Mech 2 Ch4 Moments

Q1.

A plank AB has length 6m and mass 30kg. The point C is on the plank with CB = 2m. The plank rests in equilibrium in a horizontal position on supports at A and C. Two people, each of mass 75kg, stand on the plank. One person stands at the point P of the plank, where $AP = x$ metres, and the other person stands at the point Q of the plank, where $AQ = 2x$ metres. The plank remains horizontal and in equilibrium with the magnitude of the reaction at C five times the magnitude of the reaction at A. The plank is modelled as a uniform rod and each person is modelled as a particle.

(a) Find the value of x.

(7)

(b) State two ways in which you have used the assumptions made in modelling the plank as a uniform rod.

(2)

(Total for question = 9 marks)

Q2.

A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where AS = 0.5 m and TB = 2 m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of d,

(ii) the value of M.

(7)

(Total for question = 7 marks)

Q3.

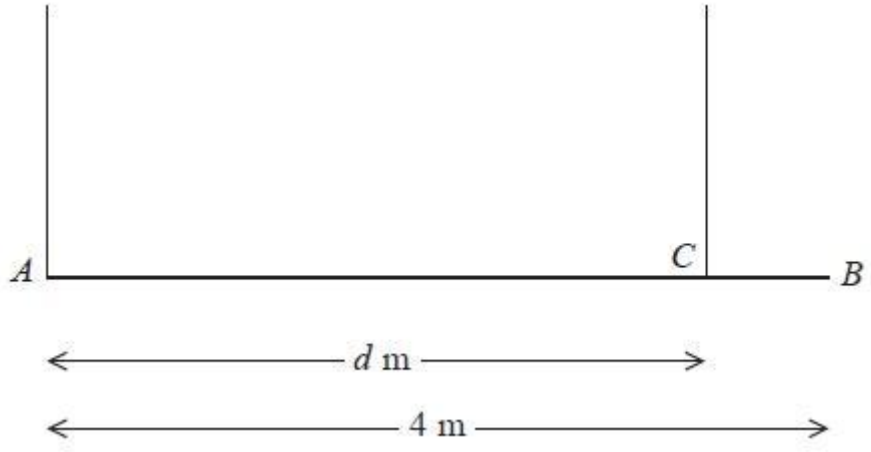


Figure 3

A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(b) Find the value of k.

(6)

(Total 12 marks)

Q4.

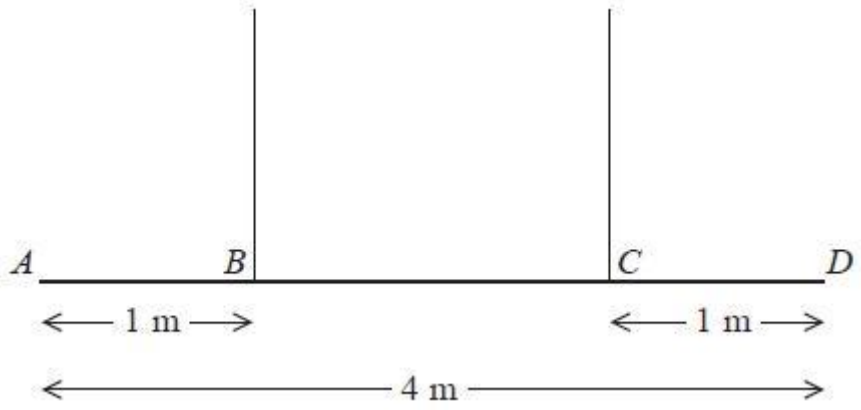


Figure 3

A non-uniform beam *AD* has weight *W* newtons and length 4 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. The ropes are attached to two points *B* and *C* on the beam, where *AB* = 1 m and *CD* = 1 m, as shown in Figure 3. The tension in the rope attached to *C* is double the tension in the rope attached to *B*. The beam is modelled as a rod and the ropes are modelled as light inextensible strings.

(a) Find the distance of the centre of mass of the beam from *A*.

(6)

A small load of weight *kW* newtons is attached to the beam at *D*. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle.

Find

(b) an expression for the tension in the rope attached to *B*, giving your answer in terms of *k* and *W*,

(3)

(c) the set of possible values of *k* for which both ropes remain taut.

(2)

(Total 11 marks)

Q5.

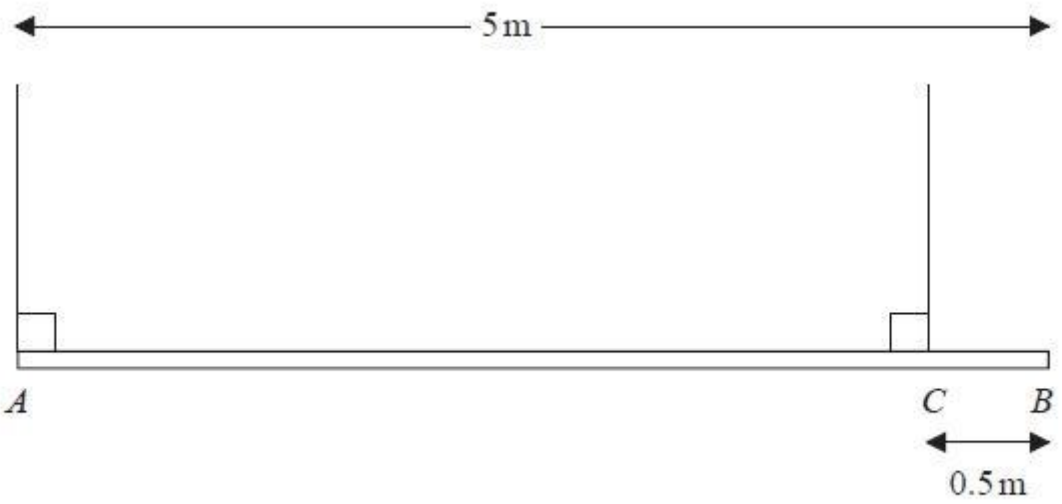


Figure 3

A beam AB has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at A and the other rope is attached to the point C on the beam where $CB = 0.5$ m, as shown in Figure 3. A particle P of mass 60 kg is attached to the beam at B and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(a) Find

(i) the tension in the rope attached to the beam at A,
(ii) the tension in the rope attached to the beam at C.

(6)

Particle P is removed and replaced by a particle Q of mass M kg at B. Given that the beam remains in equilibrium in a horizontal position,

(b) find

(i) the greatest possible value of M,
(ii) the greatest possible tension in the rope attached to the beam at C.

(6)

(Total for question = 12 marks)

Q6.

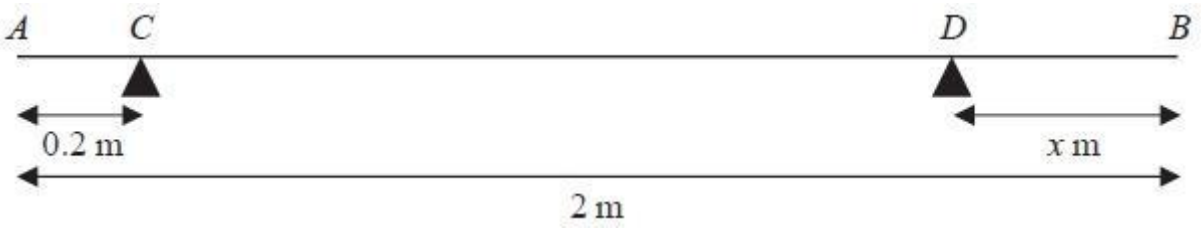


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where AC = 0.2 metres and DB = x metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C,

(a) find the value of x.

(6)

The support at D is now moved to the point E on the rod, where EB = 0.4 metres. A particle of mass m kg is placed on the rod at B, and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C,

(b) find the value of m.

(7)

(Total 13 marks)

Q7.

A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where AP = 2 m and QB = 3 m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

(Total 14 marks)

Q8.
A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where AC = 2 m and DB = 2 m. A man of mass 80 kg stands on the girder at the point P, where AP = 4 m, as shown in Figure 1.

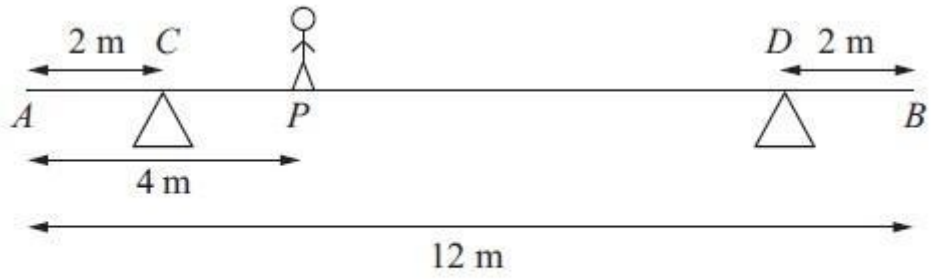


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where XB = x metres. The man remains on the girder at P, as shown in Figure 2.

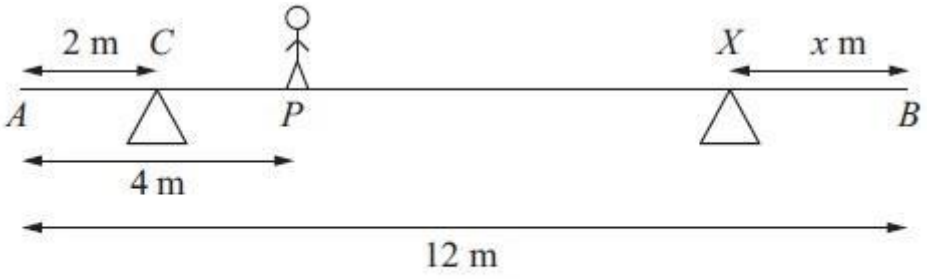


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

(Total 9 marks)

Q9.

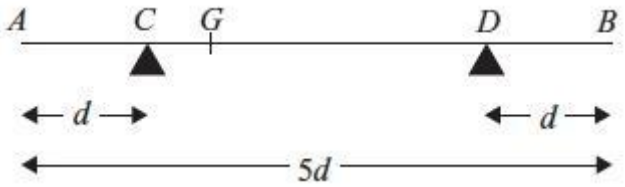


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where

$AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that GD = $\frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

(Total 9 marks)

Q10.

A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where PQ = 6 m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

(a) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

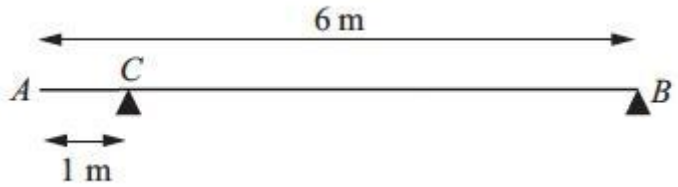
(10)

(b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)

(Total 11 marks)

Q11.



A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where AC = 1 m, and the other is at the end B, as shown in the figure above. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

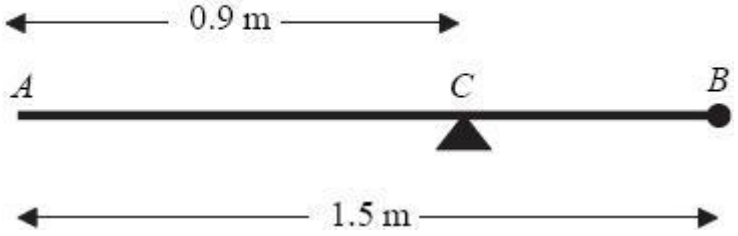
(b) Find the distance AD.

(5)

(Total 10 marks)

Q12.

Figure 2



A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where AC = 0.9 m, and the system is in equilibrium with AB horizontal, as shown in Figure 2.

(a) Show that m = 2.

(4)

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(b) Find the distance AD.

(5)

(Total 9 marks)

Q13.

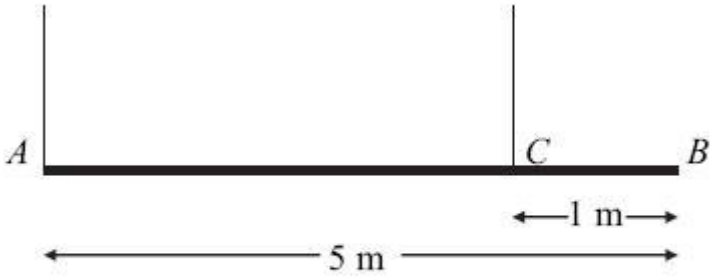


Figure 2

A beam *AB* has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to *A*, the other to the point *C* on the beam, where *BC* = 1 m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(a) Find

(i) the tension in the rope at *C*,
(ii) the tension in the rope at *A*.

(5)

A small load of mass 16 kg is attached to the beam at a point which is *y* metres from *A*. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(b) find, in terms of *y*, an expression for the tension in the rope at *C*.

(3)

The rope at *C* will break if its tension exceeds 98 N. The rope at *A* cannot break.

(c) Find the range of possible positions on the beam where the load can be attached without the rope at *C* breaking.

(3)

(Total 11 marks)

Q14.

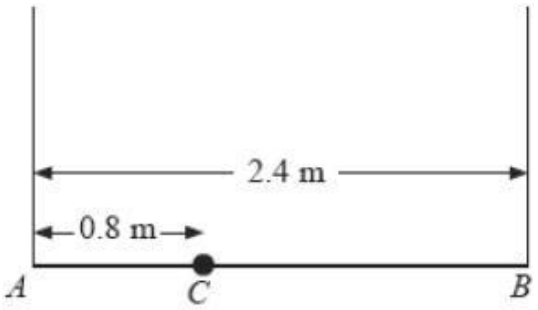


Figure 2

A plank AB has mass 12 kg and length 2.4 m. A load of mass 8 kg is attached to the plank at the point C, where AC = 0.8 m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in Figure 2. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

(a) Find the tension in the rope attached at B.

(4)

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(b) Find the distance of the centre of mass of the plank from A.

(6)

(Total 10 marks)

Q15.

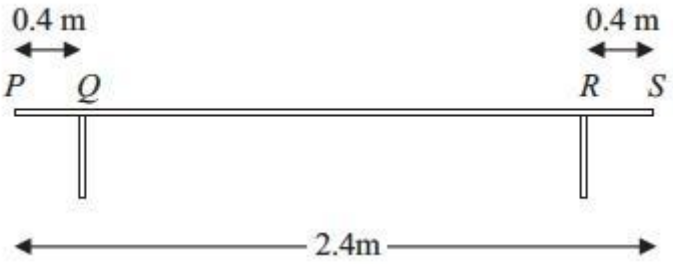


Figure 1

A bench consists of a plank which is resting in a horizontal position on two thin vertical legs. The plank is modelled as a uniform rod PS of length 2.4 m and mass 20 kg. The legs at Q and R are 0.4 m from each end of the plank, as shown in Figure 1.

Two pupils, Arthur and Beatrice, sit on the plank. Arthur has mass 60 kg and sits at the middle of the plank and Beatrice has mass 40 kg and sits at the end P. The plank remains horizontal and in equilibrium. By modelling the pupils as particles, find

(a) the magnitude of the normal reaction between the plank and the leg at Q and the magnitude of the normal reaction between the plank and the leg at R.

(7)

Beatrice stays sitting at P but Arthur now moves and sits on the plank at the point X. Given that the plank remains horizontal and in equilibrium, and that the magnitude of the normal reaction between the plank and the leg at Q is now twice the magnitude of the normal reaction between the plank and the leg at R,

(b) find the distance QX.

(6)

(Total 13 marks)

Q16.

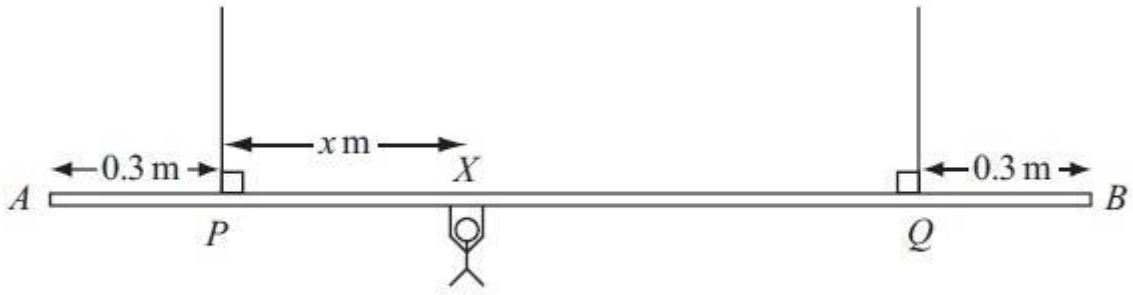


Figure 2

A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(b) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x.

(3)

(Total 12 marks)