Differential Calculus – Practice Assessment

PAPER 1:

Question 1 (5 marks)

The gradient of the tangent line to the graph of $f(x) = \frac{2x^3+kx^2-6x}{x}$ at x = 1 is 11. Find the value of k.

Question 2 (10 marks)

Write the equations for the tangent lines to the graph of $f(x) = \frac{x+2}{x-2}$ that are perpendicular to $3y - 3x = 21$

Question 3 (6 marks)

In the table below, the values of f and g and their derivatives at x = 2 and x = 3 are given.

x	f(x)	g(x)	f '(x)	g '(x)
2	5	3	7	-1
3	4	-2	5	6

a Find the gradient of $f(g(x))$ when x = 2.

b Find the gradient of $\frac{1}{[g(x)]^2}$ when x = 3

Question 4 (6 marks)

Given $f(x) = (2x^5 + 3)^9$, find the term in x^{16} in the expansion of $f'(x)$.

LONG RESPONSE:

Question 5 (13 marks)

Let $f(x) = \frac{10}{x^2+6}$

a Use the fact that $f'(x) = -\frac{20x}{(x^2+6)^2}$ to show that the second derivative is $f''(x) = \frac{60x^2-120}{(x^2+6)^3}$ [2]

b i. Find the relative maximum and minimum points for the graph of f(x). [6]

ii. Find the intervals of increase and decrease.

c Consider the $lim_{x\to\infty^+} \frac{10}{x^2+6}$ and $lim_{x\to\infty^-} \frac{10}{x^2+6}$ to help you determine the horiztonal asymptote for f(x). [2]

d Based on your results, sketch a graph of f(x). [3]

PAPER 2:

Question 6 (7 marks)

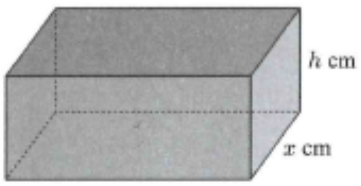
A pandemic can be modelled by the equation $C(w) = (w - 15)^3 + 4000$ where C is the number of cases and w is the number of weeks from the start of the outbreak.

a Sketch a graph representing up to 50,000 thousand cases. [3]

b Calculate the average rate of growth in cases from week 10 to week 40.
Then, add a representation of this to the graph in a. [2]

c Calculate the instantaneous rate of growth at week 25.
Then, add a representation of this to the graph in a. [2]

Question 7 (8 marks)



Radioactive waste is to be stored in a lead box with inner volume of 200 cm^3.
The base of the box has dimensions with a ratio of 2:1.

a Show that $x^2h = 100$. [2]

[3]

b Show that the inner surface area of the box is given by $A(x) = 4x^2 + \frac{600}{x}$ cm^2.

c Find the minimum surface area of the box and the dimensions that yield this minimum surface area.