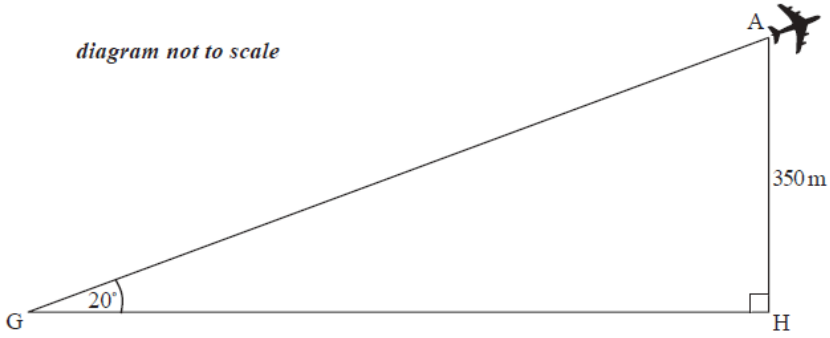
IBDP SL Mathematics

Applications & Interpretation

Year 1 – Semester 2 Final Examination - Paper 1 KEY

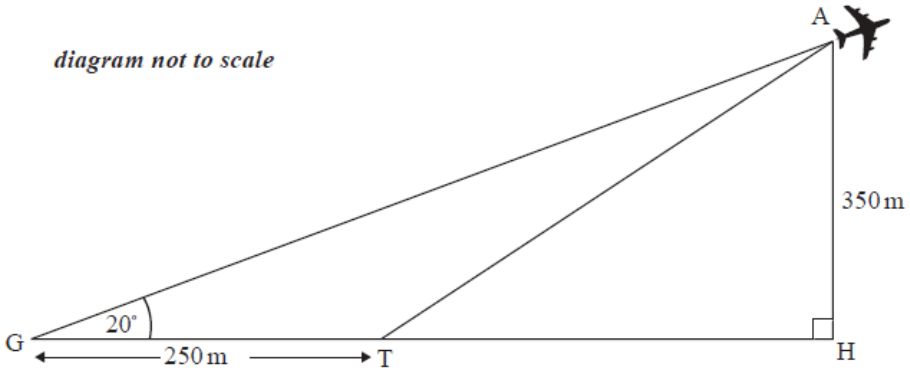
Question 1 (6 marks)

Günter is at Berlin Tegel Airport watching the planes take off. He observes a plane that is at an angle of elevation of 20° from where he is standing at point G. The plane is at a height of 350 metres. This information is shown in the following diagram.



(a) Calculate the horizontal distance, GH, of the plane from Günter. **Give your answer to the nearest metre.** *[3]*

The plane took off from a point T, which is 250 metres from where Günter is standing, as shown in the following diagram.



(b) Using your answer from part (a), calculate the angle ATH, the takeoff angle of the plane. *[3]*

(a) $\frac{350}{\tan 20^\circ}$ *(M1)*

$= 961.617\ldots$ *(A1)*
$= 962\,(\text{m})$ *(A1)*(ft) *(C3)*

Notes: Award *(M1)* for correct substitution into correct formula, *(A1)* for correct answer, *(A1)*(ft) for correct rounding to the nearest metre.
Award *(M0)(A0)(A0)* for 961 without working.

(b) $961.617\ldots - 250 = 711.617\ldots$ *(A1)*(ft)

$\tan^{-1}\left(\frac{350}{711.617\ldots}\right)$ *(M1)*

$= 26.2^\circ$ $(26.1896\ldots)$ *(A1)*(ft) *(C3)*

Notes: Accept 26.1774... from use of 3 sf answer 962 from part (a). Follow through from their answer to part (a).
Accept alternative methods.

[6 marks]

Question 2 (6 marks)

A cuboid has the following dimensions: length $= 8.7$ cm, width $= 5.6$ cm and height $= 3.4$ cm.

(a) Calculate the **exact** value of the volume of the cuboid, in cm^3. *[2 marks]*

(b) Write your answer to part (a) correct to

(i) one decimal place;

(ii) three significant figures. *[2 marks]*

(c) Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. *[2 marks]*

(a) $V = 8.7 \times 5.6 \times 3.4$ *(M1)*

Note: Award *(M1)* for multiplication of the 3 given values.

$= 165.648$ *(A1)* *(C2)*

(b) (i) 165.6 *(A1)*(ft)

Note: Follow through from their answer to part (a).

(ii) 166 *(A1)*(ft) *(C2)*

Note: Follow through from their answer to part (a).

(c) 1.66×10^2 *(A1)*(ft)*(A1)*(ft) *(C2)*

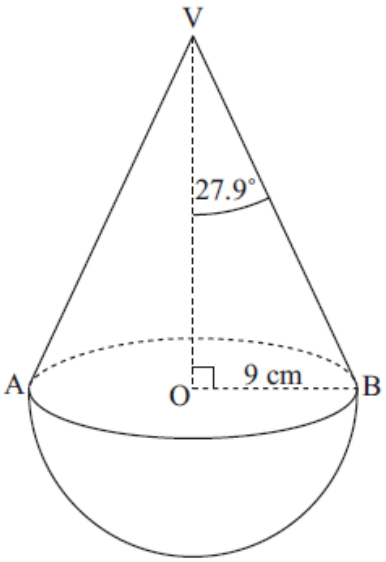
Notes: Award *(A1)*(ft) for 1.66, *(A1)*(ft) for 10^2. Follow through from their answer to part (b)(ii) only. The follow through for the index should be dependent on the value of the mantissa in part (c) and their answer to part (b)(ii).

[6 marks]

Question 3 (6 marks)

A child's wooden toy consists of a hemisphere, of radius 9 cm, attached to a cone with the same base radius. O is the centre of the base of the cone and V is vertically above O. Angle OVB is 27.9°.

diagram not to scale



(a) Calculate OV, the height of the cone. *[2]*

(b) Calculate the volume of wood used to make the toy. *[4]*

(a) $\tan 27.9^\circ = \frac{9}{OV}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in trig formula.

$OV = 17.0\,(\text{cm})\ (16.9980...)$ ***(A1)*** ***(C2)***

(b) $\frac{\pi(9)^2(16.9980...)}{3} + \frac{1}{2} \times \frac{4\pi(9)^3}{3}$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correctly substituted volume of the cone, ***(M1)*** for correctly substituted volume of a sphere divided by two (hemisphere), ***(M1)*** for adding the correctly substituted volume of the cone to *either* a correctly substituted sphere *or* hemisphere.

$= 2970\,\text{cm}^3\ (2968.63\ldots)$ ***(A1)*****(ft)** ***(C4)***

Note: The answer is $2970\,\text{cm}^3$, the units are required.

[6 marks]

Question 4 (6 marks)

The number of passengers in the first ten carriages of a train is listed below.

$$6, 8, 6, 3, 8, 4, 8, 5, p, p$$

The mean number of passengers per carriage is 5.6.

(a) Calculate the value of p. *[2 marks]*

(b) Find the median number of passengers per carriage. *[2 marks]*

If the passengers in the eleventh carriage are also included, the mean number of passengers per carriage increases to 6.0.

(c) Determine the number of passengers in the eleventh carriage of the train. *[2 marks]*

(a) $\frac{48+2p}{10}=5.6$ *(M1)*

Notes: Accept equivalent forms.
Award *(M1)* for correct substitutions in mean formula.

4 *(A1)* *(C2)*

(b) **Correctly** rearranging the list with their p *(M1)*

5.5 *(A1)*(ft) *(C2)*

Note: Follow through from their value of p in part (a).

(c) $\frac{56+x}{11}=6.0$ *(M1)*

Notes: Accept equivalent forms.
Award *(M1)* for correct substitutions in mean formula.

OR

$\frac{48+2\times\text{their part (a)}+x}{11}$ *(M1)*

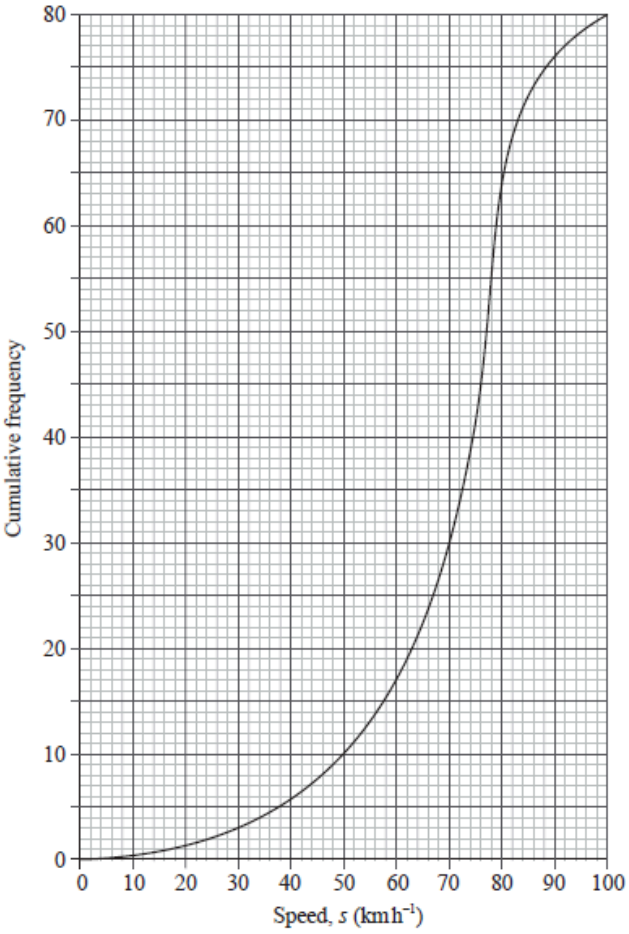
10 *(A1)*(ft) *(C2)*

Note: Follow through from their answer to part (a).

[6 marks]

Question 5 (6 marks)

The cumulative frequency graph represents the speed, s, in km h^{-1}, of 80 cars passing a speed camera.



(a) Write down the number of cars passing the camera with speed of less than or equal to $50\,\text{km h}^{-1}$. *[1]*

(b) Complete the following grouped frequency table for s, the speed of the cars passing the camera.

$s\left(\text{km h}^{-1}\right)$	$0 < s \leq 50$	$50 < s \leq 70$	$70 < s \leq 80$	$80 < s \leq 90$	$90 < s \leq 100$
Frequency			34		4

[1]

(c) Write down the mid-interval value of the $50 < s \leq 70$ interval. *[1]*

(d) Use your graphic display calculator to find an estimate of

(i) the mean speed of the cars passing the camera;

(ii) the standard deviation of the speed of the cars passing the camera. *[3]*

(a) 10 ***(A1)*** ***(C1)***

(b)

$s\left(\text{km h}^{-1}\right)$	$0 < s \leq 50$	$50 < s \leq 70$	$70 < s \leq 80$	$80 < s \leq 90$	$90 < s \leq 100$
Frequency	**10**	**20**	34	**12**	4

(A1)**(ft)** ***(C1)***

Note: Follow through from their answer to part (a).

(c) 60 ***(A1)*** ***(C1)***

(d) (i) $67.5\left(\text{km h}^{-1}\right)$ ***(A2)*****(ft)**

Notes: Award ***(M1)*** for an attempt to use the formula for the mean with at least two midpoint values consistent with their answer to part (c). Follow through from their table in part (b).

(ii) 18.6 (18.6413…) ***(A1)*****(ft)** ***(C3)***

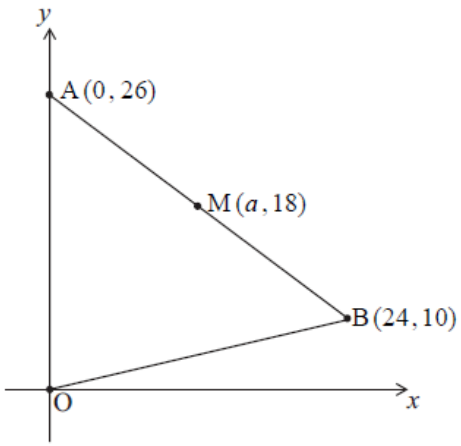
Note: Follow through from their table in part (b).

[6 marks]

Question 6 (6 marks)

The diagram shows the points $M(a, 18)$ and $B(24, 10)$. The straight line BM intersects the y-axis at $A(0, 26)$. M is the midpoint of the line segment AB.

diagram not to scale



(a) Write down the value of a. *[1]*

(b) Find the gradient of the line AB. *[2]*

(c) Decide whether triangle OAM is a right-angled triangle. Justify your answer. *[3]*

(a) 12 *(A1)* *(C1)*

Note: Award *(A1)* for $(12, 18)$.

(b) $\frac{26-10}{0-24}$ *(M1)*

Note: Accept $\frac{26-18}{0-12}$ or $\frac{18-10}{12-24}$ (or equivalent).

$= -\frac{2}{3}$ $\left(-\frac{16}{24}, -0.666666...\right)$ *(A1)* *(C2)*

Note: If either of the alternative fractions is used, follow through from their answer to part (a).
The answer is now ***(A1)*(ft)**.

(c) gradient of OM $= \frac{3}{2}$ ***(A1)*(ft)**

Note: Follow through from their answer to part (b).

$-\frac{2}{3} \times \frac{3}{2}$ *(M1)*

Note: Award *(M1)* for multiplying their gradients.

Since the product is -1, OAM is a right-angled triangle ***(R1)*(ft)**

Notes: Award the final *(R1)* only if their conclusion is consistent with their answer for the product of the gradients.
The statement that OAM is a right-angled triangle without justification is awarded no marks.

OR

$(26-18)^2 + 12^2$ and $12^2 + 18^2$ ***(A1)*(ft)**

$\left((26-18)^2 + 12^2\right) + (12^2 + 18^2) = 26^2$ *(M1)*

Note: This method can also be applied to triangle OMB.
Follow through from (a).

Hence a right angled triangle *(R1)*(ft)

Note: Award the final *(R1)* only if their conclusion is consistent with their *(M1)* mark.

OR

$OA = OB = 26\,(\text{cm})$ an isosceles triangle *(A1)*

Note: Award *(A1)* for $OA = 26\,(\text{cm})$ and $OB = 26\,(\text{cm})$.

Line drawn from vertex to midpoint of base is perpendicular to the base *(M1)*

Conclusion *(R1)* *(C3)*

Note: Award, at most *(A1)(M0)(R0)* for stating that OAB is an isosceles triangle without any calculations.

[6 marks]

Question 7 (6 marks)

In an arithmetic sequence, the fifth term, u_5, is greater than the first term, u_1. The difference between these terms is 36.

(a) Find the common difference, d. *[2 marks]*

The tenth term of the sequence is double the seventh term.

(b) (i) Write down an equation in u_1 and d to show this information.

(ii) Find u_1. *[4 marks]*

(a) $u_1 + 4d - u_1 = 36$ *(M1)*

Note: Accept equivalent forms including the use of a instead of u_1.

$(d =)\ 9$ *(A1)* *(C2)*

(b) (i) $u_{10} = 2u_7$ *(M1)*

Note: Award *(M1)* for correct use of 2 (may be implied).

$u_1 + 9d = 2[u_1 + 6d]$ *(A1)*

Notes: Accept equivalent forms.
Award *(M1)(A0)* for $a + 9d = 2[a + 6d]$.

(ii) $u_1 + 81 = 2u_1 + 108$ *(M1)*

$(u_1 =)\ -27$ *(A1)*(ft) *(C4)*

Notes: Follow through from their d found in part (a) and equation in (b)(i).
Do not penalize further use of a instead of u_1.

[6 marks]

Question 8 (6 marks)

The following diagram shows a circle with centre O and radius 3 cm.

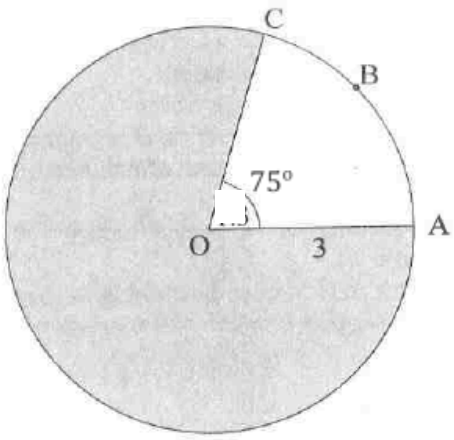


diagram not to scale

Points A, B, and C lie on the circle, and $A\hat{O}C = 75°$.

(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

a) L = (x/360)2(pi) r = (75/360)(2)(3.14)(3) = 3.93 cm.

b) A = ((360-75)/360) (pi)(r^2) = 285/360 (3.14)(3^2) = 22.4 cm^2