population proportion will be between $\widehat{p} - E$ and $\widehat{p} + E$.

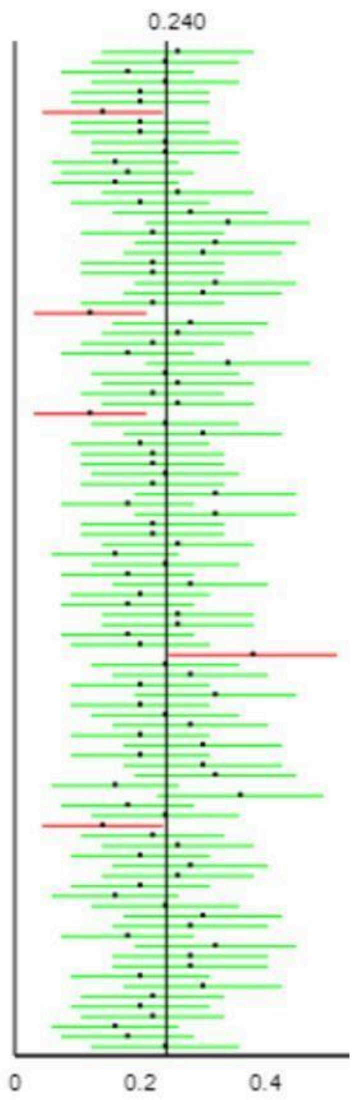
In general, the level of confidence is the proportion of intervals from $\widehat{p} - E$ to $\widehat{p} + E$ that will contain the population proportion.

H Consider a different random sample of n = 50 M&M's in which the sample proportion of blue M&M's is $\widehat{p} = 0.10$. Compute (a) the estimated standard error for sample proportions, (b) the margin of error for 95% of sample proportions, and (c) the 95% confidence interval $(\widehat{p} - E, \widehat{p} + E)$. Does this interval estimate contain the population proportion p = 0.24?

l Will all sample proportions from random samples of size n = 50 yield confidence intervals that contain the population proportion p = 0.24? Explain.

What does 95% Confidence Mean?

We just saw that two different sample proportions give two different confidence intervals. In fact, we could calculate many different confidence intervals using different samples. Most of these confidence intervals would contain the population proportion, but some would not. The level of confidence is the **proportion** of such intervals that actually contain the population proportion.



Consider the illustration to the left.

The vertical black line marks 0.24, the proportion of all M&M's that are blue. The green and red horizontal lines display 100 different confidence intervals, each constructed by finding a sample proportion from a random sample of 50 M&M's. The sample proportions are marked with black dots and are located at the center of each confidence interval. The width of each line (confidence interval) is created by adding and subtracting 1.96 standard errors to the sample proportion.

95 of these confidence intervals are colored green, indicating that they do include the population proportion.

5 of these confidence intervals are colored red, indicating that they do not include the population proportion.

This is the idea behind "95% confidence". We expect that a confidence interval created from a sample proportion **will include** the population proportion 95% of the time, and **will not** include the population proportion 5% of the time.

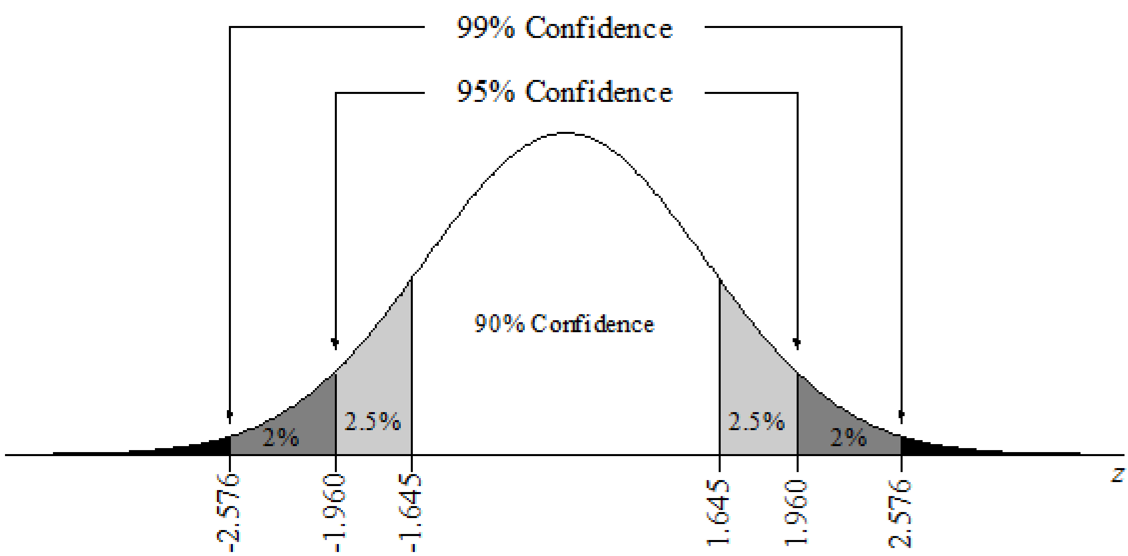
NEXT STEPS

Understanding Critical Values

If we rely strictly on the Empirical Rule, we would only be able to use 68%, 95%, and 99.7% as confidence levels. The table below displays the most commonly used critical values and their corresponding confidence levels.

Confidence Level	Critical Value
90%	$Z_c = 1.645$
95%	$Z_c = 1.960$
99%	$Z_c = 2.576$

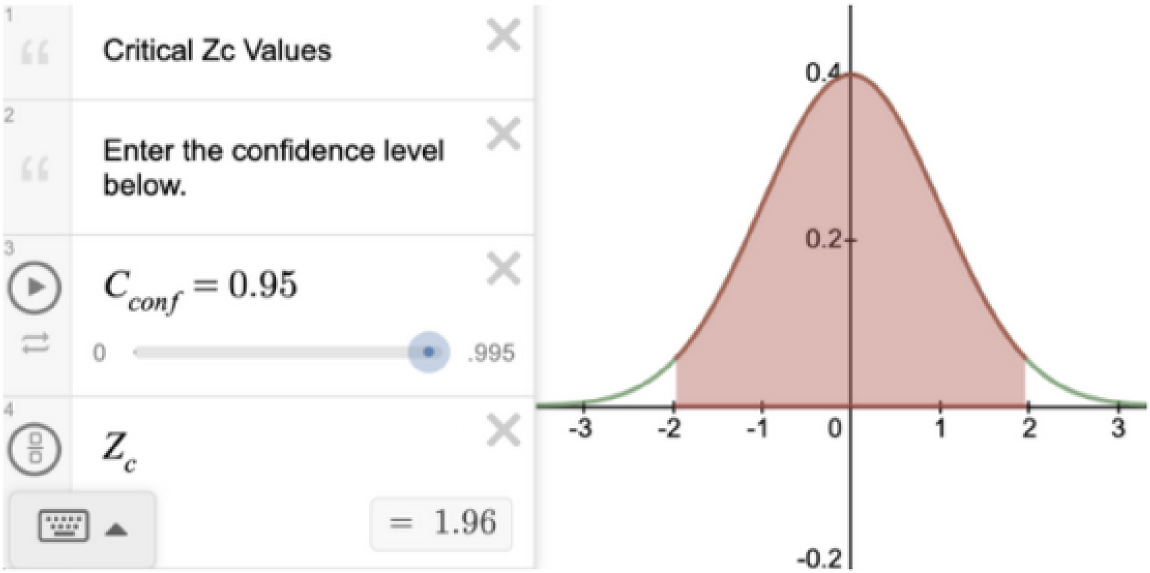
The figure below graphically depicts these critical values and the areas captured between each critical value and its negative.



While these three critical values are most common, we can find a critical value for **any** level of confidence.

Open the Desmos calculator for critical values here: https://carnegiemathpathways.org/go/desmoszcrit.

This Desmos calculator displays critical values for any confidence level you set. In the image below, the confidence level (labeled C_{conf}) is set at 0.95 for 95% confidence. The critical value, Z_C, is 1.96. Note that the critical value is displayed as a positive number. On the standard normal distribution, you can see both the locations of the positive and negative *Z*-scores.



The graph displays the central probability, showing that 95% of the values in a normal distribution are within 1.96 standard deviations of the mean. “Within” means from 1.96 standard deviations below the mean to 1.96 standard deviations above the mean.

6 Set the confidence level (labeled C_{conf}) to 0.80. What is the critical value, Z_C?

7 Describe the meaning of this critical value.

8 Experiment with various confidence levels by dragging the slider left and right. How would you describe the relationship between level of confidence and critical values?

NEXT STEPS
Constructing a Confidence Interval for a Population Proportion

9 In 2019, Experian, a consumer credit reporting company, surveyed a group of 345 U.S. millennial consumers to see what types of credit they have now and what new credit they're considering taking on. 42% of millennials surveyed reported that they had credit card debt.[6]

A Are the criteria for approximate normality satisfied? Explain your answer.

B Find the margin of error for a 95% confidence interval. Round the value to three decimal places.

C Interpret the margin of error.

D Give the 95% confidence interval.

E Write a sentence to interpret your confidence interval in context.

F Based on the confidence interval, is it reasonable to say that the majority of U.S. millennials have never had a credit card? Explain.

[6] https://www.experian.com/blogs/ask-experian/survey-less-than-half-of-millennials-say-they-have-credit-card-debt/

YOU NEED TO KNOW

Steps for Computing a Confidence Interval for a Population Proportion

(1) Verify that the random sample has at least 10 *successes* and 10 *failures*.

(2) Determine the critical value, Z_c, that corresponds to the chosen level of confidence.

(3) Compute the margin of error:

$$E = Z_c \cdot \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

(4) Compute the confidence interval. We can represent it as *estimate* ± *error*,

$$\widehat{p} \pm E$$

or as an interval, $(\widehat{p} - E, \widehat{p} + E)$. The interval represents all values between $\widehat{p} - E$ and $\widehat{p} + E$.

(5) Interpret the confidence interval in context. The **level of confidence** is the proportion of similarly constructed intervals that contain the population proportion (p). We also say that it is how confident we are that a particular interval contains the population proportion.

NEXT STEPS

Sample Size, Confidence Level and Error

10 The margin of error is influenced by sample size.

A If we increase the sample size will the margin of error increase or decrease? Explain your answer.

B Suppose the sample size from Question 9 is increased to 900, but that the sample proportion remains at $\widehat{p} = 0.42$. What is the new margin of error? Is the new margin of error greater than, less than, or equal to the margin of error you computed in Question 9?

C If we increase the sample size, does the confidence interval get wider or narrower? Explain your answer.

11 The level of confidence influences the margin of error as well. Suppose we maintain a constant sample size, but change the margin of error by adjusting the level of confidence.

A Does a smaller level of confidence make you more or less confident that the population proportion will be captured within a confidence interval?

B Suppose the confidence level from Question 9 is decreased from 95% to 90% (with the original sample size of $n = 345$). What is the new margin of error? Is the margin of error greater than, less than, or equal to the margin of error you calculated in Question 9?

C If we decrease the confidence level, does the confidence interval get wider or narrower? Explain your answer.

12 In Questions 10 and 11 we learned that there are two ways to decrease the margin of error in an estimate. We can increase the sample size or decrease the confidence level. What do you think is the best way to decrease the margin of error?

YOU NEED TO KNOW

There are two things that influence the size of the margin of error and the resulting width of the confidence interval.

- Increasing the sample size decreases the margin of error and makes the confidence interval narrower.
- Decreasing the level of confidence decreases the margin of error and makes the confidence interval narrower.

Small margins of error are nice, but not at the expense of the confidence level. As a general rule, the best way to decrease the margin of error is to increase the sample size.

LET'S SUMMARIZE

Please consider the following key points:

- A confidence interval has the form: *Point Estimate ± Margin of Error*
- A confidence interval for a population proportion has the form: $\widehat{p} \pm E$
- When estimating a population proportion, the point estimate is a sample proportion p and the margin of error E is:

$$E = Z_c \cdot \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

- The level of confidence is the proportion of similarly constructed intervals that contain the population proportion (p). We also say that it is how confident we are that a particular interval contains the population proportion.
- The margin of error of a confidence interval is impacted by the sample size and confidence level. The preferred approach to decrease the margin of error is to increase the sample size.

Exercise 6.2

1 In January 2022, the Knight Foundation wanted to explore students' views on free speech. Findings are based on a nationally representative survey of 1,023 U.S. students, conducted by the public opinion research firm Ipsos. 47% of those surveyed said that they thought free speech rights were secure.[7] We can use this information to calculate and interpret a 95% confidence interval for the proportion of U.S. students who considered free speech rights secure at that time.

A How many successes are there in the sample?

B How many failures are there in the sample?

C Are the criteria for approximate normality satisfied for a confidence interval?

D What is the sample proportion?

E Compute the margin of error for a 95% confidence interval. Round your answer to 3 decimal places.

F How would you interpret the margin of error you calculated in Question E?

G Compute the lower limit of the 95% confidence interval for the population proportion (p), of U.S. students who considered free speech rights secure in January 2022.

7 https://knightfoundation.org/press/releases/knight-ipsos-poll-college-students-covet-free-speech-rights-but-view-them-as-increasingly-fragile/

H Compute the upper limit of the 95% confidence interval for the population proportion (p), of U.S. students who considered free speech rights secure in January 2022.

I How would you interpret the confidence interval?

2 Based on your answers to Question 1 part G and H, where you found the 95% confidence interval for the proportion of U.S. students in January 2022 who considered free speech rights secure, answer the following questions.

A Based on this interval, is it possible that a majority of U.S. students considered free speech rights secure at that time? Explain.

B If someone claimed that only about 1/3 of U.S. students considered free speech rights secure, would our result support this?

3 Imagine we have conducted a survey. Among a random sample of 180 college students, 10 believed that aliens were responsible for building Stonehenge. (Stonehenge is a large circle of stones in southwest England.) We can use this information to construct a 90% confidence interval for the true proportion of all college students who believe that aliens were responsible for building Stonehenge.

A What is the sample proportion? Round the value to three decimal places.

B Compute the margin of error for a 90% confidence interval. Round your answer to three decimal places.

C Compute the lower limit of the 90% confidence interval for the true proportion of all college students who believe that aliens were responsible for building Stonehenge.

D Compute the upper limit of the 90% confidence interval for the true proportion of all college students who believe that aliens were responsible for building Stonehenge.

4 In January 2022, an education technology company surveyed 1,024 faculty members at 581 institutions (both two- and four-year) across the U.S. and asked them to share their honest feedback about how their job responsibilities and job satisfaction today compare to 3-5 years ago. 584 faculty reported spending time checking for possible plagiarism and cheating.[8] We can use this information to calculate a 99% confidence interval for the proportion of all U.S. faculty in January 2022 who were spending time upholding academic integrity.

A What is the sample proportion? Round the value to three decimal places.

B Compute the margin of error for a 99% confidence interval. Round your answer to three decimal places.

C Compute the lower limit of the 99% confidence interval for the proportion of all U.S. faculty in January 2022 who were spending time upholding academic integrity.

D Compute the upper limit of the 99% confidence interval for the proportion of all U.S. faculty in January 2022 who were spending time upholding academic integrity.

8 https://www.newamerica.org/education-policy/highered-public-opinion-hub/faces-of-faculty-the-higher-education-instructor-experience/

5 A nationally representative survey of 1,000 respondents conducted by YouGov for the Bucknell Institute for Public Policy (BIPP) found that 47% of respondents supported the idea of giving college athletes a share of the money made for the university by their sport.[9] We can use this information to calculate a 95% confidence interval for the proportion of all U.S. adults in June 2021 who felt that athletes should be given a share of the money made for the university by their sport.

A What is the sample proportion?

B Compute the margin of error for a 95% confidence interval. Round your answer to three decimal places.

C Compute the lower limit of the 95% confidence interval for the proportion of all U.S. adults in June 2021 who felt that athletes should be given a share of the money made for the university by their sport.

D Compute the upper limit of the 95% confidence interval for the proportion of all U.S. adults in June 2021 who felt that athletes should be given a share of the money made for the university by their sport.

[9] https://forthemedia.blogs.bucknell.edu/bucknell-poll-finds-mixed-support-for-college-student-athlete-compensation/

6.2 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Calculate a point estimate.	
Determine the number of standard errors (i.e. the critical value, Z_c) that corresponds to a given level of confidence.	
Compute the margin of error in a sample proportion, corresponding to a given level of confidence.	
Compute and interpret a confidence interval for a population proportion.	
Use a confidence interval to reason about a population proportion.	

6.2-S: Constructing Confidence Intervals

LEARNING GOALS

By the end of this lesson, you should understand that:

- A population parameter can be estimated by a point estimate and an interval estimate.
- A point estimate is a single value and an interval estimate is a range of values.
- A confidence interval is based on a single sample and indicates a range of values that may contain the population parameter, but only at a certain level of confidence.
- The confidence level is the percentage of all theoretical confidence intervals (from samples of size n) that contain the true parameter.

By the end of this lesson, you should be able to:

- Calculate a point estimate.
- Determine the number of standard errors (i.e. the critical value, Z_c) that correspond to a given level of confidence.
- Compute the margin of error in a sample proportion, corresponding to a given level of confidence.
- Compute a confidence interval for a population proportion.
- Correctly interpret a confidence interval for a population proportion.
- Use a confidence interval to reason about a population proportion.

INTRODUCTION

A *confidence interval* is an interval estimate of an unknown population parameter. Population parameters for very large populations, such as the proportion of all U.S. adults in a given year who favor making public 4-year colleges and universities tuition free, are impossible to determine. We can, however, estimate such a parameter with a statistic from a representative sample, and compute an interval estimate using sample data.

For example: Suppose we want to know the proportion of all U.S. adults who are in favor of making all 4-year colleges and universities tuition-free. We can't expect to hear from every U.S. adult, so we take a sample and find the proportion of our sample that are in favor of making college tuition-free. The sample proportion will probably not be equal to the exact population proportion, but it should be close. If our sample is very large, it should be very close! We utilize what we know about the sampling distribution to create a margin of error, which is the amount we suspect the sample proportion might differ from the population proportion.

We begin this lesson by examining and interpreting confidence intervals reported in the media.

Read the following study, then answer the corresponding questions:

> In May 2022, Education Next worked with the polling firm Ipsos to survey 1,784 U.S. adults' public opinion on a range of important education matters, one of which being whether 4-year public colleges and universities should be tuition free.[10] Of 1,784 respondents, 1,177 supported making 4-year public colleges and universities tuition free. Assume a margin of error of 2 percentage points, and assume the margin of error is based on a 90% confidence level.

1 Describe the sample proportion.

2 Describe the population proportion of interest.

3 Construct a confidence interval for the population proportion. Round answers to 2 decimal places.

4 Interpret the margin of error.

5 Interpret the confidence interval.

10 https://www.educationnext.org/partisan-rifts-widen-perceptions-school-quality-decline-results-2022-education-next-survey-public-opinion/#method

Read the following study, then answer the corresponding questions:

> In December 2022, the polling agency Gallup published a report based on a new analysis of the West Health-Gallup Healthcare Affordability Index and the Healthcare Value Index.[11] These results are based on survey responses of a random sample of 5,584 U.S. adults. Of the random sample of 5,584 U.S. adults, 59% said that the high costs of healthcare is a source of stress in their daily lives. Assume a 4% margin of error at the 95% confidence level.

6 Describe the sample proportion.

7 Describe the population proportion of interest.

8 Construct a confidence interval for the population proportion. Round answers to 2 decimal places.

9 Interpret the margin of error.

10 Interpret the confidence interval.

[11] https://news.gallup.com/opinion/gallup/406178/stress-prominent-among-adults-struggling-pay-care.aspx

11 Based on the confidence interval which you found above, is it appropriate to conclude that a majority of U.S. adults, aged 18 or older, feel that they have enough time to do what they want to do? Explain your answer.

NEXT STEPS

Let's review the steps for constructing a confidence interval for a population proportion.

YOU NEED TO KNOW

Steps for Computing a Confidence Interval for a Population Proportion

(1) Verify that the random sample has at least 10 *successes* and 10 *failures*.

(2) Determine the critical value, Z_c, that corresponds to the chosen level of confidence.

(3) Compute the margin of error:

$$E = Z_c \cdot \sqrt{\frac{\widehat{p}(1 - \widehat{p})}{n}}$$

(4) Compute the confidence interval. We can represent it as *estimate* ± *error*,

$$\widehat{p} \pm E$$

or as an interval, $(\widehat{p} - E, \widehat{p} + E)$. The interval represents all values between $\widehat{p} - E$ and $\widehat{p} + E$.

(5) Interpret the confidence interval in context. The **level of confidence** is the proportion of similarly constructed intervals that contain the population proportion (p). We also say that it is how confident we are that a particular interval contains the population proportion.

Read the following study, the answer the corresponding questions:

> The September 2017 Morning Consult / Politico poll referenced above collected information on registered voters' views regarding the federal government providing financial assistance to individuals with student debt. Student debt refers to debt caused by paying for college expenses. The poll found that 71% of U.S. registered voters aged 18–29 support the federal government providing debt relief to individuals with a certain amount of student financial debt. This statistic is based on a random sample of 351 U.S. registered voters aged 18–29.

12 Are the criteria for approximate normality satisfied? Explain your answer.

13 Find the margin of error for a 95% confidence interval. Round to 3 decimal places.

14 Interpret the margin of error.

15 Give the 95% confidence interval. Round your answers to 3 decimal places.

16 Write a sentence to interpret your confidence interval in context.

17 Without making any computations, explain whether a 99% confidence interval for this population proportion be wider or narrower?

TRY THESE

Read the following study, the answer the corresponding questions:

> The September 2017 Morning Consult / Politico poll found that 54% of U.S. registered voters aged 65 or older support the federal government providing debt relief to individuals with a certain amount of student financial debt. This statistic is based on a random sample of 426 U.S. registered voters aged 65 or older.

18 Construct a 95% confidence interval for the proportion of U.S. registered voters aged 65 or older who support the federal government providing debt relief to individuals with a certain amount of student financial debt. Round to 3 decimal places.

19 Interpret the confidence interval.

LET'S SUMMARIZE

In this lesson, you were introduced to confidence intervals and the relationship between confidence intervals and sampling distributions. You learned about point estimates, interval estimates, and the meaning of confidence intervals. You learned the formal process of computing a confidence interval for a population proportion. The normal distribution is used to determine critical values and margins of error.

You should now understand that:

- A population parameter can be estimated by a point estimate and an interval estimate.
- A point estimate is a single value and an interval estimate is a range of values.
- A confidence interval is based on a single sample and indicates a range of values that may contain the population parameter, but only at a certain level of confidence.
- The confidence level is the percentage of all theoretical confidence intervals (from samples of size n) that contain the true parameter.

Practice Problems 6.2-S

1 Suppose we conducted a survey about college students' plans to vote in the next midterm elections. Midterm elections are congressional elections across the U.S. which occur two years after the presidential elections. Among a random sample of 240 college students, 66 said that they plan to vote in the next midterm elections. Calculate and interpret a 90% confidence interval for the proportion of all college students who plan to vote in the next midterm elections. Round to 3 decimal places.

2 An October 2022 Morning Consult poll surveyed U.S. adults about how much of the 2002 FIFA World Cup they expected to watch.[12] In a random sample of 2,211 U.S. adults, 553 said that they planned to watch some or a lot of the FIFA 2022 World Cup. Calculate and interpret a 99% confidence interval for the proportion of all U.S. adults who plan to watch some or a lot of the FIFA 2022 World Cup. Round to 3 decimal places.

[12] https://morningconsult.com/2022/11/21/world-cup-2022-american-fan-viewership/

3 Based on the confidence interval in the prior question, would it be appropriate to conclude that fewer than 27% of all U.S. adults planned to watch some or a lot of the FIFA 2022 World Cup? Explain.

6.3: Hypothesis Tests for a Population Proportion 1

LEARNING GOALS

By the end of this collaboration, you should understand that:

- We can use a hypothetical sampling distribution to help us determine when a claim about a population proportion is reasonable.
- When we conduct a hypothesis test about a population proportion, we make an assumption about the value of the population proportion. This is the null hypothesis.
- The alternative hypothesis is a claim about the population that opposes the null hypothesis. It states that the population proportion is greater than, less than, or not equal to the value claimed by the null hypothesis.
- The sampling distribution constructed for a given hypothesis is based on the assumption that the null hypothesis is true.
- The *P*-value is the probability of randomly observing a sample proportion that is at least as extreme as the one we have, based on the assumption of the null hypothesis.
- When the *P*-value is small, it tells us that the sample proportion is statistically significant.
- When the *P*-value is less than the level of significance, we reject the null hypothesis in favor of the alternative hypothesis.

By the end of this collaboration, you should be able to:

- Given a claim about a population, choose appropriate null and alternative hypotheses.
- Determine when the sample size is large enough for a hypothesis test for a population proportion to be appropriate.
- Compute the value of the test statistic and find the associated *P*-value.
- Explain how a *P*-value is used to reach a conclusion in a hypothesis test.
- Explain what the phrase “statistically significant” means.
- Explain why rejecting the null hypothesis implies strong support for the alternative hypothesis but failing to reject the null hypothesis does not imply strong support for the null hypothesis.

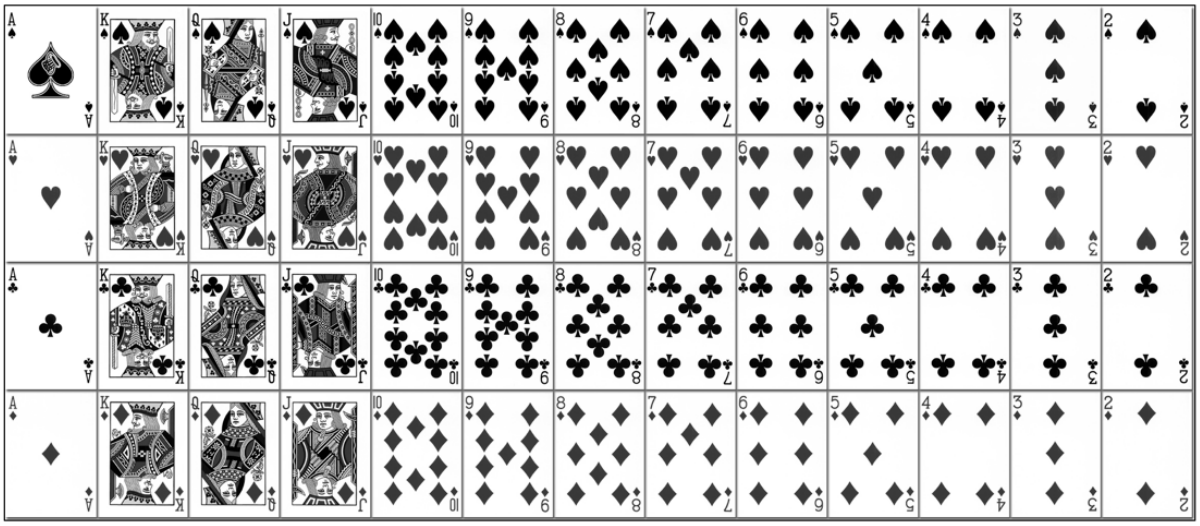
INTRODUCTION

In this collaboration, we begin a new type of statistical inference known as **hypothesis testing**. Hypothesis testing can seem awkward at first, but when you really understand it, you see that it’s actually how your mind makes decisions after being convinced by sufficient evidence.

We begin with an activity. It’s a little game, and if it goes well, you’ll win a prize!!!

We will randomly pick a card from a shuffled deck of 52 poker cards (pictured below). If it’s black, you win the prize! That’s it! And good news: you get 8 tries! Very exciting!

A deck of 52 cards is reproduced below. The black cards are in the 1st and 3rd rows.



TRY THESE 1
Using Sample Evidence to Test a Claim

1 You have just completed an activity with your instructor that involved drawing cards randomly from a shuffled deck of 52 cards. Think about the results as you answer the following questions.

A Before we began the activity, you had no reason to believe that the deck was not fair. Do you believe the deck is fair now? Why or why not?

B You have only seen 8 cards, but you are probably forming conclusions about the whole deck. Is it appropriate to draw conclusions about the whole deck based on only eight cards?

NEXT STEPS

You just made a decision about a population that you did not observe completely. Your decision was based on a small sample. In statistics, sample data help us make decisions about populations through a process known as **hypothesis testing**.

We have just conducted an informal hypothesis test. We began with an assumption about the deck. Then we gathered sample

Language Tip

A *hypothesis* is an assumption or claim. In statistics, we use sample data to test hypotheses about unknown population parameters.

data. Upon examining the data, the likelihood of what we observed led us to make a conclusion about the entire deck of cards.

Hypothesis tests include four steps: **determining hypotheses**, **collecting data**, **assessing the evidence**, and **stating a conclusion**. The following questions show how the steps fit with our reasoning during the card drawing activity.

Step 1: Determine the Hypotheses

2 Every hypothesis test makes an assumption about the value of a population parameter. That value is then challenged by the test. The parameter that we consider here is an unknown population proportion. The assumption we make about its value is called the **null hypothesis**.

A We usually assume that a deck of cards is *fair*. If we assume it is fair, what assumption are we making about the value of the population proportion of cards in the deck that are black?

p = ____________

B Based on your sample of eight cards, do you believe that the true proportion of black cards in the deck is less than, greater than, or simply different from, the value given in Question 2A above? Explain your reasoning.

The belief you expressed in Question 2B opposes the null hypothesis, so we call it the **alternative hypothesis**. Normally, we choose the alternative hypothesis before gathering data, but here we will make an exception.

We now use what we know about probability to show how strongly our evidence supports this alternative. If the evidence for the alternative hypothesis is strong, we will reject the null hypothesis.

Step 2: Collect the Data

3 Once we have identified the null and alternative hypotheses, we gather and summarize data.

A All methods of inference in statistics assume that we are gathering data from *simple random samples*. What information provided in the description of this activity ensures this?

B What was the sample proportion of randomly selected cards that were black?

$\hat{p}$ = ____________

Step 3: Assess the Evidence

4 We have summarized the sample data with a sample proportion: the proportion of black cards in the sample of 8 cards. Our job now is to determine if this sample evidence is strong enough to reject the null hypothesis in support of the alternative hypothesis.

We determine the strength of our evidence through probability. The probability, called a ***P*-value**, is computed assuming the null hypothesis is true.

> **Language Tip**
> **A *P-value* is the probability of observing a sample statistic that is *at least as extreme* as the one we gathered, assuming the null hypothesis is true.**

A Let's use the multiplication rule for independent events (the cards were drawn with replacement) to compute the *P*-value. Assuming the null hypothesis is true (that half of the cards are red), what is the probability that a randomly selected card is red?

P(*Red*) = _______________

B Use the multiplication rule to determine the probability that among 8 randomly selected cards there will be no black cards (that is, 8 red cards and 0 black cards). This probability is the *P*-value.

P-value = *P*(*Red* & *Red* & *Red* & *Red* & *Red* & *Red* & *Red* & *Red*)

= *P*(*Red*) · *P*(*Red*) · *P*(*Red*) · *P*(*Red*) · *P*(*Red*) · *P*(*Red*) · *P*(*Red*) · *P*(*Red*)

= (   ) · (   ) · (   ) · (   ) · (   ) · (   ) · (   ) · (   )

= __________ (Round to four places after the decimal)

C We computed the *P*-value assuming the null hypothesis was true. Out of 52 cards, your instructor randomly selected 8 (with replacement), and none were black. The *P*-value tells us how likely picking no black cards in 8 draws would be if half the cards are black. Does the *P*-value make you question the null hypothesis? Give a reason for your answer.

Step 4: State a Conclusion

Language Tip
When a low probability event occurs, it is *statistically significant*. The rule for how low the probability should be is the *level of significance*.

5 Statisticians use a rule about how small a probability should be in order for us to consider an event unusual, or **statistically significant**. We generally consider events that occur with a probability of around 5% or less to be unusual or significant. In such cases, we call the 5% rule the **level of significance**. Other levels of significance are sometimes used. The level of significance is referred to as alpha (α).

A We assumed that our deck was half black. Is it statistically significant if we observe a sample proportion of 0 black cards from a random sample of size 8 cards?

When the *P*-value (computed on the assumption of the null hypothesis) is **less than or equal** to the level of significance (5% = 0.05 in this example) we reject the null hypothesis and support the alternative hypothesis.

When the *P*-value (computed on the assumption of the null hypothesis) is **greater than** the level of significance (5% = 0.05 in this example) we do not reject the null hypothesis and do not support the alternative hypothesis.

B Make a decision about the null and alternative hypotheses.

C Explain what this means about the proportion of cards in the deck that are black.

D Do you know the true proportion of cards in the deck that are black? Explain.

We decided that the proportion of cards that are black is less than 0.5. This decision is about what the proportion *is not*, but be careful—we still do not know what the proportion *is*. Because of this we never conclude a hypothesis test by saying what a population proportion is. We may state that we believe it is less than, greater than, or not equal to a value. Sometimes we will say that we can't disprove an assumed value for the proportion, but this does not prove it.

TRY THESE 2

There exists a long-standing national debate on the amount of taxes that very wealthy Americans should pay. Some people argue that very wealthy Americans pay too little; others argue that they pay too much. A 2019 Gallup trend poll reported that 62% of adults in the U.S. believed that upper-income Americans pay too little in taxes[13]. Lately, however, it seems that this proportion may be on the rise. A 2020 Hill-HarrisX poll[14] of 4,441 randomly selected registered U.S. voters revealed 2,842 people felt taxes should be raised for wealthy individuals.

Let's follow the hypothesis test process at the 5% significance level to investigate the proportion of adults in the U.S. in 2020 who believe that upper-income Americans pay too little in taxes.

Step 1: Determine the Hypotheses

6 Every hypothesis test has a null hypothesis and an alternative hypothesis.

A The null hypothesis assumes that nothing has changed from 2019 to 2020. In this case we assume that there has been no change in the proportion of adults who believe that upper-income Americans pay too little in taxes. Complete the null hypothesis using this assumption. Note: p represents the proportion of U.S. adults in 2020 who believe that upper-income Americans pay too little in taxes.

Null Hypothesis: p = ______________

B The alternative hypothesis is what we try to prove. We try to prove that, somehow, things are different. In this case, we believe the proportion has *increased* from what it was previously. The alternative hypothesis claims that the population proportion is either *less than* (p < *assumed value*), *greater than* (p > *assumed value*), or *different from* (p ≠ *assumed value*) a prior value. Give the appropriate alternative hypothesis below.

Alternative Hypothesis: ______________

Step 2: Collect the Data

7 Once we have identified the null and alternative hypotheses, we gather and summarize data. We also verify that the criteria for approximate normality are met.

A Statistical inference requires random sampling. Is there evidence that the sampling process was random? Explain.

13 https://news.gallup.com/poll/1714/taxes.aspx

14 https://www.reuters.com/article/us-usa-election-inequality-poll/majority-of-americans-favor-wealth-tax-on-very-rich-reuters-ipsos-poll-idUSKBN1Z9141

B Give the sample proportion of randomly selected adults who believe that upper-income Americans pay too little in taxes. Do not round your answer. Enter in the proportion as a decimal.

$\hat{p}$ = _______________

Is a normal model appropriate for this application? The criteria for approximate normality require that $np \geq 10$ and $n(1-p) \geq 10$. Use the sample size and assumed population proportion from the null hypothesis to answer this question.

C np = _______________

D $n(1-p)$ = _______________

E Are the criteria satisfied?

Step 3: Assess the Evidence

8 We have summarized the sample data with a sample proportion. We now determine the strength of our evidence with a *P*-value. Remember, the *P*-value is the probability of observing a statistic that is at least as extreme as the one we gathered, assuming the null hypothesis is true.

Since the criteria for approximate normality are satisfied, we can use the normal distribution to determine the *P*-value. To find the *P*-value, we use the standard normal distribution. This requires that we compute the *Z*-score of the sample proportion above. To compute the *Z*-score, we need the mean and standard error of the sampling distribution of sample proportions.

A Find the mean of the sampling distribution of sample proportions.

Mean = $\mu_{\hat{p}} = p$ = ___________

B Find the standard error of the sampling distribution of sample proportions. Round your answer to three decimal places.

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ = ___________

C We can use the mean and standard error to compute the *Z*-score of our $\hat{p}$ value. Round to two places after the decimal.

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$ = _______________

D Use technology or tables to find the normal probability of observing a *Z*-score that is greater than or equal to the one computed. This is the *P*-value. Round to three places after the decimal. **Note**: If completing this problem online, follow the instructions given online to answer this question.

P-value = $P(\widehat{p} \geq$ ________ $) = P(Z \geq$ ______________) = ______________

E The null hypothesis assumes that there is no change in the proportion of adults in the U.S. who believe that upper-income Americans pay too little in taxes. But, we observed a sample proportion that was greater. Does the *P*-value indicate that the sample proportion we observed was likely or unlikely, given our population proportion?

Step 4: State a Conclusion

9 Now that we have computed a *P*-value we can make a decision.

A We have stated that when a *P*-value is less than our 5% level of significance, the sample proportion is statistically significant. This leads us to believe that the null hypothesis is unlikely to be true. What decisions should we make about the null and alternative hypothesis?

Remember, in a conclusion, we can support or fail to support the alternative hypothesis, but we never conclude that the null hypothesis is true. The only way to prove the null hypothesis is to sample the entire population! When writing your conclusion, talk about whether the data support or do not support the alternative hypothesis. The conclusion should be written *in context*.

B What does our conclusion mean with regard to the proportion of all adults in the U.S. in 2020 who believe that upper-income Americans pay too little in taxes?

LET'S SUMMARIZE

Please consider the following key points:

- A hypothesis test consists of four steps: (1) Determine the hypotheses, (2) Collect the data, (3) Assess the evidence, and (4) State a conclusion.
- In a hypothesis test for a population proportion, we use a sample proportion from a random sample to test a claim made about a population proportion.
- In a hypothesis test we make an assumption that the null hypothesis is true to create a sampling distribution. We then use the sampling distribution to determine the likelihood of observing a sample statistic like the one we found. This likelihood is called the *P*-value.
- When the *P*-value is less than the level of significance, we conclude that the observed sample statistic is *statistically significant*. This indicates that the sample statistic is *significantly* different than the assumed population parameter (and quite improbable), so we reject the null hypothesis in favor of the alternative hypothesis.

Exercise 6.3

Testing a Claim About Americans' Views on the U.S. Healthcare System

In 2008, 73% of adults in the U.S. believed the U.S. healthcare system was in a state of crisis or had major problems. Nine years later in 2017, after the implementation of numerous national and statewide attempts to improve healthcare opportunities for people throughout the country, a *Gallup.com* survey[15] reported that 72% of U.S. adults have this same negative view about the U.S. healthcare system. The Gallup survey is based on a random sample of 1,028 U.S. adults. Does this sample provide conclusive evidence that the proportion of U.S. adults in 2017 who held this view is lower than it was in 2008?

Step 1: Determine the Hypotheses

1 The null hypothesis assumes no difference in the population proportions. In this case, it assumes that the proportion of U.S. adults in 2017 who believed the U.S. healthcare system was in a state of crisis or had major problems is the same as the proportion reported in 2008. Complete the null hypothesis using this assumption. Let p represent the proportion of U.S. adults in 2017 who believed the U.S. healthcare system was in a state of crisis or had major problems.

Null Hypothesis: p = ______________

2 The alternative hypothesis is what we try to prove. We try to prove that, somehow, the proportions are different. In this hypothesis test, we want to examine whether the proportion of U.S. adults in 2017 who hold this view is *lower* than the proportion reported in 2008. Remember, the alternative hypothesis claims that the population proportion is either *less than* (p < *assumed value*), *greater than* (p > *assumed value*), or *different from* ($p \neq$ *assumed value*) a prior value. Give the appropriate alternative hypothesis below.

Alternative Hypothesis: ______________

Step 2: Collect the Data

In this step, we gather data from random samples, compute summary statistics, and verify criteria for approximate normality.

The criteria for approximate normality require that $np \geq 10$ and $n(1 - p) \geq 10$. The sample size was stated previously. Use p from the null hypothesis to verify these.

3 np = ____________

15 http://news.gallup.com/poll/223403/americans-hold-dim-view-healthcare-system.aspx?g_source=CATEGORY_WELLBEING&g_medium=topic&g_campaign=tiles

4 $n(1 - p)$ = ___________

5 Are the criteria satisfied?

6 Did the study use a representative sample?

7 You have assumed a value for the population proportion (p) above. What is the sample proportion?

$\hat{p} =$ ___________

Step 3: Assess the Evidence

8 Compute the mean of the underlying sampling distribution, assuming the null hypothesis is true. Round to two decimal places.

$\mu_{\hat{p}} = p =$ __________

9 Compute the standard error of the underlying sampling distribution, assuming the null hypothesis is true. Round the standard error to three places after the decimal.

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ __________

10 Compute the test statistic. The test statistic is the Z-score of the observed sample proportion. Round to two places after the decimal.

$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} =$ __________

11 The P-value is the probability of observing a test statistic that is at least as extreme as the one above. In this hypothesis test, the P-value is the area to the left of the test statistic. Use tables or technology to determine the P-value. Round to three places after the decimal. **Note**: If completing this problem online, follow the instructions given online to help answer this question.

P-value = __________

Step 4: State a Conclusion

When the *P*-value is less than 5%, the level of significance in this test, the sample proportion is considered statistically significant.

12 Is the sample proportion statistically significant as it differs from the assumed value of the population proportion?

13 What do you conclude regarding the null hypothesis?

14 What do you conclude regarding the alternative hypothesis?

15 Write a conclusion about this hypothesis test in plain language.

Testing a Claim About Students Enrolled in Remedial Mathematics

The Brookings Institution is a nonprofit public policy organization. In a February 2022 article, the authors report that up to 65% of U.S. community college students take at least one remedial course within six years of enrollment.[16]

A reporter for the student newspaper at a community college in Florida believes that the percentage of students taking at least one remedial course at her school is greater than 65%. She surveys a random sample of students at her school, asking the students whether they were required to take any remedial courses.

[16] https://www.brookings.edu/blog/brown-center-chalkboard/2022/02/15/rethinking-remedial-programs-to-promote-college-student-success/

Step 1: Determine the Hypotheses

16 The null hypothesis assumes a specific value for the population proportion. In this case, it assumes that the proportion of students taking at least one remedial course at the Florida college is the same as the proportion that was reported in the *New York Times*. Complete the null hypothesis using this assumption.

Null Hypothesis: p = ______________

17 The alternative hypothesis is what we try to prove. We try to prove that, somehow, the proportions are different. The reporter believes the proportion at her community college is *greater* than the proportion that was reported in the *New York Times*. Remember, the alternative hypothesis claims that the population proportion is either *less than* (p < *assumed value*), *greater than* (p > *assumed value*), or *different from* (p ≠ *assumed value*) a prior value.

Give the appropriate alternative hypothesis below.

Alternative Hypothesis: ______________

Step 2: Collect the Data

The reporter finds that in her random sample of 300 students, 210 respond that they were required to take at least one remedial course.

18 What is the sample proportion?

$\widehat{p}$ = ____________

19 Does the sample proportion alone allow us to conclude that the reporter's claim is correct? Explain.

20 Does this sampling distribution satisfy the normality criteria?

Step 3: Assess the Evidence

21 Find the mean of the sampling distribution of sample proportions.

$\mu_{\hat{p}} = p =$ __________

22 Find the standard error of the sampling distribution of sample proportions. Round your answer to three decimal places.

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ __________

23 Compute the test statistic. The test statistic is the *Z*-score of the observed sample proportion. Round to two places after the decimal.

$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} =$ ______________

24 The *P*-value is the probability of observing a test statistic that is at least as extreme as the one above. In this hypothesis test, the *P*-value is the area to the right of the test statistic.

Use technology or tables to find the *P*-value. Round to three places after the decimal. **Note**: If completing this problem online, follow the instructions given online to help answer this question.

P-value = ______________

Step 4: State a Conclusion

When the *P*-value is less than 5%, the level of significance in this test, the sample proportion is considered statistically significant.

25 Is the sample proportion statistically significant as it differs from the assumed value of the population proportion?

26 What decisions should we make about the null and alternative hypotheses?

27 Explain what your decision means in the context of this problem.

6.3 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Given a claim about a population, choose appropriate null and alternative hypotheses.	
Determine when the sample size is large enough for a hypothesis test for a population proportion to be appropriate.	
Compute the value of the test statistic and find the associated *P*-value.	
Explain how a *P*-value is used to reach a conclusion in a hypothesis test.	

6.3-S: Testing Claims about Population Parameters

LEARNING GOALS

By the end of lesson, you should understand that:

- When we conduct a hypothesis test about a population proportion, we make an assumption about the value of the population proportion. This is the null hypothesis.
- The sampling distribution constructed for a given hypothesis test is based on the assumption that the null hypothesis is true.
- The alternative hypothesis is a claim about the population that opposes the null hypothesis. It states that the population proportion is greater than, less than, or not equal to the value claimed by the null hypothesis.
- The *P*-value is the probability of randomly observing a sample proportion that is at least as extreme as the one we have, based on the assumption of the null hypothesis.
- When the *P*-value is small, it tells us that the sample proportion is statistically significant.
- When the *P*-value is less than the level of significance, we reject the null hypothesis in favor of the alternative.

By the end of the lesson, you should be able to:

- Given a claim about a population, choose appropriate null and alternative hypotheses.
- Explain the reasoning used to reach a decision in a hypothesis test.
- Explain how a *P*-value is used to reach a conclusion in a hypothesis test.
- Explain what the phrase “statistically significant” means.

INTRODUCTION

Uncovering Bias in Jury Selection

A fundamental characteristic of a fair society is that all people in the society have equal opportunities to participate in important civic activities. An example of this is jury duty. All people in a society, regardless of their gender, race, or socioeconomic status, should have equal opportunities to serve on a jury.

A journalist believed that a jury in a criminal trial showed evidence of racial bias in the juror selection process. In the region where the trial was located, out of all adults who were eligible to serve on juries, 60% were white, 20% were hispanic, 15% were black, and 5% were classified in “other” racial categories. In one trial, on an 11-person jury, the first 8 jurors selected to serve were white, and the last three jurors selected were non-white. Given the racial demographics of all eligible jurors in the region, the journalist suspected that this jury displayed evidence of racial bias.

We will perform a hypothesis test to assess whether this sample provides evidence that whites are over-represented on this jury.

Hypothesis tests include four steps: **determining hypotheses**, **collecting data**, **assessing the evidence**, and **stating a conclusion**.

Step 1: Determine the Hypotheses

Every hypothesis test begins with an assumption. In this case, we will assume that the juror selection process is fair with regards to race. This means that the *probability* that a person in a racial group is selected is equal to the *percentage* of eligible people who belong to that racial group. In this way, a person's chance of being selected is proportional to the size of their racial group in the region.

Null Hypothesis: Juror selection process is fair with regards to race. People of each race have a proportional chance of being selected.

1 Let's focus on the probability of a white juror being selected to serve. If the null hypothesis is true, what is the probability that a white juror will be selected to serve? Write your answer as a percentage.

Alternative Hypothesis: Juror selection process is unfair. Whites are over-represented on the jury, so people of each race do not have a proportional chance of being selected.

2 If the alternative hypothesis is true, what can say about the probability that a white juror will be selected to serve?

Step 2: Collect the Data

3 Once we have identified the null and alternative hypotheses, we gather and summarize data. Describe the sample in this hypothesis test.

Step 3: Assess the Evidence

We must determine if this sample evidence is sufficiently strong to lead us to reject the null hypothesis in support of the alternative hypothesis. We determine the strength of our evidence through probability. The probability, called a ***P*-value**, is computed assuming the null hypothesis is true. Assuming the juror selection process is fair with regards to race, we will find the probability that the first eight jurors selected to serve would be white.

4 We will use the multiplication rule to find this probability. We can assume that each juror selected was independent from others selected. First, assuming the null hypothesis is true, what is the probability that a single randomly selected juror will be white? Write your answer as a decimal.

P(*White*) = ______________

5 Use the multiplication rule to determine the probability of selecting 8 white jurors in a row, followed by 3 non-white jurors, assuming the null hypothesis is true. This probability is the *P*-value. Round your answer to 4 decimal places.

6 Interpret this *P*-value.

7 We computed the *P*-value assuming the null hypothesis was true. The *P*-value tells us how likely it would be to select 8 white jurors in a row if the juror selection process was fair with regards to race. Does the *P*-value make you question the null hypothesis? Give a reason for your answer.

Step 4: State a Conclusion

When the likelihood of an event occurring is unusual, the event is considered to be statistically significant. We often consider events that occur with a probability of around 5% or less to be unusual or significant. In such cases, we call the 5% rule the **level of significance**.

8 We assumed that the juror selection process was fair with regards to race. Would it be statistically significant to observe a jury in which 8 jurors selected in a row are white?

When the *P*-value (computed on the assumption of the null hypothesis) is **less than or equal** to the level of significance we reject the null hypothesis and support the alternative hypothesis.

When the *P*-value (computed on the assumption of the null hypothesis) is **greater than** the level of significance we do not reject the null hypothesis and do not support the alternative hypothesis.

9 Make a decision about the null and alternative hypotheses. What can we conclude about the juror selection process for this trial?

10 Can we conclude with absolute certainty that the juror selection process for this trial was racially biased?

NEXT STEPS

Let's say a study of adults throughout the world gathered information on people's views and attitudes about democracy in their countries. The study found that in the United States, 46% of adults are satisfied with how the U.S. democracy is working.

A student at a college in New York believes that the percentage of students at his college who are satisfied with how the U.S. democracy is working is less than 46%. He surveys a random sample of students at his school to get their views on this issue.

Step 1: Determine the Hypotheses

The null hypothesis assumes a value for the population proportion at the student's college. It assumes that the proportion of students at the college who are satisfied with how the U.S. democracy is working is the same as the proportion that was reported in the *Pew Research Center* study. Complete the null hypothesis using this assumption.

11 Complete the null hypothesis using this assumption. Write the proportion as a decimal.

Null Hypothesis: p = ______________

The alternative hypothesis is what we try to prove. The student believes the proportion at his college is *less* than the proportion that was reported in the *Pew Research Center* study. Remember, the alternative hypothesis claims that the population proportion is either *less than* (p < *assumed value*), *greater than* (p > *assumed value*), or *different from* (p ≠ *assumed value*) a prior value.

12 Give the appropriate alternative hypothesis below. Write the proportion as a decimal.

Alternative Hypothesis: ______________

Step 2: Collect the Data

The student finds that in a random sample of 250 students, 110 respond that they are satisfied with how the U.S. democracy is working.

13 What is the sample proportion? Write the proportion as a decimal.

14 At this point, can we say that the sample proportion proves that the student's claim is correct? Explain your answer.

15 Does this sampling distribution satisfy the normality criteria? Explain.

Step 3: Assess the Evidence

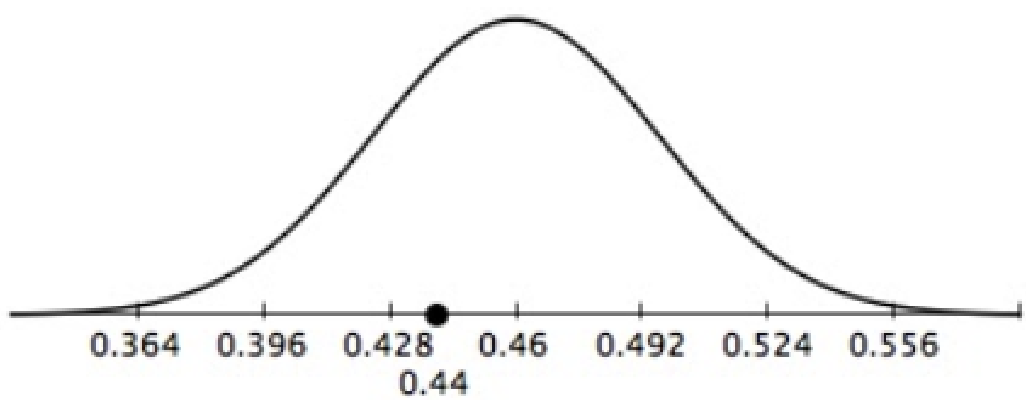
16 What is the mean of the underlying sampling distribution, assuming the null hypothesis is true. Enter the proportion as a decimal.

$\mu_{\widehat{p}} = p =$ __________

17 Compute the standard error of the underlying sampling distribution, assuming the null hypothesis is true. Round to 3 decimal places.

$\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ __________

The sampling distribution of sample proportions, with the mean from the null hypothesis, is shown on the normal curve below. The positions of one, two, and three standard errors below the mean, and the positions of one, two, and three standard errors above the mean are shown. The position of the observed sample proportion is shown as well.



18 What is the *Z*-score of the sample proportion?

$Z = \frac{\widehat{p} - \mu_{\widehat{p}}}{\sigma_{\widehat{p}}} =$ ______________

19 The *P*-value is the probability of observing a test statistic that is at least as extreme as the one above. In this hypothesis test, the *P*-value is the area to the left of the test statistic. Use technology or tables to find the *P*-value. Round to 3 decimal places.

20 The null hypothesis assumes that the proportion of students who are satisfied with how the U.S. democracy is working is the same as the proportion reported in the *Pew Research Center* study (p = 0.46). We observed a sample proportion ($\hat{p}$ = 0.44) that was lower. What does the *P*-value indicate?

Step 4: State a Conclusion

When the *P*-value is less than 5%, the level of significance in this test, the sample proportion is considered statistically significant.

21 Is the sample proportion statistically significant as it differs from the assumed value of the population proportion?

22 What decisions should we make about the null and alternative hypotheses?

23 Explain what your decision means in the context of this problem.

LET'S SUMMARIZE

In this lesson, you practiced computing and interpreting *P*-values in the context of hypothesis tests for population proportions. In the first part, you performed an informal hypothesis test regarding a claim about the fairness of a jury selection process. In this hypothesis test, you applied the multiplication rule to compute a *P*-value. In the second part of the lesson, you performed a hypothesis test for a population proportion. You explored the ideas of null and alternative hypotheses, *P*-values, statistical significance, and making decisions.

You should now understand that:

- When we conduct a hypothesis test about a population proportion, we make an assumption about the value of the population proportion. This is the null hypothesis.
- The sampling distribution constructed for a given hypothesis test is based on the assumption that the null hypothesis is true.
- The alternative hypothesis is a claim about the population that opposes the null hypothesis. It states that the population proportion is greater than, less than, or not equal to the value claimed by the null hypothesis.
- The *P*-value is the probability of randomly observing a sample proportion that is at least as extreme as the one we have, based on the assumption of the null hypothesis.
- When the *P*-value is small, it tells us that the sample proportion is statistically significant.
- When the *P*-value is less than the level of significance, we reject the null hypothesis in favor of the alternative.

Practice Problems 6.3-S

Testing a Claim About Floridians' Views on Offshore Drilling

A Pew Research Center poll asked U.S. adults on their views towards expanding offshore drilling for oil and natural gas[17]. Offshore drilling refers to oil and gas drilling off the coast of the U.S. mainland in the Atlantic Ocean, Pacific Ocean, and Gulf of Mexico. The survey reported that 51% of Americans oppose more offshore oil and gas drilling in U.S. waters.

A scientist who lives in Florida believes that the percentage of adults in Florida who oppose more offshore oil and gas drilling in U.S. waters is greater than 51%. He surveyed a random sample of Florida adults to get their views on increased offshore drilling.

Step 1: Determine the Hypotheses

1 The null hypothesis assumes that the proportion of Florida adults who oppose more offshore drilling is the same as the proportion reported in the *Pew Research Center* poll. Complete the null hypothesis using this assumption. Write the proportion as a decimal.

Null Hypothesis: p = ______________

2 The alternative hypothesis is based on the scientist's claim. The scientist believes that the proportion of Florida adults who oppose more offshore drilling is *greater* than the proportion that was reported in the *Pew Research Center* poll. Remember, the alternative hypothesis claims that the population proportion is either *less than* (p < *assumed value*), *greater than* (p > *assumed value*), or *different from* (p ≠ *assumed value*) a prior value. Give the appropriate alternative hypothesis below. Write the proportion as a decimal.

Alternative Hypothesis: ______________

Step 2: Collect the Data

The scientist finds that in a random sample of 180 Florida adults, 108 respond that they oppose more offshore drilling in U.S. waters.

3 What is the sample proportion? Write the proportion as a decimal.

[17] http://www.pewresearch.org/fact-tank/2018/01/30/more-americans-oppose-than-favor-increased-offshore-drilling/

4 What does the sample proportion tell us?

5 Does the sampling distribution satisfy the normality criteria?

Step 3: Assess the Evidence

6 What is the mean of the underlying sampling distribution, assuming the null hypothesis is true. Write the proportion as a decimal.

$\mu_{\hat{p}} = p =$ __________

7 Compute the standard error of the underlying sampling distribution, assuming the null hypothesis is true. Round to three decimal places.

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ __________

8 What is the *Z*-score of the sample proportion? Round to two decimal places.

$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} =$ ____________

9 The *P*-value is the probability of observing a test statistic that is at least as extreme as the one above. Use technology or tables to find the *P*-value. Round to three decimal places.

P-value = ___________

Step 4: State a Conclusion

10 When the *P*-value is less than 5%, the level of significance in this test, the sample proportion is considered statistically significant. Is the sample proportion statistically significant as it differs from the assumed value of the population proportion?

11 What decisions should we make about the null and alternative hypotheses?

6.4: Hypothesis Tests for a Population Proportion 2

LEARNING GOALS

By the end of this collaboration, you should understand that:

- A hypothesis test uses statistical data to test a claim about a population parameter.
- The null hypothesis states that a single population parameter equals a specific value.
- The alternative hypothesis states that a population parameter is greater than, less than, or not equal to a given value.
- Hypothesis tests are conducted under the assumption of the null hypothesis.
- The *P*-value is the probability of observing a statistic that is at least as extreme as the one obtained from our sample, assuming the null hypothesis is true.
- The smaller the *P*-value, the stronger the evidence against the null hypothesis.

By the end of this collaboration, you should be able to:

- Test claims regarding a single population proportion.
- Identify the null and alternative hypotheses in a hypothesis testing scenario.
- Compute a test statistic and determine its associated *P*-value.
- Interpret the *P*-value in a hypothesis test.
- Compare *P*-values to the level of significance in order to reject, or fail to reject, the null hypotheses.
- Interpret the results of a hypothesis test.

INTRODUCTION

In the previous unit, we learned about the four steps in the hypothesis testing process. We begin this collaboration by taking a closer look at how hypotheses can be determined from a claim or research question about a population proportion.

Determining the Hypotheses

In order to test a claim about population parameters, we create two opposing hypotheses. We call these hypotheses the **null hypothesis** (H_0) and the **alternative hypothesis** (H_a). These hypotheses are statements about the population parameter(s) we are examining. In Module 3 the parameter is the population proportion, p.

In all hypothesis tests we assume that the null hypothesis is true. The null hypothesis states that the population proportion is equal to a specific value.

$$H_0\text{: } p = \textit{assumed value}$$

The alternative hypothesis is a claim implied by the research question and is an inequality. The alternative hypothesis states that the population proportion is greater than (>), less than (<), or not equal (≠) to the value assumed in the null hypothesis.

The alternative hypothesis is represented symbolically by one of the following.

H_a: *p* < *assumed value*

H_a: *p* > *assumed value*

H_a: *p* ≠ *assumed value*

For example, research on college completion has shown that about 60% of students who begin college eventually graduate. A publication of higher education claims that the proportion for STEM (science, technology, engineering, or math) majors is higher. We will let *p* represent the proportion of all STEM majors who begin college and ultimately graduate. The null hypothesis is:

H_0: $p = 0.60$.

The alternative hypothesis is the statement that is claimed in the publication:

H_a: $p > 0.60$.

The publication claims that the population proportion is greater than 0.60, so this is the claim that researchers want to test. The researchers gather data to see if it supports this claim.

TRY THESE 1

1 A population proportion is represented by *p*. In the following questions state *in words* what *p* represents, then determine the null and alternative hypotheses for each claim below.

A About 67% of registered voters voted in the last presidential election. A student claims that less than 67% of students at our college plan to vote in the next presidential election.

p represents:

B Write the null and alternative hypotheses for the claim in part A.

H_0: ___________

H_a: ___________

C In 2013, the U.S. Department of Defense changed a policy that affected women in the armed forces. Under the new rules, women who met physical requirements could be assigned to combat positions. Many people in the U.S. were opposed to that decision. About 18% of women were against it. Researchers claimed that the proportion of U.S. men who were against the decision was different.[18]

p represents:

D Write the null and alternative hypotheses for the claim in part C.

H_0: ___________

H_a: ___________

E After President Obama's second electoral victory, a news organization claimed that a majority (more than half) of adults in the U.S. supported President Obama's gun-legislation proposals.

p represents:

F Write the null and alternative hypotheses for the claim in part E.

H_0: ___________

H_a: ___________

NEXT STEPS

Do College Students Exercise their Right to Vote?

Let's now practice the four-step hypothesis testing process. In the 2020 presidential election, about 67% of registered voters actually voted[19]. A student at a local college thinks her fellow students don't really care much about politics. She claims that less than 67% of students at her college plan to vote in the next election. To show this, she randomly samples 100 students at her college and finds that 57 plan to vote in the next election.

[18] http://www.gallup.com/poll/160124/americans-favor-allowing-women-combat.aspx

[19] https://www.census.gov/data/tables/time-series/demo/voting-and-registration/p20-585.html

2 Let's test her claim at the 5% level of significance. Remember, the level of significance is the rule for how small an event's probability should be in order to be considered statistically significant.

Step 1: Determine the Hypotheses

You determined the null and alternative hypotheses in Question 1B. Write them again below.

A H_0: ___________

B H_a: ___________

Step 2: Collect the Data

C If the underlying sampling distribution of sample proportions is approximately normal, we can use a normal probability model to determine if the sample proportion above is statistically significant. (Verify that $np \geq 10$ and $n(1 - p) \geq 10$.) Wrote these values in the table. Is there evidence that the data were gathered randomly? Are the criteria for approximate normality satisfied?

np	$n(1 - p)$	Random Selection?	Criteria Satisfied?

D We have assumed a value for the population proportion (p). Now we need the sample proportion ($\hat{p}$). Give the values of the sample size and sample proportion. Round to two places after the decimal.

n = ____________

$\hat{p}$ = ____________

Step 3: Assess the Evidence

To determine if the sample proportion is statistically significant, we compute its Z-score. In order to compute the Z-score of the sample proportion we need the mean and standard error of the underlying sampling distribution of sample proportions. Use the population proportion assumed by the null hypothesis to compute these.

E Mean = $\mu_{\hat{p}} = p =$ ________

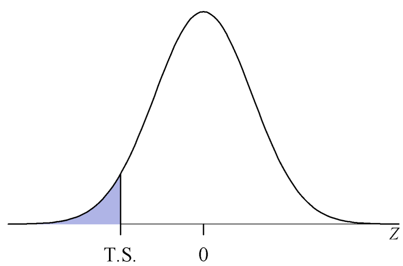
F Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ ________

G Compute the Z-score of the sample proportion identified in Question 2D above. In a hypothesis test such Z-scores are called **test statistics**. Round the Z-score to 2 decimal places.

$$Z = \frac{\widehat{p} - p}{\sigma_{\widehat{p}}} = \_\_\_\_\_\_\_\_\_\_$$

We want to compute the probability that the Z-score of this random sample proportion is at least as extreme as the one observed here. In this case, “at least as extreme” means “less than or equal”.

When the alternative hypothesis is $H_a : p <$ *assumed value*, we are conducting a **left-tailed test**. The P-value is the probability of observing a sample proportion at least as extreme as the one we observed. In this case at least as extreme means “as low or lower”. The P-value is the area to the left of the test statistic (T.S.).



H Use technology or tables to find the P-value corresponding to the test statistic computed in the previous question. Round to three decimal places. **Note**: If completing this problem online, follow the instructions given online to answer this question.

I The P-value is a probability. What probability does it represent?

J When the P-value is less than the level of significance, it indicates that the sample proportion is unusual, and therefore statistically significant. Is the sample proportion for this problem significantly less than the population proportion assumed in the null hypothesis? In this hypothesis test the level of significance is 5%.

TRY THESE 2

Step 4: State a Conclusion

Hypothesis tests are all about making decisions. We use the *P*-value to make a decision about the null hypothesis.

YOU NEED TO KNOW

- The level of significance is a rule for how unlikely a significant event should be. This level of significance is symbolized by the Greek letter α (*alpha*).
- If *P*-value ≤ α, the evidence is strong enough to lead us to reject the null hypothesis and support the alternative hypothesis.
- When *P*-value > α, the evidence is not strong enough to lead us to reject the null hypothesis. Therefore we do not support the alternative hypothesis. This does not make the null hypothesis true—we cannot prove the null hypothesis because sample data cannot reveal the true value of the population proportion.

Do College Students Exercise their Right to Vote?

Let's revisit the hypothesis test we are currently solving. In the 2020 presidential election, about 67% of registered voters actually voted. A student at a local college thinks her fellow students don't really care much about politics. She claims that less than 67% of students at her college plan to vote in the next election. To show this, she randomly samples 100 students at her college and finds that 57 plan to vote in the next election.

Let's test her claim at the 5% level of significance.

K Write the level of significance (given above as a percentage) in decimal form below.

α = __________

L When a *P*-value is less than the level of significance, α, the sample proportion is statistically significant. This leads us to believe that the null hypothesis is unlikely to be true. What decisions should we make about the null and alternative hypothesis?

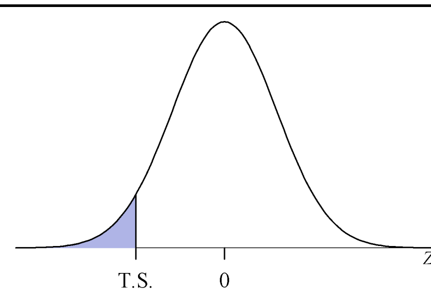
Note: The *P*-value for this hypothesis test was computed in Question 2H in the previous section.

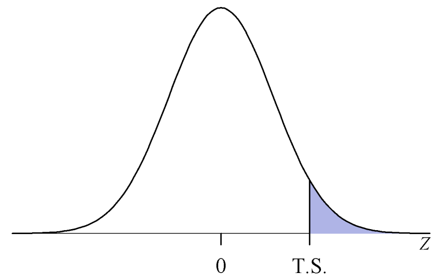
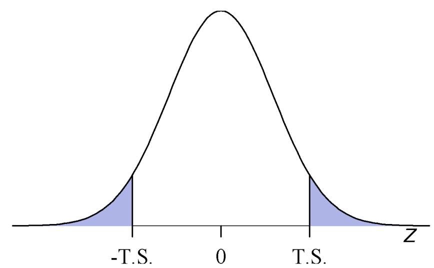
M Explain what this means in terms of the proportion of students at this college who plan to vote in the next election. Use plain language, indicating that this decision was prompted by our data. Remember that failing to reject the null hypothesis does not prove it!

NEXT STEPS
Finding *P*-Values

As explained in the previous problem, the method for computing *P*-values depends on the type of test, and the type of test depends on the alternative hypothesis.

YOU NEED TO KNOW
Computing *P*-values

When the alternative hypothesis is $H_a : p <$ *assumed value*, we are conducting a **left-tailed test**, and the *P*-value is the area to the left of the test statistic (T.S.).	

When the alternative hypothesis is H_a : p > *assumed value*, we are conducting a **right-tailed test**, and the *P*-value is the area to the right of the test statistic (T.S.).	
When the alternative hypothesis is H_a : $p \neq$ *assumed value*, we are conducting a **two-tailed test**, and the *P*-value is *twice* the area of either the tail to the right of a positive test statistic (T.S.), or the tail to the left of a negative test statistic (T.S.).	

TRY THESE 3

3 In June 2012, about 66% of U.S. adults believed that immigration is a good thing for the country. In August 2022, researchers wanted to know whether things had changed[20]. In a random sample of 1,013 U.S. adults, 709 said that immigration is a good thing for the country.

Test the claim at 1% significance that, in 2020, the percentage of U.S. adults who believed that immigration is a good thing for the country was different from 66%, the value from 2012.

Step 1: Determine the Hypotheses

A State the null hypothesis using the appropriate symbols and values.

H_0: ___________

B Give the alternative hypothesis using the appropriate symbols and values.

H_a: ___________

C Is this a right-tailed, left-tailed, or two-tailed test?

[20] https://news.gallup.com/poll/395882/immigration-views-remain-mixed-highly-partisan.aspx

Step 2: Collect the Data

D If the underlying sampling distribution of sample proportions is approximately normal, we can use a normal probability model to determine if the sample proportion above is statistically significant. Verify that $np \geq 10$ and $n(1 - p) \geq 10$. Write these values rounded to one decimal place. Are the criteria for approximate normality satisfied?

np	$n(1 - p)$	Random Selection?	Criteria Satisfied?

E We have assumed a value for the population proportion (p). Now we need the sample proportion ($\widehat{p}$). Give the values of the sample size and sample proportion. Round the sample proportion to two decimal places.

n = ____________

$\widehat{p}$ = ____________

Step 3: Assess the Evidence

F To determine if the sample proportion is statistically significant, we compute its *Z*-score. In order to compute the *Z*-score of the sample proportion we need the mean and standard error of the underlying sampling distribution of sample proportions. Use the population proportion assumed by the null hypothesis to compute these. Round the standard error to three decimal places.

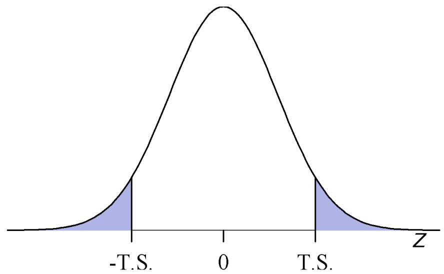
$\mu_{\widehat{p}} = p =$ ________

$\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ ________

G Compute the *Z*-score of the sample proportion identified in Question 3E above. In a hypothesis test such *Z*-scores are called **test statistics**. Round the Z-score to two decimal places.

$Z = \frac{\widehat{p} - p}{\sigma_{\widehat{p}}} =$ ____________

When the alternative hypothesis is H_a : $p \neq$ *assumed value*, we are conducting a **two-tailed test**. The *P*-value is the probability of observing a sample proportion at least as extreme as the one we observed. In this case, *at least as extreme* means greater than or equal to the observed (positive) test statistic or less than or equal to the negative of the test statistic (which is equally extreme). When the test statistic is positive, the *P*-value is **twice** the area to the right of the test statistic (T.S.).



H Use technology or tables to find the *P*-value corresponding to the test statistic computed in the previous question. Round to three decimal places. Remember, since this is a two-tailed test and the test statistic is positive, the *P*-value is twice the area in the right tail. **Note**: If completing this problem online, follow the instructions given online to answer this question.

Step 4: State a Conclusion

I What is the value of α, the level of significance? Write your answer as a decimal.

α = ___________

J When a *P*-value is less than the level of significance, α, the sample proportion is statistically significant. This leads us to believe that the null hypothesis is unlikely to be true. What decisions should we make about the null and alternative hypothesis?

K Write a statement that summarizes the conclusion of our hypothesis test.

LET'S SUMMARIZE

Please consider the following key points:

- A hypothesis test consists of four steps: (1) Determine the hypotheses, (2) Collect the data, (3) Assess the evidence, and (4) State a conclusion.
- In a hypothesis test for a population proportion, we use a sample proportion from a random sample to test a claim made about a population proportion.
- In a hypothesis test we make an assumption that the null hypothesis is true to create a sampling distribution. We then use the sampling distribution to determine the likelihood of observing a sample statistic like the one we found. This likelihood is called the *P*-value.
- When the *P*-value is less than the level of significance, we conclude that the observed sample statistic is statistically significant. This indicates that the sample statistic is significantly different than the assumed population parameter (and quite improbable), so we reject the null hypothesis in favor of the alternative hypothesis.

Exercise 6.4

After President Obama's re-election, a mass shooting in Newtown, Connecticut, prompted him to propose new laws intended to reduce gun violence. Politicians are more likely to vote for a law when a majority, or more than half of voters, support it. To test the claim (at a 5% significance level) that a majority of adults in the U.S. supported Obama's proposals, Gallup randomly sampled 1,021 adults in the U.S. and found that 541 supported the president.[21]

1 **Step 1: Determine the Hypotheses**

A Give the null hypothesis.

H_0: ____________

B Give the alternative hypothesis using the appropriate symbols.

H_a: ____________

C Is this a right-tailed, left-tailed, or two-tailed test? Explain.

2 **Step 2: Collect the Data**

A In the previous problem, you made an assumption about the population proportion (p) above. Now we need the sample size (n) and sample proportion ($\hat{p}$).

n= ____________

B Determine the sample proportion. Round your answer to three decimal places.

$\hat{p}$ = ____________

C Are the criteria for approximate normality satisfied? Explain.

[21] http://www.gallup.com/poll/159959/americans-reaction-obama-gun-proposals-positive.aspx

3 **Step 3: Assess the Evidence**

A Compute the mean and standard error of the underlying sampling distribution.

Mean = $\mu_{\hat{p}} = p =$ ___________

B Compute the standard error of the underlying sampling distribution. Round your answer to three decimal places.

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ ___________

C Compute the *Z*-score of the sample proportion (the test statistic).

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \text{\_\_\_\_\_\_\_\_\_\_}$$

D Use technology or tables to find the *P*-value. **Note**: If completing this problem online, follow the instructions given online to answer this question.

P-value = __________

E When the *P*-value is small, the sample proportion is **statistically significant**. Is the sample proportion statistically significant as it differs from the assumed value of the population proportion? Explain.

4 **Step 4: State a Conclusion**

A What is the value of α, the level of significance?

α = __________

B Compare the *P-value* to α. What do you conclude regarding the null hypothesis?

C What do you conclude regarding the alternative hypothesis?

D Write a conclusion that explains the results of this test in plain language.

National service is a term used to describe military service or service as a civilian to one's community, state or country. A politician claimed that half of all U.S. adults favor requiring young American adults to serve a mandatory year of national service. In a recent random sample of 1,006 adults, 490 stated that they believed this.

Test the claim at the 5% significance level that the proportion of U.S. adults who favor requiring young American adults to serve a mandatory year of national service is different than 50%.

5 **Step 1: Determine the Hypotheses**

A The null hypothesis assumes a value for the population proportion (p). The alternative hypothesis states that p is greater than, less than, or not equal to this value. Give the null and alternative hypotheses below.

H_0: ___________

B Give the alternative hypothesis using the appropriate symbols.

H_a: ___________

C Is this a right-tailed, left-tailed, or two-tailed test?

6 **Step 2: Collect the Data**

In this step, we gather data from random samples, compute summary statistics, and verify criteria for approximate normality.

A In the previous problem, you made an assumption about the population proportion (p) above. Now we need the sample size (n) and sample proportion $\widehat{p}$.

n = ___________

B Determine the sample proportion. Round your answer to three decimal places.

$\widehat{p} =$ ___________

C Are the normality criteria satisfied? Explain.

7 **Step 3: Assess the Evidence**

A Compute the mean and standard error of the underlying sampling distribution.

Mean = $\mu_{\widehat{p}} = p =$ __________

B Compute the standard error of the underlying sampling distribution. Round your answer to three decimal places.

Standard error = $\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ __________

C Compute the *Z*-score of the sample proportion. This is called the test statistic. Round your answer to two decimal places..

$Z = \frac{\widehat{p} - \mu_{\widehat{p}}}{\sigma_{\widehat{p}}} =$ __________

The *P*-value is the probability of observing a test statistic that is at least as extreme as the one above. On right-tail tests, the *P*-value is the area to the right of the test statistic. On left-tailed tests, the *P*-value is the area to its left. On two-tailed tests, the *P*-value is twice the area to either the right of a positive test statistic, or the left of a negative test statistic.

D Use tables or technology to compute the *P*-value below. Remember, for a two-tailed test the *P*-value is twice the area of the tail that extends beyond the test statistic. **Note**: If completing this problem online, follow the instructions given online to answer this question.

E When the *P*-value is small, the sample proportion is **statistically significant**. Is the sample proportion statistically significant as it differs from the assumed value of the population proportion?

8 **Step 4: State a Conclusion**

A What is the value of α, the level of significance?

B Compare the *P*-value to α. What do you conclude regarding the null hypothesis?

C What do you conclude regarding the alternative hypothesis?

D Write a conclusion about this hypothesis test in plain language.

6.4 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Identify the null and alternative hypotheses in a hypothesis testing scenario.	
Compute a test statistic and determine its associated *P*-value.	
Compare *P*-values to the level of significance in order to reject, or fail to reject, the null hypotheses.	
Interpret the results of a hypothesis test.	

6.4-S: Hypothesis Tests for a Population Proportion

LEARNING GOALS

By the end of this lesson, you should understand that:

- A hypothesis test consists of four steps: determining the hypotheses, collecting the data, assessing the evidence, and stating a conclusion.
- A sample statistic is statistically significant if it is unlikely to occur under the assumption of the null hypothesis.

By the end of this lesson, you should be able to:

- Implement the 4-step hypothesis test for a population proportion.
- Explain the reasoning used to reach a decision in a hypothesis test.
- Explain why rejecting the null hypothesis implies strong support for the alternative hypothesis, but failing to reject the null hypothesis does not imply strong support for the alternative hypothesis.
- Explain how a *P*-value is used to reach a conclusion in a hypothesis test.

INTRODUCTION

This lesson provides additional practice in implementing the four-step hypothesis testing process. To begin this activity, you will outline what the 4-step process entails.

In your own words, when testing a claim about a population proportion, describe what is required in each step below.

1 Step 1: Determine the Hypotheses

2 Step 2: Collect the Data

3 Step 3: Assess the Evidence

4 Step 4: State a Conclusion

TRY THESE

The National Football League (NFL) has experienced a decline in American viewership over the past five years due to Americans' concerns about NFL players and their behavior, and due to Americans' interest in other media. A Wall Street Journal/NBC News poll[22] surveyed 900 American adult men aged 18 to 49 and found that 459 of them said that they follow the NFL closely. At the 5% significance level, test the claim that the majority of American adult men aged 18 to 49 follow the NFL closely.

Complete the steps below.

5 Step 1: Determine the Hypotheses

6 Step 2: Collect the Data

[22] https://www.msn.com/en-us/sports/nfl/the-nfl-is-losing-its-core-audience-a-wsj-nbc-news-poll-finds/ar-BBIBf76

7 Step 3: Assess the Evidence

8 Step 4: State a Conclusion

9 Suppose you performed the hypothesis test above using a significance level of 0.10. How would the result of the test change? Explain your answer.

10 Suppose the hypothesis test above was performed based on a larger random sample of American men aged 18 to 49. If the sample proportion remained the same, determine a sample size which would result in the sample proportion being deemed statistically significant at the 5% significance level? Explain your answer.

NEXT STEPS

In December 2012, following the tragic Sandy Hook Elementary school shooting in Newtown, Connecticut, 33% of U.S. adults reported that they feared for their child's safety when their child was in school. This proportion represented an increase in this view among U.S. adults from the prior 5-year period, but this 33% proportion was significantly lower than the 45% proportion which was observed in March 2001 following a mass shooting in Santee, California.

In August 2017, Gallup surveyed a random sample of 233 U.S. adults and found 56 said that they feared for their child's safety when their child was in school[23]. Does the August 2017 sample show that the proportion of U.S. adults who feared for their child's safety in school (at that time) is different than the proportion in December 2012? Perform a hypothesis test at the 1% significance level. Complete the steps below.

11 Step 1: Determine the Hypotheses

12 Step 2: Collect the Data

13 Step 3: Assess the Evidence

23 http://news.gallup.com/poll/216308/parental-fear-school-safety-back-pre-newtown-level.aspx?g_source=K_12&g_medium=topic&g_campaign=tiles

14 Step 4: State a Conclusion

Testing A Claim about Adults' Views on Fake News

In June 2019, Pew Research[24] surveyed U.S. adults' on their views about current issues in politics and society. In a survey of 6127 adults, 3002 said that made-up news (fake news) is a critical problem that needs to be addressed. At the 5% significance level, test the claim that less than half of all U.S. adults think that made-up news (fake news) is a critical problem that needs to be addressed.

15 Step 1: Determine the Hypotheses

16 Step 2: Collect the Data

17 Step 3: Assess the Evidence

[24] https://www.journalism.org/2019/06/05/many-americans-say-made-up-news-is-a-critical-problem-that-needs-to-be-fixed/

18 Step 4: State a Conclusion

LET'S SUMMARIZE

In this lesson, you got additional practice computing and interpreting P-values in the context of hypothesis tests for population proportions.

You should now understand that:

- A hypothesis test consists of four steps: determining the hypotheses, collecting the data, assessing the evidence, and stating a conclusion.
- A sample statistic is statistically significant if it is unlikely to occur under the assumption of the null hypothesis.

Practice Problems 6.4-S

The political affiliation of college freshmen is an important consideration to colleges and universities, political scientists, and political parties. The table below contains information about the political affiliation of college freshmen in the U.S. at three points in time[25].

Year	% of College Freshmen who label themselves as Conservative	% of College Freshmen who label themselves as Liberal
2000	18.9	24.8
2008	20.7	31.0
2014	21.2	28.8

In 2016, from a random sample of 500 college freshmen in the United States, 101 freshmen reported being conservative and 155 freshmen reported being liberal. We will use this sample to conduct a hypothesis test.

Let's first focus on the proportion of all U.S. college freshmen in 2016 who report being conservative. At the 5% significance level, does the sample above provide evidence that the proportion of U.S. college freshmen in 2016 who report being conservative is different from the proportion in 2000?

Step 1: Determine the Hypotheses

1 Null hypothesis:

2 Alternative hypothesis:

Step 2: Collect the Data

3 Does this sampling distribution satisfy the normality criteria?

Step 3: Assess the Evidence

4 What is the *Z*-score of the observed sample proportion? Round your answer to 2 decimal places.

[25] https://www.theatlantic.com/education/archive/2017/05/the-most-polarized-freshman-class-in-half-a-century/525135/

5 What is the *P*-value? Round to three decimal places. **Note**: Use your *Z*-score.

Step 4: State a Conclusion

6 What decisions should we make about the null and alternative hypotheses, and what should we conclude?

Let's now focus on the proportion of all U.S. college freshmen in 2016 who report being liberal. At the 5% significance level, does the sample above provide evidence that the proportion of U.S. college freshmen in 2016 who report being liberal is different from the proportion in 2000?

Step 1: Determine the Hypotheses

7 Null hypothesis:

8 Alternative hypothesis:

Step 2: Collect the Data

9 Does this sampling distribution satisfy the normality criteria?

Step 3: Assess the Evidence

10 What is the *Z*-score of the observed sample proportion? Round your answer to two decimal places.

11 What is the *P*-value? Round to three decimal places. **Note**: Use your *Z*-score.

Step 4: State a Conclusion

12 What decisions should we make about the null and alternative hypotheses, and what should we conclude?

6.5: Hypothesis Tests for a Population Proportion 3

LEARNING GOALS

By the end of this collaboration, you should understand that:

- A Type I error occurs when we reject a null hypothesis that is actually true.
- A Type II error occurs when we fail to reject the null hypothesis, even though it is false.
- The significance level is the maximum allowable probability of a Type I error.
- A statistical result cannot be generalized to populations that are not represented well by the sample.
- Real-world significance is different from statistical significance.

By the end of this collaboration, you should be able to:

- Describe the Type I and Type II error for a given hypothesis test.
- Decide when a statistical result can be applied to a larger population.
- Distinguish between statistical significance and real-world significance.

INTRODUCTION

We begin this collaboration with a review of the four-step hypothesis testing process.

Step 1: Determine the Hypotheses

The null hypothesis (H_0) assumes a value for the population proportion (p). The alternative hypothesis (H_a) states that p is greater than, less than, or not equal to this value.

In the alternative hypothesis, *less-than* inequalities (<) indicate left-tailed tests, while *greater-than* inequalities (>) are for right-tailed tests. *Not-equal* inequalities (≠) are two-tailed.

Step 2: Collect the Data

Here, we verify criteria for approximate normality and summarize data from representative samples. The criteria for approximate normality require that $np \geq 10$ and $n(1 - p) \geq 10$ (using p from the null hypothesis). The summary statistics are the sample proportion and sample size, $\widehat{p}$ and n.

Step 3: Assess the Evidence

The *Z*-score of the sample proportion is called the *test statistic*. To find this *Z*-score we use the mean and standard error of the underlying sampling distribution. To compute this, we continue to use p from the null hypothesis.

Mean = $\mu_{\hat{p}} = p$

Standard error = $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}$$

The *P*-value is the probability of observing a test statistic that is at least as extreme as the one above. For right-tail tests, the *P*-value is the area to the right of the test statistic. For left-tailed tests, the *P*-value is the area to its left. For two-tailed tests, the *P*-value is twice the area to the right of a positive test statistic, or to the left of a negative test statistic.

Step 4: State a Conclusion

We use the *P*-value to make a decision about the null hypothesis.

- If *P*-value ≤ α, the evidence is strong enough to lead us to reject the null hypothesis and support the alternative hypothesis.
- When *P*-value > α, the evidence is not strong enough to lead us to reject the null hypothesis. We do not reject the null hypothesis, so we do not support the alternative hypothesis.

Once the decision is made regarding the null and alternative hypotheses, we state a conclusion that summarizes what we found about the underlying claim or research question. The conclusion should clearly address the population of interest.

TRY THESE

In 2013, the U.S. Department of Defense changed a policy that affected women in the armed forces. Under the new rules, women who met physical requirements could be assigned to combat positions.

Many people in the U.S. were opposed to that decision. About 18% of women were against it, and in January of that year, Gallup researchers claimed that the proportion for men was even higher. In a random sample of 256 U.S. men, 57 said they were opposed to allowing women to serve in combat positions.[26]

1 At 5% significance, use the steps above to test the claim that, in 2013, the proportion of men in the U.S. who were opposed to allowing women to serve in combat positions was higher than 18%.

A Step 1: Determine the Hypotheses

[26] http://www.gallup.com/poll/160124/americans-favor-allowing-women-combat.aspx

B Step 2: Collect the Data

C Step 3: Assess the Evidence

D Step 4: State a Conclusion

NEXT STEPS
Errors in Hypothesis Tests

In the previous example we supported the claim that more than 18% of men in the U.S. in 2013 opposed the policy allowing women to serve in combat positions. The evidence did support the claim, but sometimes evidence can be misleading. There is a small possibility that our conclusion was wrong.

Even if we make no errors in the testing process, our conclusion can still be wrong. There are two types of errors in conclusions.

YOU NEED TO KNOW

If we reject the null hypothesis, even though it is true, we make a **Type I Error**.

If we fail to reject the null hypothesis, even though it is false, we make a **Type II Error**.

At the conclusion of a test, we cannot know whether we have committed an error because we do not know what is actually true. We conduct tests *because* we do not know. Thus, if we reject the null hypothesis there is a chance that a Type I error may have occurred. If we fail to reject the null hypothesis, there is a chance that a Type II error may have occurred.

The following table summarizes the errors in a hypothesis test:

Is H_0 true or false?

	H_0 is True	**H_0 is False**
Reject H_0	Type I Error	Correct Decision
Fail to reject H_0	Correct Decision	Type II Error

The level of significance, α, is related to the likelihood of these errors. Consider cases where the null hypothesis is true. When α = 0.05, we reject a null hypothesis for sample data that occur less than 5% of the time by random chance. Thus, when the null hypothesis is true, we reject it 5% of the time, committing a Type I error. This does not mean a Type I error occurs in 5% of all hypothesis tests because a Type I error can only occur if the null hypothesis is true.

YOU NEED TO KNOW

The level of significance, α, is the maximum allowable probability of making a Type I error.

2 Consider the previous example about the Gallup researcher's claim. We concluded that the data support the claim that more than 18% of men in the U.S. in 2013 opposed the policy allowing women to serve in combat positions. But note—the data may have led us to the wrong conclusion! If the conclusion is not correct, and the true proportion is 0.18, what type of error would this be?

3 Consider the following null and alternative hypotheses regarding unidentified flying objects (UFOs). People often attribute UFOs to aliens or extraterrestrial (out-of-this world) beings.

H_0: The proportion of adults in the U.S. who believe in UFOs is 0.25 ($p = 0.25$).

H_a: The proportion of adults in the U.S. who believe in UFOs is less than ($p < 0.25$).

A Describe what a **Type I error** would be for these hypotheses.

B Describe what a **Type II error** would be for these hypotheses.

NEXT STEPS
Generalizing Results

Even when a study is done properly, it is important to be careful about how we generalize its results to larger groups. Statistical results should only be generalized to populations that our samples represent well.

4 When Gallup researchers conducted their survey on women in combat, they found that about 20% of U.S. adults were against the Defense Department's new policy allowing women to serve in combat positions. In the article cited above, Gallup describes the organization's sampling process as follows:

Interviews are conducted with respondents on landline telephones and cellular phones, with interviews conducted in English and Spanish. Each sample of national adults includes a minimum quota [a quota is a required amount] of 50% cell phone respondents and 50% landline respondents, with additional minimum quotas by region.

A About 94% of U.S. adults are able to speak English or Spanish. Based on the description of this study, is the sample representative of all adults in the U.S.? Explain your answer.

B To what population can we reliably generalize the results of this study?

C Gallup's study suggests that, while around 20% of those surveyed opposed the policy allowing women to serve in combat positions, 74% favored it. Do these results suggest that most people around the world would have supported women being allowed in front-line combat? Explain.

YOU NEED TO KNOW

Statistical inference should never be generalized to a population that is not represented by the sample being used.

NEXT STEPS

Statistical vs. Practical (Real-World) Significance

When we reject a null hypothesis and support its alternative, we are doing so because our sample data are statistically significant. This means that, under the assumption of the null hypothesis, what was observed in the sample was unlikely to occur by chance (as measured by the *P*-value).

Be careful, though! Even when a sample yields statistically significant results, *they may not be significant in any practical, or real-world, sense*! That is, even if there is a difference, it may not be large enough to matter.

In Unit 6.3 we tested the claim that the proportion of U.S. adults in 2017 who believed the U.S. healthcare system was in a state of crisis or had major problems was less than 73%, the proportion of U.S. adults who held this view in 2008. Our analysis was based on a random sample of 1028 adults in the U.S. from the year 2017. The sample proportion was $\widehat{p} = 0.72$, so 72% of the adults in the sample believed the U.S. healthcare system was in a state of crisis or had major problems. This is slightly different from 73%, but with a *P*-value of about 0.24 (from a test statistic of *Z* = -0.71), the difference is not statistically significant. We did not reject the null hypothesis so we did not conclude that the proportion in 2017 was less than the proportion in 2008.

5 What if we had observed the same difference ($\widehat{p} = 0.72$ vs. $p = 0.73$) from a sample of 10,000 U.S. adults? The null hypothesis is H_0: p = 0.73 and the alternative is H_a: p < 0.73.

A Compute the mean of the underlying sampling distribution, assuming the null hypothesis is true, for this revised hypothesis test.

$\mu_{\widehat{p}} = p =$ ________

B Compute the standard error of the underlying sampling distribution, assuming the null hypothesis is true, for this revised hypothesis test. Round the value to three decimal places.

Standard error = $\sigma_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} =$ ________

C Compute the test statistic for this revised hypothesis test. Round the value to two decimal places.

$$Z = \frac{\widehat{p} - \mu_{\widehat{p}}}{\sigma_{\widehat{p}}} = \_\_\_\_\_\_\_\_\_\_$$

D Use technology or tables to determine the *P*-value for this left-tailed test. Round the *P*-value to three decimal places.

P-value = ___________

E How did the *Z*-score and *P*-value change with the larger sample size? Recall that when the sample size was *n* = 1028, the *Z*-score was *Z* = -0.71 and the *P*-value was 0.24.

F Is the sample proportion statistically significant with this new sample size (using a 5% significance level)?

G In one scenario above, the difference between the 72% to 73% is statistically significant, but in another, it is not. Clearly, statistical significance is different from the sort of significance we assign to events through our own values. Do you feel that the difference between the sample proportion of U.S. adults in 2017 and the population proportion of U.S. adults in 2008 is significant in a real-world sense? Explain.

YOU NEED TO KNOW

With very large samples, even extremely small differences can be statistically significant.

With this in mind, remember that statistical significance is different from *real-world significance*. Statistical significance is measured with probability. Real-world significance is measured through our system of personal values, and is therefore difficult to measure.

---

6 At a busy intersection with no crosswalk, many pedestrians cross the street every day. The fatality rate at this intersection is slightly higher than the national rate. Testing the hypothesis that the rate at this intersection is higher than the national rate yields a *P*-value of 0.06. The hypothesis test was conducted at the 5% significance level.

A Is this higher fatality rate significant in a real-world sense? Explain.

B Given that the increased fatality rate at the busy intersection is only *slightly* statistically significant, should crosswalks and pedestrian walk lights be installed (though this could be quite expensive)? Explain.

7 Researchers conducted a study to learn whether a new anti-inflammatory medication reduces fever (on average) for patients with the flu. A large sample was gathered and the results of the study showed that, after taking the new medication, mean body temperature in patients was reduced by 0.2°F. This drop in average temperature was statistically significant, as demonstrated by a *P*-value of 0.002.

A People with the flu often have body temperatures that are 2 or 3 degrees higher than normal body temperature (98.6°F). Is the decrease in body temperature after taking the medication significant in a real-world sense? Explain.

B Knowing that there are other medications that reduce body temperatures in patients with high fever, would you consider taking this new anti-inflammatory medication if it was shown to be safe? Explain.

LET'S SUMMARIZE

Please consider the following key points:

- There are two types of errors that can occur in a hypothesis test. If we reject the null hypothesis, even though it is true, we make a **Type I Error**. If we fail to reject the null hypothesis, even though it is false, we make a **Type II Error**.
- After completing a hypothesis test, we don't know which error was made, because we don't know the value of the population proportion which the hypothesis test is based on.
- No statistical inference should be generalized to a population that is not represented by the sample being used.
- With large enough samples, extremely small differences can be statistically significant.
- An event can be statistically significant but not significant in the real-world sense. Likewise, an event can have real-world significance, but not be statistically significant.

Exercise 6.5

Consider the following claim: *Fifty years after the U.S. Supreme Court issued its opinion on Roe v. Wade, a majority of Americans support its decision.*

Roe v. Wade was a landmark decision by the United States Supreme Court that established a woman's right to have an abortion. In May of 2021, Gallup surveyed by phone 254 randomly selected adults living in the West region of the United States.

Each person was asked if they wanted the Supreme Court to overturn its 1973 Roe versus Wade decision concerning abortion.[27] Of those surveyed, 147 answered that they did not want Roe vs Wade overturned; that is, 147 support Roe v. Wade.

At the 5% significance level, test the claim that a majority of adults in the West region of the U.S. did not want to see Roe vs Wade overturned at the time of the survey. We will use the 5% significance level.

Step 1: Determine the Hypotheses

1 H_0: ___________

H_a: ___________

2 Is this a right-tailed, left-tailed, or two-tailed test?

Step 2: Collect the Data

3 Determine the sample proportion. Round your answer to three decimal places.

$\hat{p} =$___________

4 Are the normality criteria satisfied? Explain.

27 https://news.gallup.com/poll/350804/americans-opposed-overturning-roe-wade.aspx

Step 3: Assess the Evidence

5 Compute the *Z*-score of the sample proportion. This is the test statistic. Round your answer to two decimal places.

$$Z = \frac{\widehat{p} - \mu_{\widehat{p}}}{\sigma_{\widehat{p}}} = \_\_\_\_\_\_\_\_\_\_$$

6 Find the *P*-value for this hypothesis test. Round the *P*-value to two decimal places.

P-value = __________

Step 4: State a Conclusion

7 What do you conclude regarding the hypotheses?

8 Which conclusion below is most appropriate?

(i) The data support the claim that a majority of adults in the West region of the U.S. did not want to see Roe vs Wade overturned in May 2021 at the time of the survey.
(ii) The data support the claim that 50% of adults in the West region of the U.S. did not want to see Roe vs Wade overturned in May 2021 at the time of the survey.
(iii) The data do not support the claim that a majority of adults in the West region of the U.S. did not want to see Roe vs Wade overturned in May 2021 at the time of the survey.
(iv) The data do not support the claim that 50% of adults in the West region of the U.S. did not want to see Roe vs Wade overturned in May 2021 at the time of the survey.

Interpreting the Results

9 Suppose you want to know if this survey showed how adults in other regions of the United States think about Roe vs Wade. Can we assume that the results of this survey generalize to U.S. adults in other regions (East, Midwest, South) of the U.S.? Explain your answer.

10 When we conduct a hypothesis test, there is always a possibility that our sample data will lead us to the wrong conclusion. Suppose we drew the wrong conclusion in the hypothesis test above on Americans' opinions on *Roe v Wade*. What type of error would this be?

11 Explain what must be true about the population proportion for such an error to occur.

In August 2021, a poll claimed that less than one-third of U.S. adults approved of the way President Biden was handling the economy. In a random sample of 1,006 adults, 311 stated that they felt this way.

Test the study's claim at 5% significance that less than one-third (33%) of U.S. adults approved of President Biden's handling of the economy.

Step 1: Determine the Hypotheses

12 H_0: __________

H_a: __________

13 Is this a right-tailed, left-tailed, or two-tailed test?

Step 2: Collect the Data

14 Determine the sample proportion. Round your answer to three decimal places.

$\hat{p} =$__________

15 Are the normality criteria satisfied? Explain.

Step 3: Assess the Evidence

16 Compute the *Z*-score of the sample proportion. This is the test statistic. Round your answer to two decimal places.

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \_\_\_\_\_\_\_\_\_\_$$

17 Find the *P*-value for this hypothesis test. Round the *P*-value to three decimal places.

P-value = __________

Step 4: State a Conclusion

18 What do you conclude regarding the hypotheses?

19 Which conclusion below is most appropriate?

(i) The data support the claim that less than one-third (33%) of U.S. adults approved of President Biden's handling of the economy.

(ii) The data do not support the claim that less than one-third (33%) of U.S. adults approved of President Biden's handling of the economy.

Interpreting the Results

20 Think about the possibility that your conclusion in the previous hypothesis test is in error. Suppose we drew the wrong conclusion in the hypothesis test on U.S. adults' perceptions on President Biden's handling of the economy. What type of error would this be?

21 Explain what must be true about the population proportion for such an error to occur.

22 The test statistic in the previous hypothesis test led us to not support the claim that less than one-third of U.S. adults believed that Biden was doing a good job with the economy. Suppose that the sample size was doubled, and from this sample, the same sample proportion was observed. What is the new test statistic? Round the value to two decimal places.

23 How does increasing the sample size, while maintaining the same sample proportion, impact the difference between the sample proportion and the assumed population proportion?

24 Engineers are designing a new airplane. They want to select the strongest available bolts to use on the wings of the plane. They conduct stress tests of two different bolts, to find out how frequently each bolt breaks (its breakage rate). In equivalent stress tests, one bolt breaks at a slightly lower rate than the other, but after analyzing many bolts, the engineers find that the difference is barely significant (with a *P*-value of 0.07). The level of significance for the test was 0.05. Which statement below is accurate?

(i) The difference in the breakage rates is statistically significant but not significant in a real-world sense.

(ii) The difference in the breakage rates is not statistically significant and not significant in a real-world sense.

(iii) The difference in the breakage rates is statistically significant and significant in a real-world sense.

(iv) The difference in the breakage rates is not statistically significant but is significant in a real-world sense.

25 A company that manufactures tablet PC's is researching whether to upgrade the display on one of their tablets. The new display will require a large investment by the company, and will increase the price of the tablet by about $100. In a survey of a large random sample of people, the proportion of respondents who said they would buy the tablet with the new screen was 1% higher than the proportion of those who would buy the tablet with the old screen. Researchers tested the hypothesis that the proportion was higher with the new screen, and the data supported the claim with a *P*-value of 0.02. The level of significance for the test was 0.05. Which statement below is accurate?

(i) The increase in the proportion of consumers who will buy the tablet is statistically significant but not significant in a real-world sense.

(ii) The increase in the proportion of consumers who will buy the tablet is not statistically significant and not significant in a real-world sense.

(iii) The increase in the proportion of consumers who will buy the tablet is statistically significant and significant in a real-world sense.

(iv) The increase in the proportion of consumers who will buy the tablet is not statistically significant but is significant in a real-world sense.

A survey based on a random sample of 425 high school students showed that 103 of the students smoked cigarettes at least once during the past year. The researcher wants to test if this result provides strong evidence that more than 20% of all high school students smoked cigarettes during the past year. The research conducts the test using a 5% significance level. Complete all four-steps in the hypothesis testing process. Then answer the questions below.

26 What is the test statistic in this hypothesis test? Round the value to two decimal places.

27 What is the *P*-value in this hypothesis test? Round the value to three decimal places.

28 Write a concluding statement for this hypothesis test.

6.5 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Describe the Type I and Type II error for a given hypothesis test.	
Decide when a statistical result can be applied to a larger population.	
Distinguish between statistical significance and real-world significance.	