

Differential Calculus – Practice Assessment

PAPER 1:

Question 1 (5 marks)

The gradient of the tangent line to the graph of $f(x) = \frac{2x^3 + kx^2 - 6x}{x}$ at $x = 1$ is 11. Find the value of k .

Question 2 (10 marks)

Write the equations for the tangent lines to the graph of $f(x) = \frac{x+2}{x-2}$ that are perpendicular to $3y - 3x = 21$

Question 3 (6 marks)

In the table below, the values of f and g and their derivatives at $x = 2$ and $x = 3$ are given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	3	7	-1
3	4	-2	5	6

a Find the gradient of $f(g(x))$ when $x = 2$.

b Find the gradient of $\frac{1}{[g(x)]^2}$ when $x = 3$

Question 4 (6 marks)

Given $f(x) = (2x^5 + 3)^9$, find the term in x^{16} in the expansion of $f'(x)$.

LONG RESPONSE:

Question 5 (13 marks)

Let $f(x) = \frac{10}{x^2+6}$

a Use the fact that $f'(x) = -\frac{20x}{(x^2+6)^2}$ to show that the second derivative is $f''(x) = \frac{60x^2-120}{(x^2+6)^3}$ [2]

b i. Find the relative maximum and minimum points for the graph of $f(x)$. [6]

ii. Find the intervals of increase and decrease.

c Consider the $\lim_{x \rightarrow \infty^+} \frac{10}{x^2+6}$ and $\lim_{x \rightarrow \infty^-} \frac{10}{x^2+6}$ to help you determine the horizontal asymptote for $f(x)$. [2]

d Based on your results, sketch a graph of $f(x)$. [3]

PAPER 2:

Question 6 (7 marks)

A pandemic can be modelled by the equation $C(w) = (w - 15)^3 + 4000$ where C is the number of cases and w is the number of weeks from the start of the outbreak.

a Sketch a graph representing up to 50,000 thousand cases. [3]

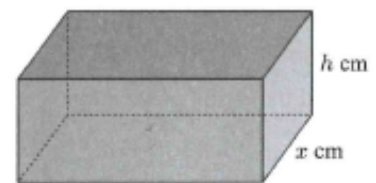
b Calculate the average rate of growth in cases from week 10 to week 40. [2]
Then, add a representation of this to the graph in a.

c Calculate the instantaneous rate of growth at week 25. [2]
Then, add a representation of this to the graph in a.

Question 7 (8 marks)

Radioactive waste is to be stored in a lead box with inner volume of 200 cm^3 .

The base of the box has dimensions with a ratio of 2:1.



a Show that $x^2h = 100$. [2]

[3]

b Show that the inner surface area of the box is given by $A(x) = 4x^2 + \frac{600}{x}$ cm².

c Find the minimum surface area of the box and the dimensions that yield this minimum surface area.